Available online at www.elixirpublishers.com (Elixir International Journal)

Statistics

Elixir Statistics 97 (2016) 42391-42401

ISSN: 2229-712X

Estimation of Population Proportion of a Sensitive Attribute Using Bayesian Approach: Theory and Application

Adepetun, A.O.[1,*] and Adewara, A.A.[2]

[1]Department of Statistics, Federal University of Technology, PMB 704, Akure, Ondo State, Nigeria.

[2]Department of Statistics, University of Ilorin, PMB 1515, Ilorin, Kwara State, Nigeria.

ARTICLE INFO

Article history:

Received: 11 September 2015;

Received in revised form:

21 August 2015;

Accepted: 31 August 2015;

Keywords

ProposedBayesian estimators (PBEs),
Alternative beta priors (ABPs),
Sensitive attribute,
Mean Square Error (MSE),
Absolute Bias.

ABSTRACT

In this paper,Bayesian estimators of the population proportion of a sensitive attribute were developed when real life data were gathered through the administration of survey questionnaires on an induced abortion on 300 matured women in some selected hospitals in the metropolis. Using both the Kumaraswamy (KUMA) and the Generalised (GLS) beta distributions as alternative beta priors, efficiency of the proposed Bayesian estimators was established for a wide interval of the values of the population proportion. We observed that for small, medium as well as large sample sizes, the proposed Bayesian estimators were better than the conventional Bayesian estimator proposed by Hussain and Shabbir [10] when a simple beta prior was used.

Introduction

Direct questioningabout a sensitive attribute such as induced abortion, use of drug, tax evasion, etc.in a human population survey is a strenuous exercise. A survey statistician may receivewrong responses from the survey respondents when he/she uses direct questioningtechnique. Due to many reasons, information about prevalence of sensitiveattributes in the population becomes essential. Warner [24]was the first to put forward acomplicatedmethod of survey to collect information in relation to sensitve attributes byensuring privacy and anonymity to the respondents. To date, numerous developments and improvements on Warner's Randomized Response Technique (RRT) have beendeveloped by many researchers. Greenberg et al. [8], Mangat and Singh [16],Mangat [15], Singh et al. [21], Christofides [7], Kim and Warde [14], Adebola and Adepetun [2], Adebola and Adepetun [3], Adepetun and Adebola [4] are someof the many to be cited. In some situations, priorinformation about the unknown parameter may be available and can be combined with thesample information for the estimationof that unknown parameter. This is known as the Bayesianapproach of estimation. Work done by researchers on Bayesian analysis of Randomizedresponse models are not very elaborate, however, attempts have been made on theBayesian analysis of Randomized response techniques. Winkler and Franklin [25], Pitz[20], Spurrier and Padgett [22], O'Hagan [18], Oh [19], Migon and Tachibana[17], Unnikrishnan and Kunte [23], Bar-Lev and Bobovich [5], Barabesi andMarcheselli [6],Kim et al. [13], Hussain and Shabbir [10,11],Hussain and Shabbir [12],Adepetun and Adewara [1], are the major references on the Bayesiananalysis of the Randomized Response Techniques. The paper is arranged as follows. In Section 2, we present Hussain and Shabbir [10] Randomized Response Technique (RRT) followed by our proposed alternativeBayesianestimation of population proportion in section 3.Section 4 contains the numerical consideration and comparison of results. Section 5 is the conclusion. The appendix is an attached copy of the administered survey questionnaire on an induced abortion respectively.

The Existing Bayesian Technique of Estimation

Hussain and Shabbir [10] in their referred paper presented a Bayesian estimation to the Randomized Response Technique (RRT) put forward by Hussain and Shabbir [9] using a simple beta prior distribution to estimate the population proportion of respondents possessing sensitive attribute.

Assume the simple beta prior is defined as follows

$$f(\pi) = \frac{1}{B(a,b)}\pi^{a-1}(1-\pi)^{b-1} \; ; \; 0 < \pi < 1 \tag{1}$$

Where (a, b) are the shape parameters of the distribution and π is the population proportion of respondents possessing the sensitive attribute.

Let $X = \sum x_i$ be the total number of the women who have committed an abortion for a particular sample of size n selected from the population with simple random sampling with replacement sampling. Then the conditional distribution of X given π is

$$f(X|\pi) = \frac{n!}{x!\,(n-x)!}\phi^{x}(1-\phi)^{n-x} \tag{2}$$

where ϕ is the probability of "yes response" to the sensitive attribute which was defined as

$$\phi = \frac{\alpha}{\alpha+\beta}\left(P_1\pi + (1-P_1)(1-\pi)\right) + \frac{\beta}{\alpha+\beta}\left(P_2\pi + (1-P_2)(1-\pi)\right) \tag{3}$$

where P_1 is the preset probability of "yes" response to the sensitive attribute and (α, β) are non-zero constants such that $P_1 + P_2 = 1$ respectively

