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Some results on n-edge magic labeling of special graphs

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Cubic graph,
Nauru graph.

ABSTRACT

Let G (V, E) be a graph with vertex set V= V(G) and edge set E= E(G) of order p and size q. 0 –edge magic labeling[4], 1-edge magic labeling[7] and n-edge magic was introduced and extended by [6, 9,10].This article discussed and found generalisation of order and size n- edge magic labeling of special graphs such as caterpillar graph, Mӧubius- Kantor graph, Hypercube graph, etc,.

1. Introduction

Let G (V, E) be a finite, simple, without loop, planer and undirected graph. Labeling of a graph G is a mapping that carries graph elements to integers. The origin of this labeling is introduced by Kotzig and Rosa[1,2]. Dealing with labeling have domain either the set of all vertices, or the set of all edges, or the set of all vertices and edges, respectively. This named as vertex labeling, or an edge labeling, or a total labeling, depending on the graph elements that are being labeled. Magic graph defined many others, it is helping to unsolved applications. Edge anti magic labeling is the motivation of 0-edge magic and 1 – edge magic labelling. In 2013, [6] introduced n –edge to Pn, Cn (n being odd +ve integer). In [9,10] extended this n- edge magic labeling to labeling to Bistar, $K_{1,t} \circ P_{t-1}$, Möbius Ladder (n $\equiv$ 0 (mod 2) and wheel Graph, Ladder graph , Friendship graph, Armed Crown graph, G = $P_{t+1} \odot K_{1,}$. This article n –edge magic labeling extend to some other class like caterpillar graph, Mӧubius- Kantor graph, Hypercube graph, splitting graph, prism graph, web graph, Desargues graph, Franklin graph, cubic graph, Nauru graph.

2. Preliminaries

An edge-magic labeling of a (p; q)-graph G is a bijective function f : V(G) ∪ E(G)→{1; 2; : : : ; p + q} such that f(u) + f(v) + f(uv)=k is a constant for any edge uv of G. In such a case, G is said to be edge-magic and k is called the valence of f. 0-Edge Magic Labeling: Let G = (V, E) be a graph where V = {v_i, 1 ≤ i ≤ n}, and E = {v_i v_{i+1},1 ≤ i ≤ n}. Let f : V→ {-1, 1}, and f * : E → {0}, such that all uv Є E, f *(uv) = f (u) + f (v) = 0 then the labeling is said to be 0- Edge Magic labeling.

A (p, q) graph G is said to be (1,0) edge-magic with the common edge count k if there exists a bijection f : V(G) → {1, …, p} such that for all e = uv Є E(G), f(u) + f(v) = k. It is said to be (1, 0) edge anti-magic if for all e = (u,v) Є E(G), f(u) + f(v) are distinct.

A (p,q) graph G is said to be (0,1) vertex-magic with the common vertex count k if there exists a bijection f: E(G) → {1, …, q} such that for each u Є V(G). i.e, Σ f(e) =k for all e = uv Є E(G) with v ϵ V(G). It is said to be (0, 1) vertex-antimagic if for each u Є V (G), e Є Σ f(e) are distinct for all e = uv Є E(G) with v ϵ V(G).

Let G= (V, E) be a graph where V= {vi, 1 ≤ i ≤ t} and E = {v_i v_{i+1},1 ≤ i ≤ t-1}. Let f: V → {-1, 2} and f*: E → {1} such that for all uv Є E, f*(u v) = f (u) + f (v) =1 then the labelling is said to be 1-Edge Magic Labeling**.**

A (p,q) graph G is said to be (1,1) edge-magic with the common edge count k if there exists a bijection f : V(G) ∪ E(G) → {1, …, p+q} such that f(u) + f(v) +f(e) = k for all e = uv Є E(G). It is said to be (1,1) edge-antimagic if f(u) + f(v) + f(e) are distinct for all e = uv Є E(G).

Let G=(V, E) be a graph where V= { vi, 1≤ i ≤ t } and E = {vi v i+1,1≤ i ≤ t-1}. Let f: V → {-1, n+1} and f*: E → {n} such that for all uv E, f*(u v) = f (u) + f (v) = n then the labeling is said to be n-Edge Magic Labeling.

