Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 96 (2016) 41389-41393

ISSN: 2229-712X

Connected Total Dominating Sets and Connected Total Domination Polynomials of Extended Grid Graphs

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ARTICLE INFO

Article history:

Received: 24 May 2016;

Received in revised form: 29 June 2016;

Accepted: 1 July 2016;

Keywords

Extended grid graph,
Connected total dominating set,
Connected total domination number,
Connected total domination polynomial.

ABSTRACT

Let G be a simple connected graph of order n. Let $D_{ct}(G, i)$ be the family of connected total dominating sets of G with cardinality i. The polynomial $D_{ct}(G, x) = \sum_{i=\gamma_{ct}(G)}^{n} d_{ct}(G, i)\, x^i$ is called the connected total domination polynomial of G. In this paper, we study some properties of connected total domination polynomials of the Extended grid graph G_n. We obtain a recursive formula for $d_{ct}(G_n, i)$. Using this recursive formula, we construct the connected total domination polynomial $D_{ct}(G_n, x) = \sum_{i=n-2}^{2n} d_{ct}(G_n, i)\, x^i$, of G_n, where $d_{ct}(G_n, i)$ is the number of connected total dominating sets of G_n with cardinality i and some properties of this polynomial have been studied.

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1. Introduction

Let G = (V, E) be a simple graph of order n. For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighbourhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$. The maximum degree of the graph G is denoted by $\Delta(G)$ and the minimum degree is denoted by $\delta(G)$.

A set S of vertices in a graph G is said to be a total dominating set if every vertex $v \in V$ is adjacent to an element of S.

A total dominating set S of G is called a connected total dominating set if the induced subgraph $\langle S \rangle$ is connected.

The minimum cardinality taken over all connected total dominating sets S of G is called the connected total domination number of G and is denoted by $\gamma_{ct}(G)$.

A connected total dominating set with cardinality $\gamma_{ct}(G)$ is called γ_{ct} - set. We denote the set {1, 2, . . ., 2n -1, 2n} by [2n], throughout this paper.

2. Connected Total Dominating Sets of Extended Grid Graphs

Consider two paths $[u_1u_2\ldots u_n]$ and $[v_1v_2\ldots v_n]$. Join each pair of vertices u_i,v_i ; u_i,v_{i+1} ; v_i,u_{i+1}, i = 1, 2,…, n. The resulting graph is an Extended grid graph.

Le G_n be an Extended grid graph with 2n vertices. Label the vertices of G_n as given in Figure 1.

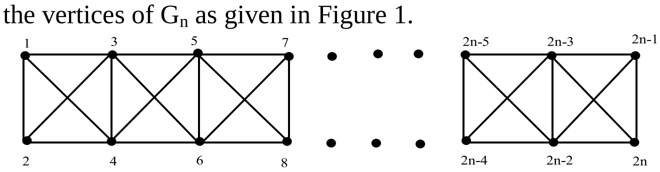

Figure 1. Extended Grid Graph G_n.

Then, $V(G_n)$ = { 1,2,3,…,2n-3,2n-2, 2n-1, 2n} and $E(G_n)$ = {(1,3)(3,5), (5,7),…, (2n-5, 2n-3), (2n-3, 2n-1), (2,4), (4,6), (6,8),...,(2n-4, 2n-2), (2n-2, 2n), (1,2),(3,4),(5,6),...,(2n-3,2n-2),(2n-1,2n),(1,4),(3,6)(5,8),...,(2n-5,2n-2),(2n-3,2n), (2,3),(4,5),(6,7),...,(2n-4,2n-3),(2n-2,2n-1)}.

For the construction of the connected total dominating sets of the Extended grid graphs G_n, we need to investigate the connected total dominating sets of G_n - {2n}. In this section, we investigate the connected total dominating sets of G_n. Let $D_{ct}(G_{n,i})$ be the family of connected total dominating sets of G_n with cardinality i. We shall find the recursive formula for $d_{ct}(G_n, i)$.

