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On Decomposition of Neo-Curvature Tensor Field in Finsler Recurrent Spaces

U.S.Negi

Department of Mathematics, H.N.B. Garhwal (Central) University, Campus Badshashi-thaul, Tehri Garhwal-249199, U.K. India.

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ABSTRACT

Takano [1967] has studied the decomposition of curvature tensor in a recurrent space. Chandra [1972] has defined Neo-covariant derivative and its applications. In this paper, I have studied decomposition of Neo-curvature tensor field in Finsler recurrent spaces and several theorems have been established.

Introduction

MSC2010: 53C15, 53C50, 53C55, 53C80.

Let F_m be a subspace of the Finsler space F_n. The metric tensors $g_{\alpha\beta}(u,\dot{u})$, $g_{ij}(x.\dot{x})$ of the spaces F_m and F_n are such that, given by Rund [1959].

$$g_{\alpha\beta}(u,\dot{u}) = g_{ij}(x,\dot{x})X^i X^j, \qquad X^i = \frac{\partial x^i}{\partial u_\alpha} \tag{1.1}$$

The neo-covariant differentiation of any vector field $Y^\alpha(u,\dot{u})$ with respect to u^β is given by Chandra [1972].

$$\overset{n}{\underset{\beta}{\nabla}} Y^\alpha = \partial_\beta Y^\alpha + \dot{\partial}_\delta Y^\alpha \partial_\beta \dot{u}^\delta + F^\alpha_{\beta\gamma} Y^\gamma \tag{1.2}$$

where $\overset{n}{\underset{\beta}{\nabla}}$ is the notation for the neo-covariant differentiation, $F^\alpha_{\beta\gamma}$ is the neo-connection of F_m, ∂_β and $\dot{\partial}_\beta$ denote $\partial/\partial u^\beta$ and $\partial/\partial \dot{u}^\beta$ respectively.

Now differentiating (1.2) neo-covariantly with to u^γ and commuting the indices β and γ, we have

$$\overset{n}{\underset{\beta\gamma}{\nabla}} Y^\alpha - \overset{n}{\underset{\beta\gamma}{\nabla}} Y^\alpha = N^\alpha_{\delta\beta\gamma} Y^\delta \tag{1.3}$$

where $N^\alpha_{\delta\beta\gamma}(u,\dot{u})$ is the neo-curvature tensor field given by Chandra [1972].

The neo-curvature tensor field $N^\alpha_{\delta\beta\gamma}$ satisfies the following identities given by Chandra [1972]

$$N^\alpha_{\delta\beta\gamma} + N^\alpha_{\delta\gamma\beta} = 0 \tag{1.4a}$$

$$N^\alpha_{\delta\beta\gamma} + N^\alpha_{\beta\gamma\delta} + N^\alpha_{\gamma\delta\beta} = 0 \tag{1.4b}$$

$$\overset{n}{\underset{\emptyset}{\nabla}} N^\alpha_{\delta\beta\gamma} + \overset{n}{\underset{\beta}{\nabla}} N^\alpha_{\delta\gamma\emptyset} + \overset{n}{\underset{\gamma}{\nabla}} N^\alpha_{\delta\emptyset\beta} = 0 \tag{1.4c}$$

Let $T^\alpha_{\beta\gamma}$ be any tensor field. The following commutation formula will be used in sequel

$$\overset{n}{\underset{\emptyset\theta}{\nabla}} T^\alpha_{\beta\gamma} + \overset{n}{\underset{\theta\emptyset}{\nabla}} T^\alpha_{\beta\gamma} = T^\varepsilon_{\beta\gamma} N^\alpha_{\varepsilon\emptyset\theta} - T^\alpha_{\varepsilon\gamma} N^\varepsilon_{\beta\emptyset\theta} - T^\alpha_{\beta\varepsilon} N^\varepsilon_{\gamma\emptyset\theta} \tag{1.5}$$

Tele: 9758037370
E-mail address: usnegi7@gmail.com

Definition (1.1): The subspace F_m is said to be neo-recurrent if the neo-curvature tensor field $N^{\alpha}_{\delta\beta\gamma}$ satisfies the relation

$$\overset{n}{\underset{\emptyset}{\nabla}} N^{\alpha}_{\delta\beta\gamma} - v_{\emptyset} \, N^{\alpha}_{\delta\beta\gamma} = 0, \tag{1.6}$$

Where $v_{\emptyset}$ is non-zero neo-recurrent vector field.