Theorem 1 Pt , Ct , sun graph St , G$\odot K_1$, double star graph Sm,t admits n-Edge Magic Labeling for all t [6]

Theorem 2 Let G = Bn,n be a Bistar graph, and $K_{1,t} \circ P_{t-1}$ and Wn then G admits n-Edge Magic Labeling and M_n (Mobius Ladder) (Labeling n $\equiv$ 0 (mod 2)) [10].

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Theorem 3 Let G be Ladder graph(2n, n+2(n-1)), Friendship graph, Armed Crown graph $C_t \odot P_m$, $P_{t+1} \odot K_{1,t}$ admits n-edge magic labeling [9].

Here listed few graphs definition for future work.

A Caterpillar or Caterpillar tree is a tree in which all the vertices are within distance 1 of a central path.

The Möbius– Kantor graph is symmetric bipartite cubic graph with 16 vertices and 24 edges.

The Hypercube Graph Q_n is the graph formed from the vertices and edges of an n-dimensional hypercube. Q_n has 2^n vertices, $2^{n-1}n$ edges, and is a regular graph with n edges touching each vertex.

The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges, the smallest non-hamiltonian polyhedral graph.

A Splitting graph of Cycle C_t admits n-Edge Magic Labeling for $n \equiv 0 \pmod 2$, either t is even.

Cartesian product of cycle and path graph($Y_n = C_m \times P_n$) are called Prism graph denoted by Y_t. Sometimes also called a circular ladder graph and denoted CL_n is a graph corresponding to the skeleton of as n-prism. Prism graphs are called both planar and polyhedral.

The Web graph describes the directed links between pages of World Wide Web. A graph, in general, consists of several vertices, some pairs connected by edges. In a directed graph, edges are directed lines or arcs. The web graph is a directed graph, whose vertices correspond to the pages of the WWW and a directed edge connects page X to page Y if there exists a hyperlink on page X referring to page Y.

The Desargues graph is a distance-transitive cubic graph with 20 vertices and 30 edges.

The Franklin graph a 3-regular graph with 12 vertices and 18 edges.

A cubic graph is a graph in which all vertices have degree three. In other words a cubic graph is a 3-regular graph. Cubic graphs are also called trivalent graphs.

The Nauru graph is a symmetric bipartite cubic graph with 24 vertices and 36 edges.

3. Main Results of n-edge magic labeling of special graphs

Theorem 4 A Caterpillar graph admits n-Edge Magic Labeling for all t.

Proof: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le t\}$ and $E = \{v_i v_{i+1}, 1 \le i \le t-1\}$. Let f: $V \rightarrow \{-1, n+1\}$ Such that

$f(u_i) = -1$ for $1 \le i \le t$,

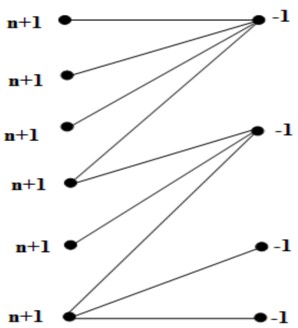
$f(v_i) = n+1$ for $1 \le i \le t$,

we have, $f^*(u_i v_i) = -1 + (n+1) = n$ if $1 \le i \le t$,

$f^*(v_i u_i) = (n+1) + (-1) = n$ if $1 \le i \le t$.

Hence the proof.

Example: n-Edge Magic Labeling for a Caterpillar Graph



Theorem 5 A Möbius–Kantor M (16 , 24) graph admits n-Edge Magic Labeling.

Proof: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le t\}$ and $E = \{v_i v_{i+1}, 1 \le i \le t-1\}$.