Lemma 2.1[7]

$\gamma_{ct}(P_n) = n - 2$.

Lemma 2.2

For every $n \in N$ and $n \geq 4$,

(i) $\gamma_{ct}(G_n) = n - 2$.

(ii) $\gamma_{ct}(G_n - \{2n\}) = n - 2$.

(iii) $D_{ct}(G_n, i) = \phi$ if and only if $i < n - 2$ or $i > 2n$.

(iv) $D_{ct}(G_n - \{2n\}, i) = \phi$ if and only if $i < n - 2$ or $i > 2n - 1$.

Proof

(i) Clearly {3,5,7,9,…,2n –3} is a minimum connected total dominating set for G_n. If n is even or odd it contains n-2 elements. Hence, $\gamma_{ct}(G_n) = n - 2$.

(ii) Clearly { 3,5,7,9,…, 2n –3} is a minimum connected total dominating set for $G_n - \{2n\}$. If n is even or odd it contains n – 2 elements.

Hence, $\gamma_{ct}(G_n - \{2n\}) = n - 2$.

(iii) follows from (i) and the definition of connected total dominating set.

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(iv) follows from (ii) and the definition of connected total dominating set.

Lemma 2.3

(i) If $D_{ct}(G_{n-2}-\{2n-4\}, i-1)=\phi$, $D_{ct}(G_{n-1}, i-1)=\phi$, and $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, then $D_{ct}(G_{n-2}, i-1)=\phi$.

(ii) If $D_{ct}(G_{n-2}-\{2n-4\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, then $D_{ct}(G_{n-2}, i-1)\neq\phi$.

(iii) If $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}-\{2n\}, i-1)\neq\phi$, then $D_{ct}(G_n, i)\neq\phi$.

(iv) If $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, and $D_{ct}(G_n-\{2n\}, i-1)=\phi$, then $D_{ct}(G_n, i)\neq\phi$.

(v) If $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, $D_{ct}(G_{n-1}, i-1)=\phi$, and $D_{ct}(G_n-\{2n\}, i-1)=\phi$, then $D_{ct}(G_n, i)=\phi$.

Proof

(i) Since, $D_{ct}(G_{n-2}-\{2n-4\}, i-1)=\phi$, $D_{ct}(G_{n-1}, i-1)=\phi$, and

$D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, by Lemma 2.2 (iii) & (iv), we have,

$i-1<n-4$ or $i-1>2n-5$,

$i-1<n-3$ or $i-1>2n-2$,

and $i-1<n-3$ or $i-1>2n-3$.

Therefore, $i-1<n-4$ or $i-1>2n-3$.

Therefore, $i-1<n-4$ or $i-1>2n-4$ holds.

Hence, $D_{ct}(G_{n-2}, i-1)=\phi$.

(ii) Since, $D_{ct}(G_{n-2}-\{2n-4\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, by Lemma 2.2 (iii) & (iv), we have,

$n-4\leq i-1\leq 2n-5$, $n-3\leq i-1\leq 2n-2$, and

$n-3\leq i-1\leq 2n-3$.

Suppose, $D_{ct}(G_{n-2}, i-1)=\phi$, then $i-1<n-4$ or $i-1>2n-4$. Suppose, $i-1<n-4$, then $D_{ct}(G_{n-1}, i-1)=\phi$, a contradiction.

Suppose, $i-1>2n-4$, then $i-1>2n-5$ holds, which implies

$D_{ct}(G_{n-2}-\{2n-4\}, i-1)=\phi$, a contradiction.

Therefore, $D_{ct}(G_{n-2}, i-1)\neq\phi$.

(iii) Since, $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}-\{2n\}, i-1)\neq\phi$, by Lemma 2.2 (iii) & (iv), we have ,

$n-3\leq i-1\leq 2n-3$,

$n-3\leq i-1\leq 2n-2$ and

$n-2\leq i-1\leq 2n-1$.

Suppose, $D_{ct}(G_n, i)=\phi$, then, by Lemma 2.2 (iii), we have $i<n-2$ or $i>2n$.