Tele:
E-mail address: aoadepetun@futa.edu.ng

$$f(X|\pi) = \binom{n}{x}\left[\frac{\pi\big((2P_1-1)(\alpha-\beta)\big)+\beta P_1+\alpha P_2}{\alpha+\beta}\right]^x\left[1-\frac{\pi\big((2P_1-1)(\alpha-\beta)\big)+\beta P_1+\alpha P_2}{\alpha+\beta}\right]^{n-x}$$

On simplification, we have

$$f(X|\pi) = \binom{n}{x}\left[\frac{\big((2P_1-1)(\alpha-\beta)\big)}{\alpha+\beta}\right]^n(\pi+F)^x(1-\pi+H)^{n-x}$$

where

$$F=\frac{\beta P_1+\alpha P_2}{(2P_1-1)(\alpha-\beta)};\quad H=\frac{3P_1(\beta-\alpha)+3\alpha}{(2P_1-1)(\alpha-\beta)}$$

Setting

$$A=\binom{n}{x}\left[\frac{\big((2P_1-1)(\alpha-\beta)\big)}{\alpha+\beta}\right]^n$$

$$f(X|\pi)=A\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^i(1-\pi)^j \tag{4}$$

for $x = 0,1,2,\ldots,n$

Thus, the joint probability density functions (pdf) of X and π was

$$f(X,\pi)=D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^i\pi^{a-1}(1-\pi)^j(1-\pi)^{b-1} \tag{5}$$

where

$$D=\frac{\binom{n}{x}}{B(a,b)}\left[\frac{\big((2P_1-1)(\alpha-\beta)\big)}{\alpha+\beta}\right]^n$$

Now the marginal distribution of X can be obtained by integrating the joint distribution of X and π over π. Thus the marginal distribution of X was given by

$$f(X)=\int_0^1 f_g(X,\pi)\,d\pi = D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\int_0^1\pi^{a-1+i}(1-\pi)^{b-1+j}\,d\pi$$

$$f(X)=D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}B(a+i,b+j) \tag{6}$$

The posterior distribution of π given X wasdefined as

$$f(\pi|X)=\frac{f(X,\pi)}{f(X)} \tag{7}$$

$$f(\pi|X)=\frac{D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^i\pi^{a-1}(1-\pi)^j(1-\pi)^{b-1}}{D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}B(a+i,b+j)}$$

$$f(\pi|X)=\frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{a-1+i}(1-\pi)^{b-1+j}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}B(a+i,b+j)} \tag{8}$$

Under the squared error loss function, the Bayes estimator of π which is the posterior mean of (8) was given by

$$\hat{\pi}_{SH}=\int_0^1\pi f(\pi|X)d\pi = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\int_0^1\pi^{a+i}(1-\pi)^{b+j-1}\,d\pi}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}B(a+i,j+1)}$$

$$=\frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}B(a+i+1,b+j)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}B(a+i,j+1)} \tag{9}$$

The Bias of $\hat{\pi}_{SH}$ as well as its Mean Square Error (MSE) was given by

$$B(\hat{\pi}_{SH})=\hat{\pi}_{SH}-\pi \tag{10}$$

$$MSE(\hat{\pi}_{SH})=\sum_{x=0}^{n}(\hat{\pi}_{SH}-\pi)^2\phi^x(1-\phi)^{n-x} \tag{11}$$

The Proposed Bayesian Techniques of Estimation

In this section, we propose an alternative Bayesian estimation to Hussain and Shabbir [9] Randomized Response Technique using both the Kumaraswamy (KUMA) and the Generalised (GLS) beta prior distributions as our alternative beta prior distributions in addition to the simple beta prior distribution used by Hussain and Shabbir [10].

Estimation of π using Kumaraswamy prior

The Kumaraswamy prior distribution of π is given as

$$f(\pi)=bc\pi^{c-1}(1-\pi^c)^{b-1}\ ;b,c>0 \tag{12}$$

Using the Kumaraswamy prior in (12), the joint probability density function of X and π is derived as

$$f(X,\pi) = bcE\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{i}(1-\pi)^{j}\,(1-\pi^{c})^{b-1}\pi^{c-1} \quad (13)$$

where $E = \binom{n}{x}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right]^{n}$

The marginal probability density function (pdf) of X can be obtained as

$$f(X) = \int_0^1 f(X,\pi)\,d\pi \quad (14)$$

$$= bcE\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\int_0^1(1-\pi)^j\,\pi^{ck+i+c-1}d\pi$$

$$= abE\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{a-1}{k}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}B(ck+c+i,j+1) \quad (15)$$

Similarly, the posterior distribution as usual is obtained as follows

$$f(\pi|X) = \frac{f(X,\pi)}{f(X)}$$

$$= \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}(1-\pi)^j\,\pi^{ck+i+c-1}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(ck+c+i,j+1)} \quad (16)$$

Under the Square error loss, we proceed to obtain the posterior mean which is the Bayes estimator as follows

$$\hat{\pi}_{KH} = \int_0^1 \pi\, f(\pi|X)\,d\pi \quad (17)$$

Considering the fact that

$$\int_0^1 \pi\,(1-\pi)^j\pi^{ck+i+c-1}d\pi = \int_0^1 \pi^{ck+i+c}\,(1-\pi)^j\,d\pi = B(ck+i+c+1,j+1)$$