Let $f : V \rightarrow \{-1, n+1\}$ such that

$$f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd}, \\ n+1, & \text{if } i \text{ is even}. \end{cases} \quad \text{for } 1 \le i \le 16,$$

we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1 + (n+1) = n & \text{if } i \text{ is odd}, \\ (n+1) + (-1) = n & \text{if } i \text{ is even}. \end{cases}$$

$f^*(v_i v_{i+5}) = -1 + (n+1) = n$ if $i = 5, 7$

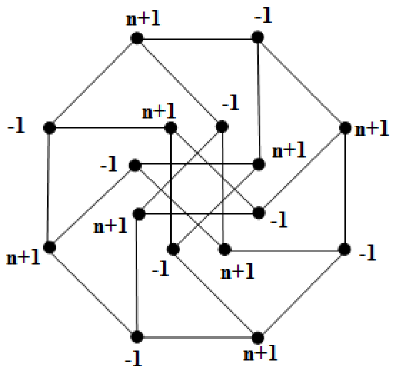
$f^*(v_i v_{i+7}) = -1 + (n+1) = n$ if $i = 1, 9$

$f^*(v_i v_{i+9}) = (n+1) + (-1) = n$ if $i = 2, 4, 6$

$f^*(v_i v_{i+13}) = -1 + (n+1) = n$ if $i = 1, 3$

Hence the proof.

Example: n-Edge Magic Labeling for A Möbius–Kantor Graph M(16,24)



Theorem 6 A Hypercube graph Q (16 , 32) admits n-Edge Magic Labeling.

Proof: Let G = (V, E) be a graph where $V = \{v_i, 1 \le i \le t\}$ and $E = \{v_i v_{i+1}, 1 \le i \le t-1\}$. Let f: $V \rightarrow \{-1, n+1\}$ such that

$$f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd}, \\ n+1, & \text{if } i \text{ is even}. \end{cases}$$

$$f(u_i) = \begin{cases} n+1, & \text{if } i \text{ is even}, \\ -1, & \text{if } i \text{ is odd}. \end{cases} \quad \text{for } 1 \le i \le 8$$

$$f^*(u_i u_{i+1}) = \begin{cases} -1 + (n+1) = n & \text{if } i \text{ is odd}, \\ (n+1) + (-1) = n & \text{if } i \text{ is even}. \end{cases}$$

$$f^*(v_i u_{i+1}) = \begin{cases} -1 + (n+1) = n & \text{if } i \text{ is odd}, \\ (n+1) + (-1) = n & \text{if } i \text{ is even}. \end{cases}$$

$$f^*(u_i v_{i+1}) = \begin{cases} -1 + (n+1) = n & \text{if } i \text{ is odd}, \\ (n+1) + (-1) = n & \text{if } i \text{ is even}. \end{cases}$$

$$f^*(v_i v_{i+3}) = \begin{cases} -1 + (n+1) = n & \text{if } i \text{ is odd}, \\ (n+1) + (-1) = n & \text{if } i \text{ is even}. \end{cases}$$

$$f^*(v_i v_{i+5}) = \begin{cases} -1 + (n+1) = n & \text{if } i = 1,3 \\ (n+1) + (-1) = n & \text{if } i = 2. \end{cases}$$

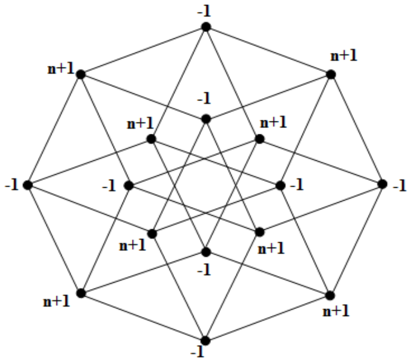
$f^*(u_i v_{i+7}) = -1 + (n+1) = n$ if $i = 1$,

$f^*(v_i u_{i+7}) = -1 + (n+1) = n$ if $i = 1$,

$f^*(u_i u_{i+7}) = -1 + (n+1) = n$ if $i = 1$

Hence the proof.

Example: n-Edge Magic Labeling for a hypercube Graph Q(16,32)



Theorem 7 A Herschel graph H (11,18) admits n-Edge Magic Labeling.

Proof: Let G = (V, E) be a graph where V = $\{v_i, 1 \leq i \leq t\}$ and E = $\{v_i v_{i+1}, 1 \leq i \leq t-1\}$.