Suppose, $i<n-2$, then $i-1<n-3$, which implies $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, a contradiction.

Suppose, $i>2n$ then $i-1>2n-1$, which implies $D_{ct}(G_{n-1}-\{2n\}, i-1)=\phi$, a contradiction.

Therefore, $D_{ct}(G_n, i)\neq\phi$.

(iv) Since, $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}, i-1)\neq\phi$,

by Lemma 2.2 (iii) & (iv), we have,

$n-3\leq i-1\leq 2n-3$ and $n-3\leq i-1\leq 2n-2$.

Suppose, $D_{ct}(G_n, i)=\phi$, then, by Lemma 2.2 (iii), we have $i<n-2$ or $i>2n$.

Suppose, $i<n-2$ then $i-1<n-3$ which implies $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, a contradiction.

Suppose, $i>2n$, then $i-1>2n-1$.

Therefore, $i-1>2n-2$ holds, which implies $D_{ct}(G_{n-1}, i-1)=\phi$, a contradiction.

Hence, $D_{ct}(G_n, i)\neq\phi$.

(v) Since, $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, $D_{ct}(G_{n-1}, i-1)=\phi$ and

$D_{ct}(G_n-\{2n\}, i-1)=\phi$, by Lemma 2.2 (iii) & (iv), we have,

$i-1<n-3$ or $i-1>2n-3$,

$i-1<n-3$ or $i-1>2n-2$, and

$i-1<n-2$ or $i-1>2n-1$.

Therefore, $i-1<n-3$ or $i-1>2n-1$.

Therefore, $i<n-2$ or $i>2n$ holds.

Hence, $D_{ct}(G_n, i)=\phi$.

Lemma 2.4

Suppose that $D_{ct}(G_n, i)\neq\phi$, then for every $n\in N$,

(i) $D_{ct}(G_n-\{2n\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)=\phi$, and $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, if and only if $i=2n$.

(ii) $D_{ct}(G_n-\{2n\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, if and only if $i=2n-1$.

(iii) $D_{ct}(G_n-\{2n\}, i-1)\neq\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, and $D_{ct}(G_{n-2}, i-1)=\phi$,

if and only if $i=2n-2$.

(iv) $D_{ct}(G_n-\{2n\}, i-1)=\phi$, $D_{ct}(G_{n-1}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}-\{2n-2\}, i-1)\neq\phi$, if and only if $n=k+2$ and $i=k$ for some $k\in N$.

Proof

(i) $(\Rightarrow)$ since, $D_{ct}(G_{n-1}, i-1)=\phi$, and $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, by Lemma 2.2(iii)& (iv), we have, $i-1<n-3$ or $i-1>2n-2$ and $i-1<n-3$ or $i-1>2n-3$.

Therefore, $i-1<n-3$ or $i-1>2n-2$.

Suppose, $i-1<n-3$, then $i<n-2$ which implies $D_{ct}(G_n, i)=\phi$, a contradiction.

Therefore, $i-1>2n-2$.

Therefore, $i-1\geq 2n-1$. (1)

Also, since, $D_{ct}(G_n-\{2n\}, i-1)\neq\phi$, by Lemma 2.2 (iv), we have,

$n-2\leq i-1\leq 2n-1$. (2)

From (1) and (2), $i-1=2n-1$.

Therefore, $i=2n$.

$(\Leftarrow)$ follows from Lemma 2.2 (iii) & (iv).

(ii) $(\Rightarrow)$ Since, $D_{ct}(G_n-\{2n\}, i-1)\neq\phi$, and $D_{ct}(G_{n-1}, i-1)\neq\phi$,

by Lemma 2.2 (iii) & (iv), we have,

$n-2\leq i-1\leq 2n-1$ and

$n-3\leq i-1\leq 2n-2$.

Therefore, $n-2\leq i-1\leq 2n-2$. (3)

Also, since, $D_{ct}(G_{n-1}-\{2n-2\}, i-1)=\phi$, by Lemma 2.2 (iv), we have, $i-1<n-3$ or $i-1>2n-3$.