Therefore,

$$\hat{\pi}_{KH} = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(ck+i+c+1,j+1)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(ck+i+c,j+1)} \quad (18)$$

As a result, the Bias of $\hat{\pi}_{KH}$ as well as its Mean Square Error is also given by

$$B(\hat{\pi}_{KH}) = \hat{\pi}_{KH} - \pi \quad (19)$$

$$MSE(\hat{\pi}_{KH}) = \sum_{x=0}^{n}(\hat{\pi}_{KH}-\pi)^2\phi^x(1-\phi)^{n-x} \quad (20)$$

Estimation of π using Generalised Beta prior

The Generalised Beta prior is defined as

$$f(\pi) = \frac{c}{B(a,b)}\pi^{ac-1}(1-\pi^c)^{b-1}; \quad a,b,c > 0 \quad (21)$$

Where a, b, c are the shape parameters of the prior distribution as given in equation (21)

By binomial series expansion, we know that

$$(1-\pi^c)^{b-1} = \sum_{k=0}^{b-1}(-1)^k\binom{b-1}{k}(\pi^c)^k$$

consequently

$$f(\pi) = \frac{c}{B(a,b)}\sum_{k=0}^{b-1}(-1)^k\binom{b-1}{k}\pi^{c(k+a)-1}$$

As a result, the joint density function of π and X with Generalized beta prior is

$$f(X,\pi) = G\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}(1-\pi)^j\pi^{c(a+k)+i-1} \quad (22)$$

where

$$G = \frac{c}{B(a,b)}\binom{n}{x}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right]^n$$

The marginal probability density function (pdf) of X can then be obtained from (22) as

$$f(X) = \int_0^1 f(X,\pi)\,d\pi \quad (23)$$

$$= G\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i,j+1) \quad (24)$$

Similarly, we obtained the posterior distribution of π given X as

$$f(\pi|X) = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}(1-\pi)^j\pi^{c(a+k)+i-1}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i,j+1)} \quad (25)$$

In the same manner, under the square error loss, the posterior mean which is otherwise known as the Bayes estimator is given by

$$\hat{\pi}_{GH} = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i+1,j+1)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}B(c(k+a)+i,j+1)} \quad (26)$$

The Bias of $\hat{\pi}_{GH}$ and its Mean Square Error (MSE) are respectively given by

$$B(\hat{\pi}_{GH}) = \hat{\pi}_{GH} - \pi \quad (27)$$

$$MSE(\hat{\pi}_{GH}) = \sum_{x=0}^{n}(\hat{\pi}_{GH} - \pi)^2\ \phi^x(1-\phi)^{n-x} \quad (28)$$

Numerical consideration and comparison of Results

Here, we present the numerical consideration as well as comparativestudy of our results with the existing Hussain and Shabbir [10] using the real life data obtained from the administered survey questionnaires on an induced abortion under the samevalues of parameters in the estimators using sample sizes 25, 100 and 250 respectively. To overcome the associated computational difficulties, we wrote computer programs using available statistical software to generate our results. To minimizespaces, we present few results in tables and figures as follows:

Table 1a. Mean Square Errors (MSEs) for Hussain and Shabbir [9]
RRT at $n = 25, x = 11,\ \alpha = 1,\ \beta = 10,\ P_1 = 0.1, P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	4.225819E-10	1.017346E-08	1.607482E-08
0.2	2.767500E-12	6.719137E-09	1.164096E-08
0.3	2.968604E-10	3.978724E-09	7.921015E-09
0.4	1.304861E-09	1.952218E-09	4.914973E-09
0.5	3.026768E-09	6.396192E-10	2.622838E-09
0.6	5.462583E-09	4.092771E-11	1.044610E-09
0.7	8.612305E-09	1.561435E-10	1.802902E-10
0.8	1.247593E-08	9.852666E-10	2.987716E-11
0.9	1.705347E-08	2.528297E-09	5.933715E-10

Table 1b. Absolute Bias for Hussain and Shabbir [9]RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.10880517	0.53386124	0.67106898
0.2	0.00880517	0.43386124	0.57106898
0.3	0.09119483	0.33386124	0.47106898
0.4	0.19119483	0.23386124	0.37106898
0.5	0.29119483	0.13386124	0.27106898
0.6	0.39119483	0.03386124	0.17106898
0.7	0.49119483	0.06613876	0.07106898
0.8	0.59119483	0.16613876	0.02893102
0.9	0.69119483	0.26613876	0.12893102