Let f:V⟶{-1,n+1} such that

$$f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd,} \\ n+1, & \text{if } i \text{ is even.} \end{cases} \quad \text{for } 1 \leq i \leq 11,$$

We have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$$f^*(v_i v_{i+5}) = \begin{cases} -1+(n+1) = n & \text{if } i = 3, \\ (n+1)+(-1) = n & \text{if } i = 6. \end{cases}$$

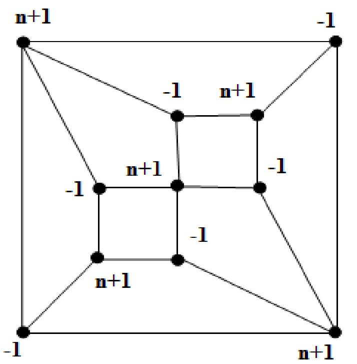
$$f^*(v_i v_{i+9}) = \begin{cases} -1+(n+1) = n & \text{if } i = 1, \\ (n+1)+(-1) = n & \text{if } i = 2. \end{cases}$$

$f^*(v_i v_{i+3}) = (n+1)+(-1) = n$ if i = 2,4,6

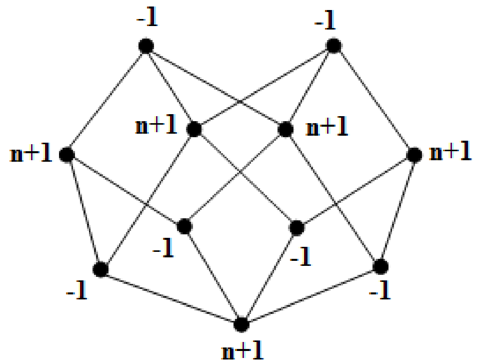
$f^*(v_i v_{i+7}) = -1+(n+1) = n$ if i = 1.

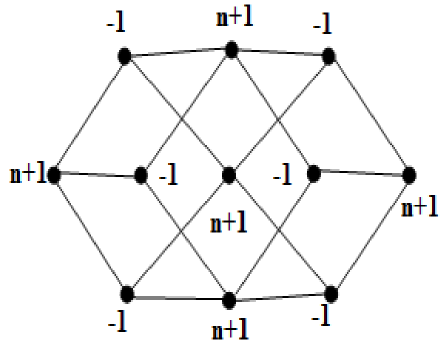
Hence the proof.

Example n-Edge Magic Labeling for a Herschel Graph H(11,18)



and





Theorem 8 A Splitting graph of Cycle C_t admits n-Edge Magic Labeling for n ≡ 0 (mod 2), either t is even.

Proof: Let G = (V, E) be a graph where V= $\{v_i, 1 \leq i \leq t\}$ and E = $\{v_i v_{i+1}, 1 \leq i \leq t-1\}$.

Let f:V⟶{-1,n+1} such that

$$f(u_i) = \begin{cases} -1, & \text{if } i \text{ is odd,} \\ n+1, & \text{if } i \text{ is even.} \end{cases}$$

for $1 \leq i \leq t$,

we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$$f^*(u_i v_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$$f^*(v_i u_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

This completes the proof.

Lemma 9 If n-Edge Magic Labeling for Splitting Cycle C_t is in the form of 2t vertices with 3t edges (for all t = 4,6,8,…..n).

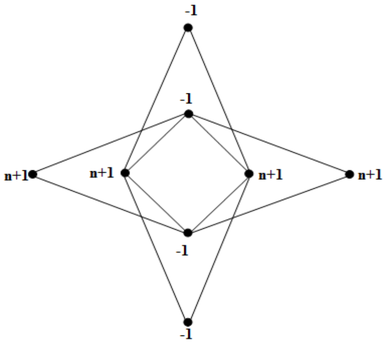
Proof: Proof given by induction method.

Generalised form of splitting cycle C_t.

n-Edge Magic Labeling for Splitting Graph C_t exist t > 3, when t is even.