Suppose, $i-1<n-3$, then $i-1<n-2$, which implies, $D_{ct}(G_n, i)=\phi$, a contradiction.

Therefore, $i-1>2n-3$.

Therefore, $i-1\geq 2n-2$. (4)

From (3) and (4), we have, $i-1=2n-2$.

Therefore, $i=2n-1$.

($\Leftarrow$) follows from Lemma 2.2 (iii) & (iv).

(iii) Since, $D_{ct}(G_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(G_{n-1}, i-1) \neq \phi$ and $D_{ct}(G_{n-1} - \{2n-2\}, i-1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have,

$n-2 \le i-1 \le 2n-1$,
$n-3 \le i-1 \le 2n-2$ and
$n-3 \le i-1 \le 2n-3$.

Therefore, $n-2 \le i-1 \le 2n-3$. (5)

Also, since, $D_{ct}(G_{n-2}, i-1) = \phi$, by Lemma 2.2 (iii), we have,

$i-1 < n-4$, or $i-1 > 2n-4$.

Suppose, $i-1 < n-4$, then $i-1 < n-3$.

Therefore, $i < n-2$, which implies $D_{ct}(G_n, i) = \phi$, a contradiction.

Therefore, $i-1 > 2n-4$.

Therefore, $i-1 \ge 2n-3$. (6)

From (5) and (6), we have, $i-1 = 2n-3$.

Therefore, $i = 2n-2$.

($\Leftarrow$) follows from Lemma 2.2 (iii) & (iv).

(iv) Since, $D_{ct}(G_{n-1}, i-1) \neq \phi$ and $D_{ct}(G_{n-1} - \{2n-2\}, i-1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have,

$n-3 \le i-1 \le 2n-2$ and
$n-3 \le i-1 \le 2n-3$.

Therefore, $n-3 \le i-1 \le 2n-3$.

Therefore, $n-2 \le i \le 2n-2$. (7)

Also, since, $D_{ct}(G_n - \{2n\}, i-1) = \phi$, by Lemma 2.2 (iv), we have,

$i-1 < n-2$ or $i-1 > 2n-1$.

Suppose, $i-1 > 2n-1$, then $i > 2n$, which implies $D_{ct}(G_n, i) = \phi$, a contradiction.

Therefore, $i-1 < n-2$.

Therefore, $i < n-1$. (8)

From (7) and (8), we have, $n-2 \le i < n-1$. (9)

When $n \neq k+2$, we get an inequality of the form $s \le i < s$, which is not possible. When $n = k+2$, we have $s \le i < s+1$. Therefore (9) holds. In this case $i = k$.

Conversely, assume $n = k+2$ and $i = k$.

Therefore $n-2 = k$ and
$i-1 = k-1$.
$k-1 < k = n-2$.

Therefore,
$D_{ct}(G_n - \{2n\}, i-1) = \phi$.

Also,
$D_{ct}(G_{n-1} - \{2n-2\}, i-1) = D_{ct}(G_{k+1} - \{2(k+1)\}, k-1) \neq \phi$.

and $D_{ct}(G_{n-1}, i-1) = D_{ct}(G_{k+1}, k-1) \neq \phi$.

Theorem 2.5

For every $n \ge 4$,

(i) If $D_{ct}(G_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(G_{n-1}, i-1) = \phi$, and $D_{ct}(G_{n-1} - \{2n-2\}, i-1) = \phi$, then $D_{ct}(G_n, i) = \{X \cup \{2n\} / X \in D_{ct}(G_n - \{2n\}, i-1)\}$.

(ii) If $D_{ct}(G_n - \{2n\}, i-1) = \phi$, $D_{ct}(G_{n-1}, i-1) \neq \phi$, and $D_{ct}(G_{n-1} - \{2n-2\}, i-1) \neq \phi$, then

$D_{ct}(G_n, i) = \{\{X_1 \cup \{2n-2\} / X_1 \in D_{ct}(G_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-3\} / X_2 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\}\}$.