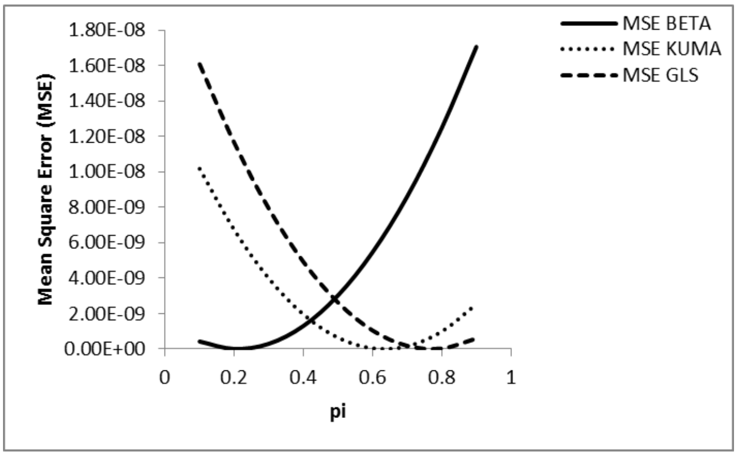

Figure 1a. Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

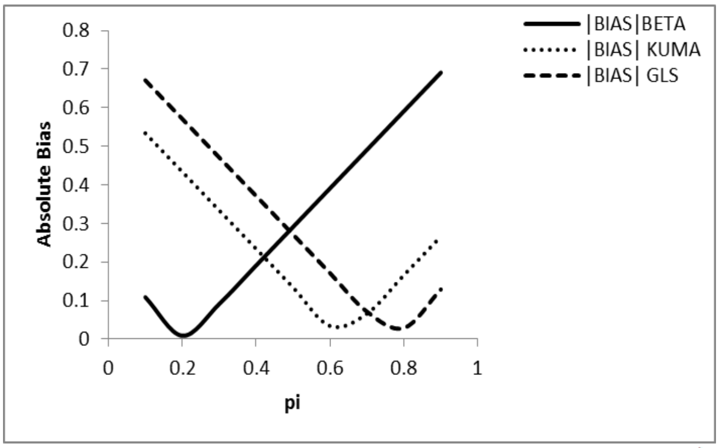

Figure 1b. Absolute Bias for Hussain and Shabbir [9] RRT at $n = 25, x = 11,$ $\alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

Comment: When $n = 25, P_1 = 0.1,$ the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi \leq 0.4$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.4 < \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.7 < \pi < 1$ respectively.

Table 2a. Mean Square Errors (MSEs) for Hussain and Shabbir [9]
RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	3.980527E-09	1.327131E-08	1.886529E-08
0.2	1.953481E-09	9.275222E-09	1.403224E-08
0.3	6.403423E-10	5.993041E-09	9.913099E-09
0.4	4.111079E-11	3.424768E-09	6.507863E-09
0.5	1.557865E-10	1.570402E-09	3.816535E-09
0.6	9.843696E-10	4.299438E-10	1.839114E-09
0.7	2.526860E-09	3.392482E-12	5.756007E-10
0.8	4.783257E-09	2.907485E-10	2.599432E-11
0.9	7.753562E-09	1.292012E-09	1.902953E-10

Table 2b. Absolute Bias for Hussain and Shabbir [9] RRT at $n = 25, x = 11,$ $\alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.33393689	0.60974884	0.72698568
0.2	0.23393689	0.50974884	0.62698568
0.3	0.13393689	0.40974884	0.52698568
0.4	0.03393689	0.30974884	0.42698568
0.5	0.06606311	0.20974884	0.32698568
0.6	0.16606311	0.10974884	0.22698568
0.7	0.26606311	0.00974884	0.12698568
0.8	0.36606311	0.09025116	0.02698568
0.9	0.46606311	0.19025116	0.07301432

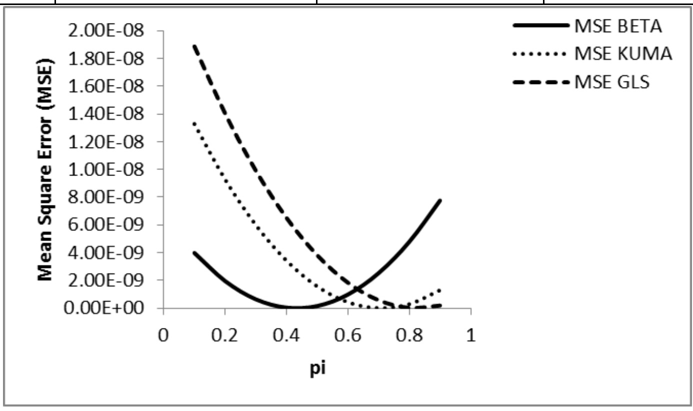

Figure 2a. Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

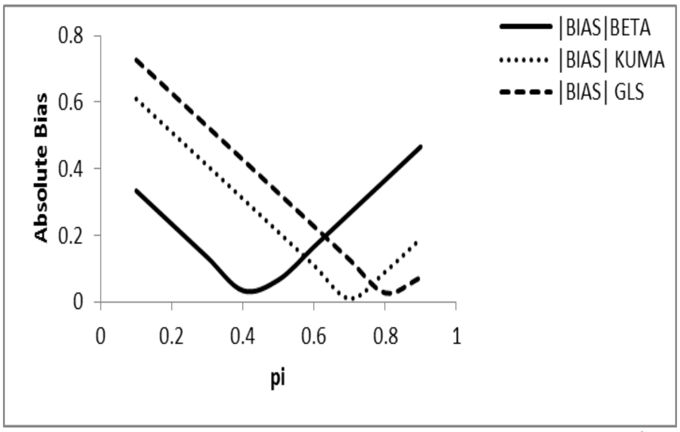

Figure 2b. Absolute Bias for Hussain and Shabbir [9] RRT at $n = 25, x = 11, \alpha = 1,$ $\beta = 10, P_1 = 0.2, P_2 = 0.8$

Comment: When $n = 25, P_1 = 0.2,$ the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.7 < \pi < 1$ respectively.