Case 1: If t = 4

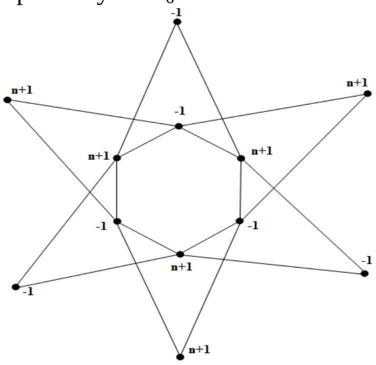
A Splitting Graph of Cycle C_4 is



Therefore n-Edge Magic Labeling for Splitting Cycle C_t exist 8 vertices with 12 edges since (t = 4).

Case 2:If t = 6

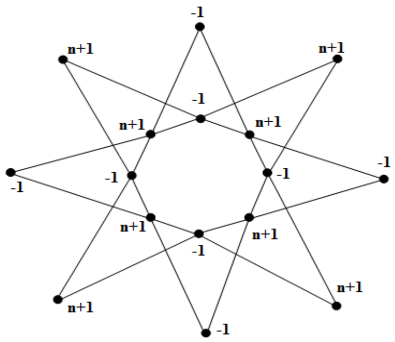
A Splitting Graph of Cycle C_6 is



Therefore n-Edge Magic Labeling for Splitting Cycle C_t exist 12 vertices with18 edges since (t = 6).

Case 3: If t = 8

A Splitting Graph of Cycle C_8 is



Therefore n-Edge Magic Labeling for Splitting Cycle C_t exist 16 vertices with 24 edges since(t = 8).

Continuing this process, we get

Case n: If t = n

For n-Edge Magic Labeling for Splitting Cycle C_t is in the form of 2t vertices with 3t edges (for all t = 4,6,8,…..n).

Theorem 10 Let G be a Prism Graph Y_t then G admits n-Edge Magic Labeling for all t is even.

Proof: Let G = (V, E) be a graph where V = {v_i, $1\leq i\leq t$} and E = {$v_i\, v_{i+1}$, $1\leq i\leq t-1$}.

Let f: V → { -1, n + 1 } such that

$$f(v_i) = \begin{cases} -1\ , & if\ i\ is\ odd, \\ n+1\ , & if\ i\ is\ even. \end{cases}$$

$$f(u_i) = \begin{cases} -1\ , & if\ i\ is\ even, \\ n+1\ , & if\ i\ is\ odd. \end{cases}$$

for $1 \leq i \leq t$,

We have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & if\ i\ is\ odd, \\ (n+1)+(-1) = n & if\ i\ is\ even. \end{cases}$$

$$f^*(u_i u_{i+1}) = \begin{cases} -1+(n+1) = n & if\ i\ is\ even, \\ (n+1)+(-1) = n & if\ i\ is\ odd. \end{cases}$$

$$f^*(u_i v_i) = \begin{cases} -1+(n+1) = n & if\ i\ is\ even, \\ (n+1)+(-1) = n & if\ i\ is\ odd. \end{cases}$$

This completes the proof.

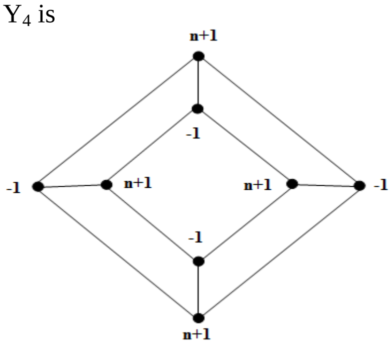
Lemma 11 Let n-Edge Magic Labeling for a Prism Graph Y_t is in the form of 2t vertices with 3t edges (for all t = 4,6,8,…n).

Proof: Proof given by induction method.

Generalised form of Prism Graph Y_t is

Case 1: If t = 4

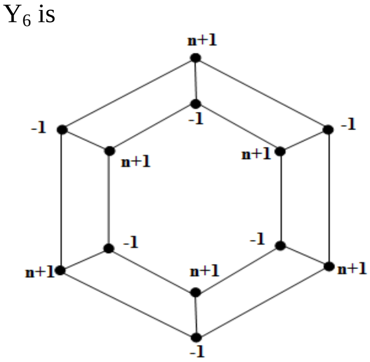
Prism Graph Y_4 is



Therefore n-Edge Magic Labeling for a Prism Graph Y_t is in the form of 8 vertices with 12 edges (Since t = 4).