(iii) If $D_{ct}(G_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(G_{n-1}, i-1) \neq \phi$, and $D_{ct}(G_{n-1} - \{2n-2\}, i-1) \neq \phi$, then,

$D_{ct}(G_n, i) =$

$$\begin{cases} \{X_1 \cup \{2n\} / X_1 \in D_{ct}(G_n - \{2n\}, i-1)\} \cup \\ \{X_2 \cup \{2n-1\} / X_2 \in D_{ct}(G_{n-1}, i-1)\} \cup \\ \{X_2 \cup \{2n-2\} / X_2 \in D_{ct}(G_{n-1}, i-1)\} \cup \\ \{X_3 \cup \{2n-2\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\} \cup \\ \{X_3 \cup \{2n-3\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\}. \end{cases}$$

Proof

(i) Since, $D_{ct}(G_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(G_{n-1}, i-1) = \phi$ and $D_{ct}(G_{n-1} - \{2n-2\}, i-1) = \phi$, by Theorem 2.4 (i), $i = 2n$.

Therefore, $D_{ct}(G_n, i) = D_{ct}(G_n, 2n) = \{[2n]\}$ and $D_{ct}(G_n - \{2n\}, i-1) = D_{ct}(G_n - \{2_n\}, 2n-1) = \{[2n-1]\}$, we have the result.

(ii) Let $Y_1 = \{X_1 \cup \{2n-2\} / X_1 \in D_{ct}(G_{n-1}, i-1)\}$ and $Y_2 = \{X_2 \cup \{2n-3\} / X_2 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\}$.

Obviously, $Y_1 \cup Y_2 \subseteq D_{ct}(G_n, i)$ (1)

Now, let $Y \in D_{ct}(G_n, i)$.

If $2n-2 \in Y$, then atleast one of the vertices labeled $2n-4$ or $2n-5$ is in Y. In either cases, $Y = \{X_1 \cup \{2n-2\}\}$ for some $X_1 \in D_{ct}(G_{n-1}, i-1)$.

Therefore, $Y \in Y_1$.

If $2n-3 \in Y$, then atleast one of the vertices labeled $2n-4$ or $2n-5$ is in Y. In either cases, $Y = \{X_2 \cup \{2n-3\}\}$ for some $X_2 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)$.

Therefore, $Y \in Y_2$.

Therefore, $D_{ct}(G_n, i) \subseteq Y_1 \cup Y_2$.

From (1) and (2), we have,

$D_{ct}(G_n, i) =$

$$\begin{cases} \{X_1 \cup \{2n-2\} / X_1 \in D_{ct}(G_{n-1}, i-1)\} \cup \\ \{X_2 \cup \{2n-3\} / X_2 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\} \end{cases}$$

(iii) Let $Y_1 = \{X_1 \cup \{2n\} / X_1 \in D_{ct}(G_n - \{2n\}, i-1)\}$.

$Y_2 =$

$$\begin{cases} \{X_2 \cup \{2n-1\} / X_2 \in D_{ct}(G_{n-1}, i-1)\} \cup \\ \{X_2 \cup \{2n-2\} / X_2 \in D_{ct}(G_{n-1}, i-1)\}. \end{cases}$$

$Y_3 =$

$$\begin{cases} \{X_3 \cup \{2n-2\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\} \cup \\ \{X_3 \cup \{2n-3\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\}. \end{cases}$$

Obviously, $Y_1 \cup Y_2 \cup Y_3 \subseteq D_{ct}(G_n, i)$ (3)

Now, Let $Y \in D_{ct}(G_n, i)$.

If $2n \in Y$, then atleast one of the vertices labeled 2n-1 or 2n-2 or 2n-3 is in Y. In each cases, $Y = \{X_1 \cup \{2n\}\}$ for some $X_1 \in D_{ct}(G_n - \{2n\}, i-1)$.

Therefore, $Y \in Y_1$.