Table 3a. Mean Square Errors (MSEs) for Hussain and Shabbir [9]
RRT at $n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	7.321979E-33	6.766773E-34	5.777443E-33
0.2	5.326008E-32	2.931831E-32	4.789110E-33
0.3	1.413718E-31	1.001335E-31	4.597435E-32
0.4	2.716570E-31	2.131223E-31	1.293332E-31
0.5	4.441158E-31	3.682846E-31	2.548655E-31
0.6	6.587482E-31	5.656206E-31	4.225715E-31
0.7	9.155541E-31	8.051300E-31	6.324510E-31
0.8	1.214534E-30	1.086813E-30	8.845041E-31
0.9	1.555687E-30	1.410670E-30	1.178731E-30

Table 3b. Absolute Bias for Hussain and Shabbir [9] RRT at
$n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.05892628	0.01791371	0.05234349
0.2	0.15892628	0.11791371	0.04765651
0.3	0.25892628	0.21791371	0.14765651
0.4	0.35892628	0.31791371	0.24765651
0.5	0.45892628	0.41791371	0.34765651
0.6	0.55892628	0.51791371	0.44765651
0.7	0.65892628	0.61791371	0.54765651
0.8	0.75892628	0.71791371	0.64765651
0.9	0.85892628	0.81791371	0.74765651

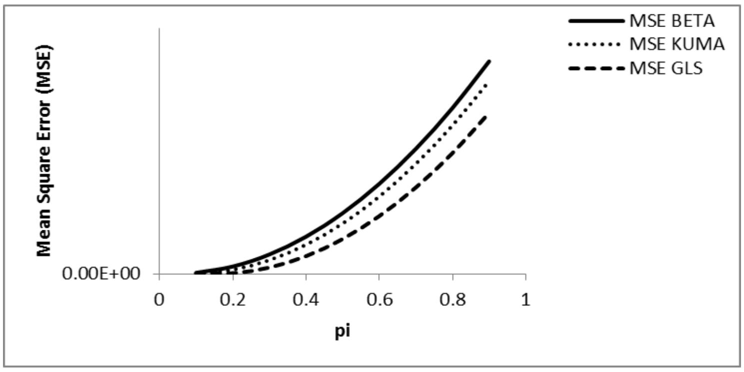

Figure 3a. Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 100,$ $x = 43, \quad \alpha = 1, \quad \beta = 10, \quad P_1 = 0.1, P_2 = 0.9$

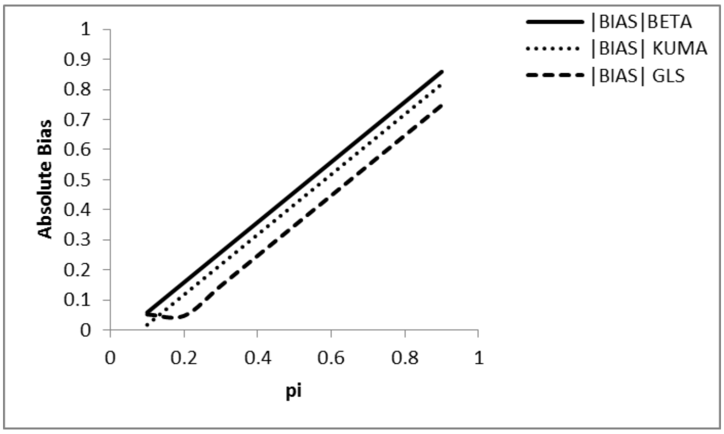

Figure 3b. Absolute Bias for Hussain and Shabbir [9] RRT at $n = 100, x = 43,$ $\alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

Comment: When $n = 100, P_1 = 0.1,$ the proposed estimators are better than the conventional estimator when π lies within the range $0.1 \leq \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.1 \leq \pi < 1$ respectively.

Table 4a. Mean Square Errors (MSEs) for Hussain and Shabbir [9]
RRT at $n = 100, x = 43, \ \alpha = 1, \ \beta = 10, \ P_1 = 0.2, P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	2.791151E-31	4.914985E-31	6.731825E-31
0.2	1.467660E-31	3.089766E-31	4.559816E-31
0.3	5.659053E-32	1.686283E-31	2.809543E-31
0.4	8.588608E-33	7.045355E-32	1.481005E-31
0.5	2.760257E-33	1.445236E-32	5.742033E-32
0.6	3.910548E-32	6.247455E-34	8.913720E-33
0.7	1.176243E-31	2.897070E-32	2.580680E-33
0.8	2.383166E-31	9.949022E-32	3.842121E-32
0.9	4.011825E-31	2.121833E-31	1.164353E-31