Case 2: If t = 6

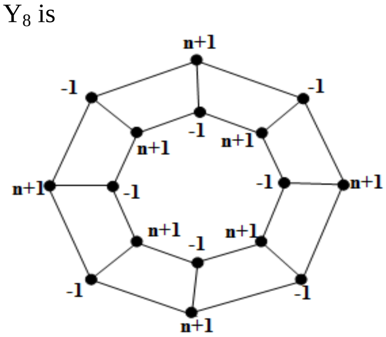
Prism Graph Y_6 is



Therefore n-Edge Magic Labeling for a Prism Graph Y_t is in the form of 12 vertices with 18 edges (Since t = 6).

Case 3: If t = 8

Prism Graph Y_8 is



Therefore n-Edge Magic Labeling for a Prism Graph Y_t is in the form of 16 vertices with 24 edges (Since t = 8).
Continuing this process, we get
Case n:
If t = n
For n-Edge Magic Labeling for a Prism Graph Y_t is in the form of 2t vertices with 3t edges (for all t = 4,6,8,…n).

Theorem 12 Let G be a Web Graph W_t then G admits n-Edge Magic Labeling for all t is even.
Proof: Let G = (V, E) be a graph where V= $\{v_i, 1 \le i \le t\}$ and E = $\{v_i v_{i+1}, 1 \le i \le t-1\}$. Let f: V → { -1, n + 1 } such that

$$f(v_i) = \begin{cases} -1, & if\ i\ is\ odd, \\ n+1, & if\ i\ is\ even. \end{cases}$$

$$f(u_i) = \begin{cases} n+1, & if\ i\ is\ odd, \\ -1, & if\ i\ is\ even. \end{cases}$$

$$f(w_i) = \begin{cases} -1, & if\ i\ is\ odd, \\ n+1, & if\ i\ is\ even. \end{cases}$$

for $1 \le i \le t$,
we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & if\ i\ is\ odd, \\ (n+1)+(-1) = n & if\ i\ is\ even. \end{cases}$$

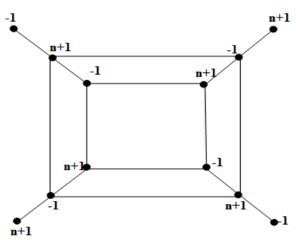
$$f^*(u_i u_{i+1}) = \begin{cases} -1+(n+1) = n & if\ i\ is\ even, \\ (n+1)+(-1) = n & if\ i\ is\ odd. \end{cases}$$

$$f^*(u_i v_i) = \begin{cases} -1+(n+1) = n & if\ i\ is\ even, \\ (n+1)+(-1) = n & if\ i\ is\ odd. \end{cases}$$

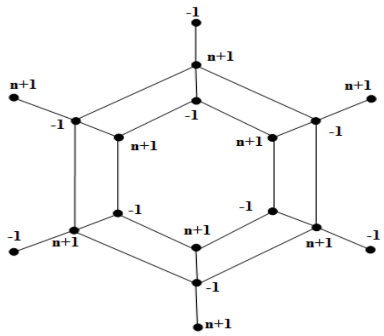
$$f^*(u_i w_i) = \begin{cases} -1+(n+1) = n & if\ i\ is\ even, \\ (n+1)+(-1) = n & if\ i\ is\ odd. \end{cases}$$

This completes the proof.

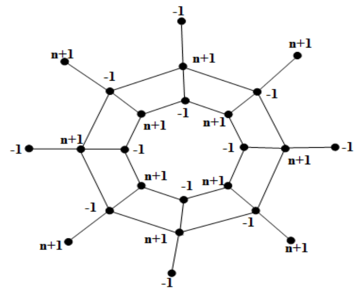
Lemma 13 Let G be a Web Graph W_t then G admits n-Edge Magic Labeling for all t is even.
Proof: Proof given by induction method.
Generalised form of Web Graph W_t
Case 1: If t = 4
Web Graph W_4 is



Therefore n-Edge Magic Labeling for a Web Graph W_t is in the form of 12 vertices with 16 edges (Since t = 4).
Case 2: If t = 6
Web Graph W_6 is



Therefore n-Edge Magic Labeling for a Web Graph W_t is in the form of 18 vertices with 24 edges (Since t = 6).
Case 3: If t = 8
Web Graph W_8 is



Therefore n-Edge Magic Labeling for a Web Graph W_t is in the form of 24 vertices with 32 edges (Since t = 8).
Continuing this process, we get
Case n:
If t = n
For n-Edge Magic Labeling for a Web Graph W_t is in the form of 6t vertices with 8t edges (for all t = 2,3,4,…n).