The neo-curvature tensor field in this case is called neo-recurrent curvature tensor field.
In view of (1.6), (1.4c) yields

$$v_{\emptyset} \, N^{\alpha}_{\delta\beta\gamma} + v_{\beta} \, N^{\alpha}_{\delta\gamma\emptyset} + v_{\gamma} \, N^{\alpha}_{\delta\emptyset\beta} = 0.$$

2. Decomposition of neo-curvature tensor field in finsler recurrent spaces.

Let us consider the decomposition of the neo-curvature tensor field $N^{\alpha}_{\delta\beta\gamma}$ is the form

$$N^{\alpha}_{\delta\beta\gamma} = W_{\delta} \, X^{\alpha}_{\beta\gamma} \tag{2.1}$$

Where $X^{\alpha}_{\beta\gamma}(u.\dot{u})$ is decomposition tensor field and $W_{\delta}(u,\dot{u})$ is a non-zero vector-field.

Theorem (2.1): Under the decomposition (2.1), the tensor fields $X^{\alpha}_{\beta\gamma}$ satisfies the identities

$$X^{\alpha}_{\beta\gamma} + X^{\alpha}_{\gamma\beta} = 0, \tag{2.2a}$$

$$W_{\delta} \, X^{\alpha}_{\beta\gamma} + W_{\beta} \, X^{\alpha}_{\gamma\delta} + W_{\gamma} \, X^{\alpha}_{\delta\beta} = 0 \tag{2.2b}$$

$$v_{\emptyset} \, X^{\alpha}_{\beta\gamma} + v_{\beta} \, X^{\alpha}_{\gamma\emptyset} + v_{\gamma} \, X^{\alpha}_{\emptyset\beta} = 0 \tag{2.2c}$$

Theorem (2.2): The necessary and sufficient condition that the tensor field $X^{\alpha}_{\beta\gamma}$ behaves like a neo-recurrence tensor field of the first order is that the vector field W_{δ} be neo-covariant constant.

Proof: Taking neo-covariant differentiation of (2.1) with respect to $u^{\emptyset}$, making use of (1.6) and (2.1) in resulting equation and simplifying, we have

$$W_{\delta}\left(\overset{n}{\underset{\emptyset}{\nabla}} X^{\alpha}_{\beta\gamma} - v_{\emptyset} X^{\alpha}_{\beta\gamma}\right) = -\left(\overset{n}{\underset{\emptyset}{\nabla}} W_{\delta}\right) X^{\alpha}_{\beta\gamma}, \tag{2.3}$$

Which proves the theorem.

Theorem (2.3): If the vector field W_{δ} be neo-covariant constant, then under the decomposition (2.1), the tensor $\overset{n}{\underset{[\theta}{\nabla}} v_{\emptyset]}$ is neo-recurrent, where the square bracket denotes the skew symmetric part.

Proof: Since

$$\overset{n}{\underset{\emptyset}{\nabla}} W_{\delta} = 0,$$

then from (2.3), we have

$$\overset{n}{\underset{\emptyset}{\nabla}} X^{\alpha}_{\beta\gamma} = v_{\emptyset} X^{\alpha}_{\beta\gamma} \tag{2.4}$$

Differentiating (2.4) neo-covariantly with respect to u^{θ} and using (2.4), we have

$$\overset{n}{\underset{\emptyset\theta}{\nabla}} X^{\alpha}_{\beta\gamma} = \left(\overset{n}{\underset{\theta}{\nabla}} v_{\emptyset} + v_{\theta} v_{\emptyset}\right) X^{\alpha}_{\beta\gamma}. \tag{2.5}$$

Commuting the indices θ **and** $\emptyset$ in (2.5) and using (1.5), we have

$$\left(\overset{n}{\underset{\theta}{\nabla}} v_{\emptyset} - \overset{n}{\underset{\emptyset}{\nabla}} v_{\theta}\right) X^{\alpha}_{\beta\gamma} = X^{\varphi}_{\beta\gamma} N^{\alpha}_{\varphi\emptyset\theta} - X^{\alpha}_{\varphi\gamma} N^{\varphi}_{\beta\emptyset\theta} - X^{\alpha}_{\beta\varphi} N^{\varphi}_{\gamma\emptyset\theta} \tag{2.6}$$