Table 4b. Absolute Bias for Hussain and Shabbir [9] RRT at $n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.36381991	0.48278740	0.56501661
0.2	0.26381991	0.38278740	0.46501661
0.3	0.16381991	0.28278740	0.36501661
0.4	0.06381991	0.18278740	0.26501661
0.5	0.03618009	0.08278740	0.16501661
0.6	0.13618009	0.01721260	0.06501661
0.7	0.23618009	0.11721260	0.03498339
0.8	0.33618009	0.21721260	0.13498339
0.9	0.43618009	0.31721260	0.23498339

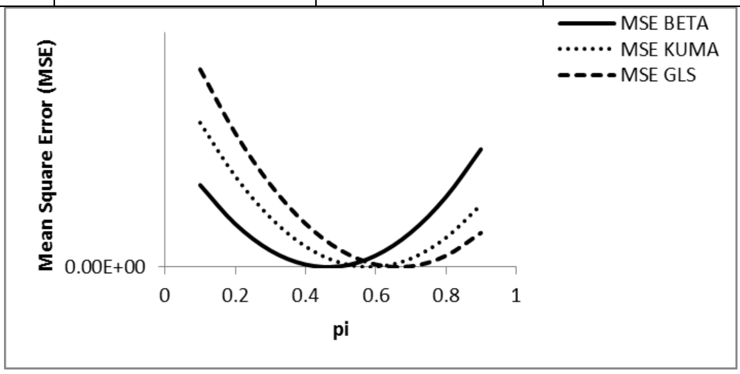

Figure 4a. Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 100,$ $x = 43, \quad \alpha = 1, \quad \beta = 10, \quad P_1 = 0.2, P_2 = 0.8$

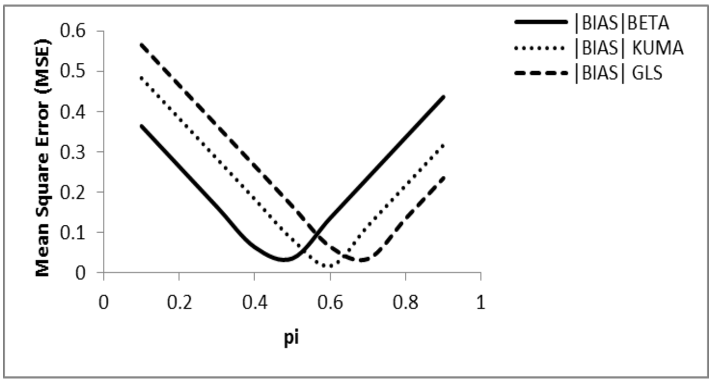

Figure 4b. Absolute Bias for Hussain and Shabbir [9] RRT at $n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

Comment: When $n = 100, P_1 = 0.2$, the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.6 < \pi < 1$ respectively

Table 5a. Mean Square Errors (MSEs) for Hussain and Shabbir [9]
RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	6.762010E-77	4.132263E-77	9.538537E-78
0.2	3.327631E-76	2.705011E-76	1.717961E-76
0.3	7.986527E-76	7.004263E-76	5.348004E-76
0.4	1.465289E-75	1.331098E-75	1.098551E-75
0.5	2.332672E-75	2.162517E-75	1.863049E-75
0.6	3.400802E-75	3.194682E-75	2.828293E-75
0.7	4.669678E-75	4.427594E-75	3.994285E-75
0.8	6.139301E-75	5.861253E-75	5.361022E-75
0.9	7.809671E-75	7.495658E-75	6.928507E-75

Table 5b. Absolute Bias for Hussain and Shabbir [9] RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.08207837	0.06416302	0.03082703
0.2	0.18207837	0.16416302	0.13082703
0.3	0.28207837	0.26416302	0.23082703
0.4	0.38207837	0.36416302	0.33082703
0.5	0.48207837	0.46416302	0.43082703
0.6	0.58207837	0.56416302	0.53082703
0.7	0.68207837	0.66416302	0.63082703
0.8	0.78207837	0.76416302	0.73082703
0.9	0.88207837	0.86416302	0.83082703