Now suppose that, $2n-1 \in Y$, $2n \notin Y$, then atleast one of the vertices labeled $2n-2$ or $2n-3$ is in Y.

In each cases, $Y = \{X_2 \cup \{2n-1\}\}$ for some $X_2 \in D_{ct}(G_{n-1}, i-1)$.

Now suppose that, $2n-2 \in Y$ and $2n, 2n-1 \notin Y$ then atleast one of the vertices labeled $2n-3$ or $2n-4$ is in Y.

If $2n-3 \in Y$, then

$Y = \{X_3 \cup \{2n-2\}\}$ for some $X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)$.

If $2n-4 \in Y$, then

$Y = \{X_2 \cup \{2n-2\}\}$ for some $X_2 \in D_{ct}(G_{n-1}, i-1)$.

Therefore, $Y \in Y_2$ or $Y \in Y_3$.

Now suppose that, $2n-3 \in Y$ and $2n, 2n-1, 2n-2 \notin Y$, then $2n-4$ is in Y.

In this case, $Y = \{X_3 \cup \{2n-3\}\}$ for some $X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)$.

Therefore, $Y \in Y_3$.

Hence, $D_{ct}(G_n,i) \subseteq Y_1 \cup Y_2 \cup Y_3$. (4)

From (3) and (4) we have,

$D_{ct}(G_n,i) =$

$$\begin{cases} \{X_1 \cup \{2n\}\} / X_1 \in D_{ct}(G_n - \{2n\}, i-1) \cup \\ \{X_2 \cup \{2n-1\}\} / X_2 \in D_{ct}(G_{n-1}, i-1) \cup \\ \{X_2 \cup \{2n-2\}\} / X_2 \in D_{ct}(G_{n-1}, i-1) \cup \\ \{X_3 \cup \{2n-2\}\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1) \cup \\ \{X_3 \cup \{2n-3\}\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1). \end{cases}$$

Theorem 2.6

If $D_{ct}(G_n,i)$ is the family of connected total dominating sets of G_n with cardinality i, where $i \geq n-2$, then $d_{ct}(G_n,i) = d_{ct}(G_n - \{2n\}, i-1) + d_{ct}(G_{n-1}, i-1) + d_{ct}(G_{n-1} - \{2n-2\}, i-1)$.

Proof

We consider all the three cases given in Theorem 2.5.

By Theorem 2.5 (i), we have,

$D_{ct}(G_n,i) = \{X \cup \{2n\} / X \in D_{ct}(G_n - \{2n\}, i-1)\}$.

Since, $D_{ct}(G_{n-1}, i-1) = \phi$ and

$D_{ct}(G_{n-1} - \{2n-2\}, i-1) = \phi$, we have dct $(G_{n-1}, i-1) = 0$ and $d_{ct}(G_{n-1} - \{2n-2\}, i-1) = 0$.

Therefore, $d_{ct}(G_n,i) = d_{ct}(G_n - \{2n\}, i-1)$.

By Theorem 2.5 (ii), we have,

$D_{ct}(G_n, i) =$

$$\begin{cases} \{X_1 \cup \{2n-2\} / X_1 \in D_{ct}(G_{n-1}, i-1)\} \cup \\ \{X_2 \cup \{2n-3\} / X_2 \in D_{ct}(G_{n-1} - \{2n-2\},\ i-1)\}. \end{cases}$$

Since, $D_{ct}(G_n - \{2n\}, i-1) = \phi$, we have $d_{ct}(G_n - \{2n\}, i-1) = 0$.

Therefore, $d_{ct}(G_n,i) = d_{ct}(G_{n-1}, i-1) + d_{ct}(G_{n-1} - \{2n-2\}, i-1)$.