Theorem 14 A Desargues graph D (20,30) admits n-Edge Magic Labeling.
Proof: Let G = (V, E) be a graph where V = $\{v_i, 1 \le i \le t\}$ and E = $\{v_i v_{i+1}, 1 \le i \le t-1\}$.
Let f: V → { - 1 , n + 1 } such that

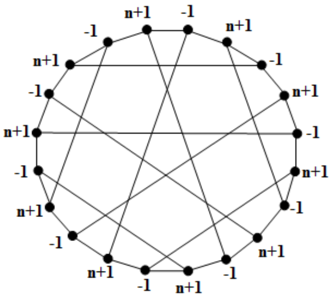
$$f(v_i) = \begin{cases} -1, & if\ i\ is\ odd, \\ n+1, & if\ i\ is\ even. \end{cases} \quad \text{for } 1 \le i \le 20,$$

we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & if\ i\ is\ odd, \\ (n+1)+(-1) = n & if\ i\ is\ even. \end{cases}$$

$f^*(v_i v_{i+5})$ = (n + 1) + (- 1) = n if i = 2,6,10,14.
$f^*(v_i v_{i+9})$ = (n + 1) + (- 1) = n if i = 4 , 8.
$f^*(v_i v_{i+11})$ = - 1 + (n + 1) = n if i = 1,5,9.
$f^*(v_i v_{i+15})$ = - 1 + (n + 1) = n if i = 3.
$f^*(v_i v_{i+19})$ = - 1 + (n + 1) = n if i = 1.
Hence the proof.

Example n-Edge Magic Labeling for a Desargues Graph D(20,30).



Theorem 15 A Franklin graph F (12,18) admits n-Edge Magic Labeling.

Proof: Let G = (V,E) be a graph where V = $\{v_i, 1 \le i \le t\}$ and E = $\{v_i v_{i+1}, 1 \le i \le t\text{-}1\}$.

Let f: V → { - 1 , n + 1 } such that $f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd,} \\ n+1, & \text{if } i \text{ is even.} \end{cases}$ for $1 \le i \le 12$,

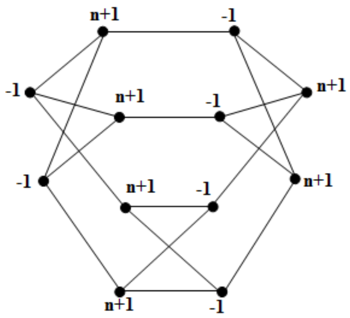
we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$f^*(v_i v_{i+5}) = (n+1)+(-1) = n$ if $i = 2,6,10$.
$f^*(v_i v_{i+7}) = -1+(n+1) = n$ if $i = 1,3,5$.
$f^*(v_i v_{i+11}) = -1+(n+1) = n$ if $i = 1$.

Hence the proof.

Example: n-Edge Magic Labeling for a Franklin Graph F (12,18).



Theorem 16 A Cubical Graph C (8,12) admits n-Edge Magic Labeling.

Proof: Let G = (V,E) be a graph where V = $\{v_i, 1 \le i \le t\}$ and E = $\{v_i v_{i+1}, 1 \le i \le t\text{-}1\}$.