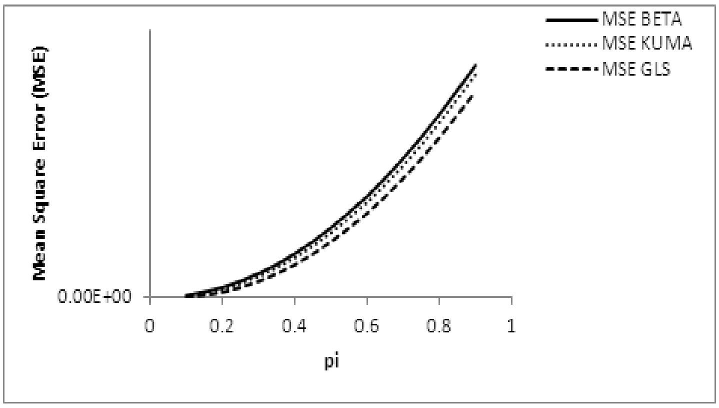

Figure 5a. Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

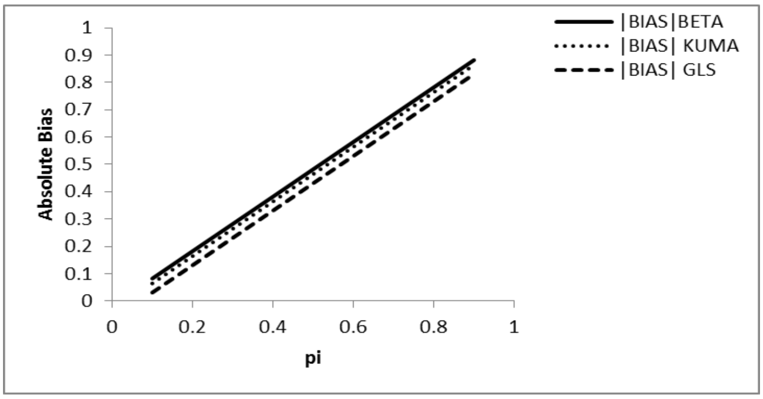

Figure 5b. Absolute Bias for Hussain and Shabbir [9] RRT at
$n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

Comment: When $n = 250, P_1 = 0.1$, the proposed estimators are better than the conventional estimator when π lies within the range $0.1 \leq \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.1 \leq \pi < 1$ respectively.

Table 6a. Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at
$n = 250, x = 106, \quad \alpha = 1, \quad \beta = 10, \quad P_1 = 0.2, P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	6.286780E-77	1.617733E-75	2.278710E-75
0.2	3.221153E-76	9.121848E-76	1.422586E-75
0.3	7.821096E-76	4.073836E-76	7.672089E-76
0.4	1.442850E-75	1.033292E-76	3.125784E-76
0.5	2.304338E-75	2.144654E-80	5.869454E-77
0.6	3.366572E-75	9.746041E-77	5.557399E-78
0.7	4.629553E-75	3.956461E-76	1.531669E-76
0.8	6.093281E-75	8.945784E-76	5.015232E-76
0.9	7.757756E-75	1.594257E-75	1.050626E-75

Table 6b. Absolute Bias for Hussain and Shabbir [9] RRT at
$n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.07914162	0.401461737	0.47646975
0.2	0.17914162	0.301461737	0.37646975
0.3	0.27914162	0.201461737	0.27646975
0.4	0.37914162	0.101461737	0.17646975
0.5	0.47914162	0.001461737	0.07646975
0.6	0.57914162	0.098538263	0.02353025
0.7	0.67914162	0.198538263	0.12353025
0.8	0.77914162	0.298538263	0.22353025
0.9	0.87914162	0.398538263	0.32353025

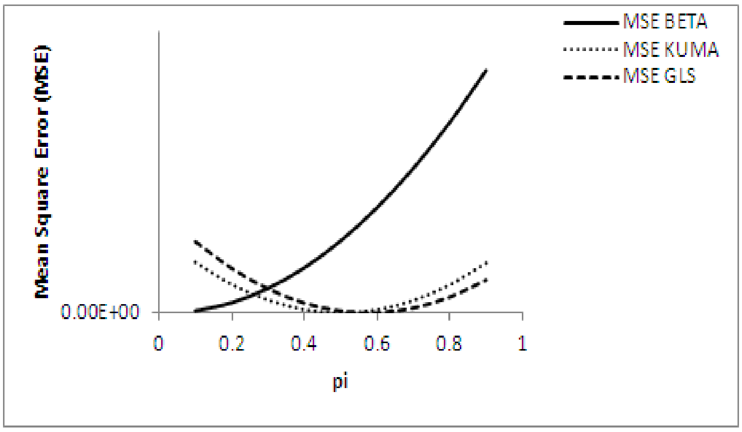

Figure 6a. Mean Square Errors (MSEs) for Hussain and Shabbir [9] RRT at
$n = 250, x = 106, \quad \alpha = 1, \quad \beta = 10, \quad P_1 = 0.2, P_2 = 0.8$

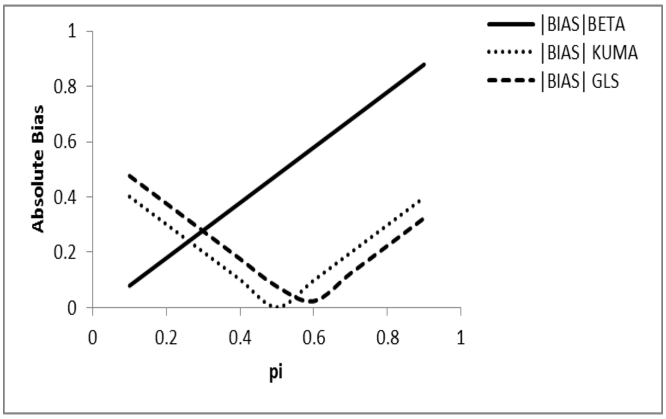

Figure 6b. Absolute Bias for Hussain and Shabbir [9] RRT at
$n = 250, x = 106, \alpha = 1, \ \beta = 10, P_1 = 0.2, P_2 = 0.8$

Comment: When $n = 250, P_1 = 0.2,$ the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.3$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.2 < \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.5 < \pi < 1$ respectively.