By Theorem 2.5 (iii), we have ,

$D_{ct}(G_n,i) =$

$$\begin{cases} \{X_1 \cup \{2n\}\} / X_1 \in D_{ct}(G_n - \{2n\}, i-1)\} \cup \\ \{X_2 \cup \{2n-1\}\} / X_2 \in D_{ct}(G_{n-1}, i-1)\} \cup \\ \{X_2 \cup \{2n-2\}\} / X_2 \in D_{ct}(G_{n-1}, i-1)\} \cup \\ \{X_3 \cup \{2n-2\}\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\} \cup \\ \{X_3 \cup \{2n-3\}\} / X_3 \in D_{ct}(G_{n-1} - \{2n-2\}, i-1)\}. \end{cases}$$

Therefore, $d_{ct}(G_n,i) = d_{ct}(G_n - \{2n\}, i-1) + d_{ct}(G_{n-1}, i-1) + d_{ct}(G_{n-1} - \{2n-2\}, i-1)$.

3.Connected Total Domination Polynomials of Extented Grid Graphs.

Definition 3.1

Let $D_{ct}(G_n, i)$ be the family of connected total dominating sets of G_n with cardinality i and let $d_{ct}(G_n, i) = |D_{ct}(G_n, i)|$. Then the connected total domination Ploynomial $D_{ct}(G_n, x)$ of G_n is defined as,

$$D_{ct}(G_n, x) = \sum_{i=\gamma_{ct}(G_n)}^{2n} d_{ct}(G_n, i)\, x^i.$$

Theorem 3.2

For every $n \geq 5$,

$D_{ct}(G_n, x) = x[D_{ct}(G_n - \{2n\}, x) + D_{ct}(G_{n-1}, x) + D_{ct}(G_{n-1} - \{2n-2\}, x)]$, with initial values,

$D_{ct}(G_2 - \{4\}, x) = 3x^2 + x^3$.

$D_{ct}(G_2, x) = 6x^2 + 4x^3 + x^4$.

$D_{ct}(G_3 - \{6\}, x) = 7x^2 + 9x^3 + 5x^4 + x^5$.

$D_{ct}(G_3, x) = 9x^2 + 16x^3 + 14x^4 + 6x^5 + x^6$.

$D_{ct}(G_4 - \{8\}, x) = 4x^2 + 16x^3 + 25x^4 + 19x^5 + 7x^6 + x^7$.

$D_{ct}(G_4, x) = 4x^2 + 20x^3 + 41x^4 + 44x^5 + 26x^6 + 8x^7 + x^8$.

$D_{ct}(G_5 - \{10\}, x) = 8x^3 + 36x^4 + 66x^5 + 63x^6 + 33x^7 + 9x^8 + x^9$.

Proof

We have,

$d_{ct}(G_n, i) = d_{ct}(G_n - \{2n\}, i-1) + d_{ct}(G_n - 1, i-1) + d_{ct}(G_{n-1} - \{2n-2\}, i-1)$.

Therefore,

$d_{ct}(G_n, i)\, x^i = d_{ct}(G_n - \{2n\}, i-1)\, x^i + d_{ct}(G_n - 1, i-1)\, x^i + d_{ct}(G_{n-1} - \{2n-2\}, i-1)\, x^i$.

$\sum d_{ct}(G_n, i)\, x^i = \sum d_{ct}(G_n - \{2n\}, i-1)\, x^i + \sum d_{ct}(G_n - 1, i-1)\, x^i + \sum d_{ct}(G_{n-1} - \{2n-2\}, i-1)\, x^i$.

$\sum d_{ct}(G_n, i)\, x^i = x\sum d_{ct}(G_n - \{2n\}, i-1)\, x^{i-1} + x\sum d_{ct}(G_{n-1}, i-1)x^{i-1} + x\sum d_{ct}(G_{n-1} - \{2n-2\}, i-1)\, x^{i-1}$.

$D_{ct}(G_n, x) = x\, D_{ct}(G_n - \{2n\}, x) + x\, D_{ct}(G_{n-1}, x) + x\, D_{ct}(G_{n-1} - \{2n-2\}, x)$.