Again differentiating (2.6) neo-covariantly with respect to u^{ε} and making use of (1.6), (2.4) and (2.6), we have the **result**

Theorem (2.4): If the vector field W_{δ} be the neo-covariant constant, then under the decomposition (2.1), the recurrence vector field $v_{\emptyset}$ satisfies the relation

$$v_{\emptyset}\left(\overset{n}{\underset{\varphi}{\nabla}} v_{\theta} - \overset{n}{\underset{\theta}{\nabla}} v_{\varphi}\right) + v_{\theta}\left(\overset{n}{\underset{\emptyset}{\nabla}} v_{\varphi} - \overset{n}{\underset{\varphi}{\nabla}} v_{\emptyset}\right) + v_{\varphi}\left(\overset{n}{\underset{\theta}{\nabla}} v_{\emptyset} - \overset{n}{\underset{\emptyset}{\nabla}} v_{\theta}\right) = 0 \tag{2.7}$$

Proof: Differentiating (2.5) neo-covariantly with respect to u^{φ} and using (2.4), we have

$$\overset{n}{\underset{\emptyset\theta\varphi}{\nabla}} X^{\alpha}_{\beta\gamma} = [\overset{n}{\underset{\theta\varphi}{\nabla}} v_{\emptyset} + \left(\overset{n}{\underset{\varphi}{\nabla}} v_{\emptyset}\right) v_{\theta} + v_{\emptyset}\left(\overset{n}{\underset{\varphi}{\nabla}} v_{\emptyset}\right) + + v_{\emptyset}\ (\overset{n}{\underset{\theta}{\nabla}} v_{\emptyset}) + v_{\varphi}\, v_{\emptyset}\, v_{\theta}\,] X^{\alpha}_{\beta\gamma} \qquad (2.8)$$

Commuting the indices θ and φ in (2.8), we have

$$\overset{n}{\underset{\emptyset\theta\varphi}{\nabla}} X^{\alpha}_{\beta\gamma} - \overset{n}{\underset{\emptyset\varphi\theta}{\nabla}} X^{\alpha}_{\beta\gamma} = [\left(\overset{n}{\underset{\theta\varphi}{\nabla}} v_{\emptyset} - \overset{n}{\underset{\varphi\theta}{\nabla}} v_{\emptyset}\right) + + v_{\emptyset}\left(\overset{n}{\underset{\varphi}{\nabla}} v_{\theta} - \overset{n}{\underset{\theta}{\nabla}} v_{\varphi}\right)] X^{\alpha}_{\beta\gamma} \qquad (2.9)$$

Which may be written as

$$\overset{n}{\underset{\theta\varphi}{\nabla}} (\overset{n}{\underset{\emptyset}{\nabla}} X^{\alpha}_{\beta\gamma}) - \overset{n}{\underset{\varphi\theta}{\nabla}}\left(\overset{n}{\underset{\emptyset}{\nabla}} X^{\alpha}_{\beta\gamma}\right) = [(\overset{n}{\underset{\theta\varphi}{\nabla}} v_{\emptyset} - \overset{n}{\underset{\varphi\theta}{\nabla}} v_{\emptyset}) + v_{\emptyset}\ ((\overset{n}{\underset{\varphi}{\nabla}} v_{\theta}) - (\overset{n}{\underset{\theta}{\nabla}} v_{\emptyset}))] X^{\alpha}_{\beta\gamma} \qquad (2.10)$$

Applying (1.5) and (2.4) in (2.10) and simplifying, we have

$$v_{\emptyset}(X^{\delta}_{\beta\gamma}\, N^{\alpha}_{\delta\theta\varphi} - X^{\alpha}_{\delta\gamma}\, N^{\delta}_{\beta\theta\varphi} - X^{\alpha}_{\beta\delta}\, N^{\delta}_{\gamma\theta\varphi}) = v_{\emptyset}\ (\overset{n}{\underset{\varphi}{\nabla}} v_{\theta} - \overset{n}{\underset{\theta}{\nabla}} v_{\varphi}) X^{\alpha}_{\beta\gamma} \qquad (2.11)$$