Let f: V → { - 1 , n + 1 } such that $f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd,} \\ n+1, & \text{if } i \text{ is even.} \end{cases}$ for $1 \le i \le 8$,

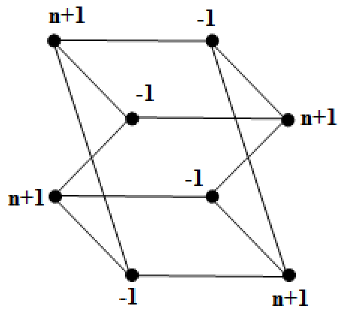
we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$f^*(v_i v_{i+3}) = -1+(n+1) = n$ if $i = 1,3,5$.
$f^*(v_i v_{i+5}) = (n+1)+(-1) = n$ if $i = 2$.
$f^*(v_i v_{i+7}) = -1+(n+1) = n$ if $i = 1$.

Hence the proof.

Example:n-Edge Magic Labeling for a Cubical Graph C (8,12)



Theorem 17 A Nauru Graph N (24,36) admits n-Edge Magic Labeling.

Proof: Let G = (V,E) be a graph where V = $\{v_i, 1 \le i \le t\}$ and E = $\{v_i v_{i+1}, 1 \le i \le t\text{-}1\}$.

Let f: V → { - 1 , n + 1 } such that $f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd,} \\ n+1, & \text{if } i \text{ is even.} \end{cases}$ for $1 \le i \le 24$,

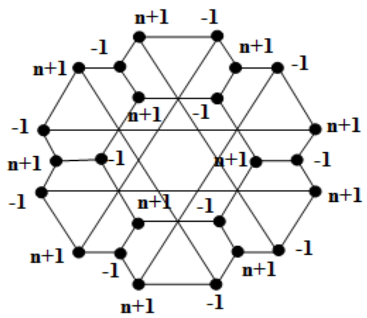
we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$f^*(v_i v_{i+5}) = -1+(n+1) = n$ if $i = 1,7,13,19$.
$f^*(v_i v_{i+7}) = -1+(n+1) = n$ if $i = 3,9,15$.
$f^*(v_i v_{i+9}) = -1+(n+1) = n$ if $i = 5,11$.
$f^*(v_i v_{i+15}) = (n+1)+(-1) = n$ if $i = 2,8$.
$f^*(v_i v_{i+17}) = (n+1)+(-1) = n$ if $i = 4$.
$f^*(v_i v_{i+23}) = -1+(n+1) = n$ if $i = 1$.

Hence the proof.

Example: n-Edge Magic Labeling for a Nauru Graph N (24,36).



Theorem 18 A Pappus Graph P (18 , 27) admits n-Edge Magic Labeling.

Proof: Let G = (V,E) be a graph where V = $\{v_i, 1 \le i \le t\}$ and E = $\{v_i v_{i+1}, 1 \le i \le t-1\}$.

Let f: V → { - 1 , n + 1 }

Such that $f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd,} \\ n+1, & \text{if } i \text{ is even.} \end{cases}$ for $1 \le i \le 18$,

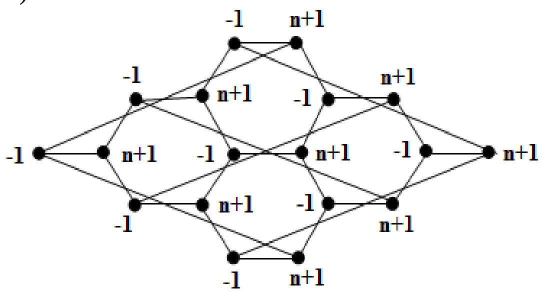
we have

$$f^*(v_i v_{i+1}) = \begin{cases} -1+(n+1) = n & \text{if } i \text{ is odd,} \\ (n+1)+(-1) = n & \text{if } i \text{ is even.} \end{cases}$$

$f^*(v_i v_{i+5}) = (n+1)+(-1) = n$ if $i = 4,10$.
$f^*(v_i v_{i+7}) = -1+(n+1) = n$ if $i = 1,5,7,11$.
$f^*(v_i v_{i+11}) = (n+1)+(-1) = n$ if $i = 2,6$.
$f^*(v_i v_{i+13}) = -1+(n+1) = n$ if $i = 3$.

$f^*(v_i v_{i+17}) = -1 + (n+1) = n$ if $i = 1$.
Hence the proof.

Example: n-Edge Magic Labeling for a Pappus Graph P(18,27).



4. **Conclusion**

The author extends the work to derived graph and arc routing graphs.

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