Therefore,

$D_{ct}(G_n, x) = x[D_{ct}(G_n - \{2n\}, x) + D_{ct}(G_{n-1}, x) + D_{ct}(G_{n-1} - \{2n-2\}, x)]$ with initial values,

$D_{ct}(G_2 - \{4\}, x) = 3x^2 + x^3$.

$D_{ct}(G_2, x) = 6x^2 + 4x^3 + x^4$.

$D_{ct}(G_3 - \{6\}, x) = 7x^2 + 9x^3 + 5x^4 + x^5$.

$D_{ct}(G_3, x) = 9x^2 + 16x^3 + 14x^4 + 6x^5 + x^6$.

$D_{ct}(G_4 - \{8\}, x) = 4x^2 + 16x^3 + 25x^4 + 19x^5 + 7x^6 + x^7$.

$D_{ct}(G_4, x) = 4x^2 + 20x^3 + 41x^4 + 44x^5 + 26x^6 + 8x^7 + x^8$.

$D_{ct}(G_5 - \{10\}, x) = 8x^3 + 36x^4 + 66x^5 + 63x^6 + 33x^7 + 9x^8 + x^9$.

Example 3.3

$D_{ct}(G_4 - \{8\}, x) = 4x^2 + 16x^3 + 25x^4 + 19x^5 + 7x^6 + x^7$.

$D_{ct}(G_4, x) = 4x^2 + 20x^3 + 41x^4 + 44x^5 + 26x^6 + 8x^7 + x^8$.

$D_{ct}(G_5 - \{10\}, x) = 8x^3 + 36x^4 + 66x^5 + 63x^6 + 33x^7 + 9x^8 + x^9$.

By Theorem 3.2, we have,

$D_{ct}(G_5, x) = x[4x^2 + 16x^3 + 25x^4 + 19x^5 + 7x^6 + x^7 + 4x^2 + 20x^3 + 41x^4 + 44x^5 + 26x^6 + 8x^7 + x^8 + 8x^3 + 36x^4 + 66x^5 + 63x^6 + 33x^7 + 9x^8 + x^9] = 8x^3 + 44x^4 + 102x^5 + 129x^6 + 96x^7 + 42x^8 + 10x^9 + x^{10}$.

In the following Theorem we obtain some properties of $d_{ct}(G_n, i)$.

Theorem 3.4

The following properties hold for the coefficients of $D_{ct}(G_n, x)$ for all n.

(i) $d_{ct}(G_n, 2n) = 1$, for every $n \geq 2$.

(ii) $d_{ct}(G_n, 2n-1) = 2n$, for every $n \geq 2$.

(iii) $d_{ct}(G_n, 2n-2) = 2[n^2 - n + 1]$, for every $n \geq 2$.

(iv) $d_{ct}(G_n, n-2) = 8 \times 2^{n-5}$, for every $n \geq 5$.

Table 1. $d_{ct}(G_n, i)$ and $d_{ct}(G_n – \{2n\}, i)$ for $2 \leq n \leq 9$.

i / n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
G_2-{4}	3	1															
G_2	6	4	1														
G_3-{6}	7	9	5	1													
G_3	9	16	14	6	1												
G_4-{8}	4	16	25	19	7	1											
G_4	4	20	41	44	26	8	1										
G_5-{10}	0	8	36	66	63	33	9	1									
G_5	0	8	44	102	129	96	42	10	1								
G_6-{12}	0	0	16	80	168	192	129	51	11	1							
G_6	0	0	16	96	248	360	321	180	62	12	1						
G_7-{14}	0	0	0	32	176	416	552	450	231	73	13	1					
G_7	0	0	0	32	208	592	968	1002	681	304	86	14	1				
G_8-{16}	0	0	0	0	64	384	1008	1520	1452	912	377	99	15	1			
G_8	0	0	0	0	64	448	1392	2528	2972	2364	1289	476	114	16	1		
G_9-{18}	0	0	0	0	0	128	832	2400	4048	4424	3276	1666	575	129	17	1	
G_9	0	0	0	0	0	128	960	3232	6448	8472	7700	4942	2241	704	146	18	1

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