Cyclic permutation of the indices $\emptyset, \theta$ and φ in (2.11) yield two more relations. On adding these three relations and making use of (1.4a) and (1.7)
we have (2.7)

Theorem (2.5): Under the decomposition (2.1) the decomposition tensor field $X^{\alpha}_{\beta\gamma}$, satisfies the relation

$$(\overset{n}{\underset{\emptyset\theta\varphi}{\nabla}} X^{\alpha}_{\beta\gamma} - \overset{n}{\underset{\emptyset\varphi\theta}{\nabla}} X^{\alpha}_{\beta\gamma}) + (\overset{n}{\underset{\theta\varphi\emptyset}{\nabla}} X^{\alpha}_{\beta\gamma} - \overset{n}{\underset{\theta\varphi\emptyset}{\nabla}} X^{\alpha}_{\beta\gamma}) + (\overset{n}{\underset{\varphi\emptyset\theta}{\nabla}} X^{\alpha}_{\beta\gamma} - \overset{n}{\underset{\emptyset\varphi\theta}{\nabla}} X^{\alpha}_{\beta\gamma}) = 0 \qquad (2.12)$$

Provided that the vector field W_{δ} be neo-covariant constant.

Proof: In view of commutation formula (1.5), equation (2.9) yields

$$\overset{n}{\underset{\emptyset\theta\varphi}{\nabla}} X^{\alpha}_{\beta\gamma} - \overset{n}{\underset{\emptyset\varphi\theta}{\nabla}} X^{\alpha}_{\beta\gamma} = = -v_{\delta}\, N^{\alpha}_{\emptyset\theta\varphi}\, X^{\alpha}_{\beta\gamma} + v_{\emptyset}\ (\overset{n}{\underset{\varphi}{\nabla}} v_{\theta} - \overset{n}{\underset{\theta}{\nabla}} v_{\varphi}) X^{\alpha}_{\beta\gamma} \qquad (2.13)$$

Interchanging the indices $\emptyset, \theta$ and φ cyclically in (2.13) and adding all the three equations and using (1.4b) and (2.7), we have (2.12).

Now considering the decomposition of the tensor field $X^{\alpha}_{\beta\gamma}$ in the form

$$X^{\alpha}_{\beta\gamma} = U^{\alpha}\, Y_{\beta\gamma} \qquad (2.14)$$

Where $U^{\alpha}(u,\dot{u})$ is any non-zero vector field and $Y_{\beta\gamma}(u, \dot{u})$ is non-zero tensor field.

Under the decomposition (2.14), the theorem (2.1) yields the following results:

(a) $Y_{\beta\gamma} + Y_{\gamma\beta} = 0$ (2.15a)

(b) $W_\delta Y_{\beta\gamma} + W_\beta Y_{\gamma\delta} + W_\gamma Y_{\delta\beta} = 0$ (2.15b)

(c) $v_\emptyset Y_{\beta\gamma} + v_\beta Y_{\gamma\emptyset} + v_\gamma Y_{\emptyset\beta} = 0$ (2.15c)

We may establish the following theorem

Theorem (2.6): If the subspace F_m undergoes the decompositions (2.1) and (2.14) and the vector-field W_δ is neo-covariant constant then the tensor $Y_{\beta\gamma}$ behaves like neo-recurrent tensor field of the first order provided that U^α is neo-covariant constant.

Proof: Differentiating (2.14) neo-covariantly with respect to $u^\emptyset$, we have

$$\overset{n}{\underset{\varphi}{\nabla}} X^\alpha_{\beta\gamma} = (\overset{n}{\underset{\emptyset}{\nabla}} U^\alpha) Y_{\beta\gamma} + U^\alpha (\overset{n}{\underset{\emptyset}{\nabla}} Y_{\beta\gamma}) \tag{2.16}$$

Since W_δ is neo-covariant constant, hence using (2.4) and (2.14) in (2.16), we have

$$(\overset{n}{\underset{\emptyset}{\nabla}} Y_{\beta\gamma} - v_\emptyset Y_{\beta\gamma}) U^\alpha = -(\overset{n}{\underset{\emptyset}{\nabla}} U^\alpha) Y_{\beta\gamma} \tag{2.17}$$

from which we get the theorem.

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