Results and Discussions

From the results presented in tables and figures 4.7.1a to 4.7.6b respectively, when $n = 25, P_1 = 0.1,$ the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi \leq 0.4$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.4 < \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.7 < \pi < 1.$

When $n = 25, P_1 = 0.2,$ the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.7 < \pi < 1.$

When $n = 100, 250, P_1 = 0.1,$ the proposed estimators are better than the conventional estimator when π lies within the range $0.1 \leq \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.1 \leq \pi < 1.$

When $n = 100, P_1 = 0.2,$ the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.6 < \pi < 1.$

When $n = 250, P_1 = 0.2,$ the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.3$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.2 < \pi < 1.$ However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.5 < \pi < 1$ respectively.

Conclusion

We have developed the alternative Bayesian estimation of the population proportion when real life data were gathered through the administration of survey questionnaires on an induced abortion on 300 matured women in some selected hospitals in the metropolisusing both Kumaraswamy (KUMA) and Generalised (GLS) Beta priors as our alternative beta prior distributions in addition to simple Beta prior distribution used by Hussain and Shabbir [10]. We observed clearly from the results presented in tables and figures above, that for small, intermediate as well as large sample sizes, the proposedBayesian estimators outperformed that of Hussain and Shabbir [10].

Acknowledgment

The authors thank the members of academic staff in the Department of Statistics, Federal University of Technology, Akure, Ondo State, Nigeria for their useful contributions during the preparation of this manuscript.

References

[1] Adepetun, A.O.,Adewara, A.A. (2014): *Bayesian Analysis of Kim and Warde Randomized Response Technique Using Alternative Priors*. American Journal of Computational and Applied Mathematics, 4(4): 130-140.

[2] Adebola, F.B. and Adepetun,A.O. (2011): *A new Tripartite Randomized Response Technique*. Journal of the Nigerian Association of Mathematical Physics, Volume 19:pp 119-122.

[3] Adebola,F.B. and Adepetun,A.O. (2012): *On a Qualitative Comparison of the Proposed Randomized Response Technique with Hussain and Shabbir (2007)*. International Journal of Mathematical Theory and Modeling, Volume 2:pp 61-67.

[4] Adepetun, A.O. and Adebola, F.B. (2014): *On the Relative Efficiency of the Proposed Reparametized Randomized Response Model*. International Journal of Mathematical Theory and Modeling, Volume 4:pp 58-67.

[5] Bar-Lev, S.K. Bobovich, E. and Boukai, B. (2003):*A common conjugate priorstructure for several randomized response models*. *Test*, 12(1), 101-113.

[6] Barabesi, L., Marcheselli, M. (2006): *A practical implementation and Bayesian estimation in Franklin's randomized response procedure*. Communication in Statistics- Simulation andComputation, 35, 365-573.

[7] Christofides, T.C. (2003): *A generalized randomized response technique*. Metrika, 57,195-200.

[8] Greenberg, B., Abul-Ela, A., Simmons, W., Horvitz, D. (1969): *The unrelated question randomized response: theoretical framework*. Journal of the American StatisticalAssociation, 64, 529-539.

[9] Hussain, Z. and Shabbir, J. (2007):*Randomized use of Warner's randomized response model*. InterStat: April # 7. http://interstat.statjournals.net/INDEX/Apr07.html

[10] Hussain, Z., Shabbir, J. (2009a): *Bayesian estimation of population proportion of a sensitive characteristic using simple Beta prior*. Pakistan Journal of Statistics, 25(1), 27-35.

[11] Hussain, Z., Shabbir, J. (2009b): *Bayesian Estimation of population proportion in Kim and Warde (2005) Mixed Randomized Response using Mixed Prior Distribution*. Journal ofprobability and Statistical Sciences, 7(1), 71-80.

[12] Hussain, Z., Shabbir, J. (2012): *Bayesian Estimation of population proportion in Kim and Warde Mixed Randomized Response Technique*. Electronic Journal of Applied Statistical Analysis, Vol. 5, Issue 2, 213 – 225.

[13] Kim, J. M., Tebbs, J. M., An, S. W. (2006): *Extension of Mangat's randomized response model*. Journal of Statistical Planning and Inference, 36(4), 1554-1567.

[14] Kim, J.M. and Warde, D.W. (2004): *A stratified Warner's Randomized ResponseModel*. J. Statist. Plann. Inference, 120(1-2), 155-165.

[15] Mangat, N.S. (1994): *An improved randomized response strategy*. J. Roy. Statist. Soc.Ser. B, 56(1), 93-95.

[16] Mangat, N.S. and Singh, R. (1990):*An alternative randomized response procedure*. Biometrika.77, 439-442.

[17] Migon, H., Tachibana, V. (1997): *Bayesian approximations in randomized response models*. Computational Statistics and Data Analysis, 24, 401-409.

[18] O'Hagan, A. (1987): *Bayes linear estimators for randomized response models*. Journal of the American Statistical Association, 82, 580-585.

[19] Oh, M. (1994): *Bayesian analysis of randomized response models*: *a Gibbs sampling approach*. Journal of the Korean Statistical Society, 23, 463-482.

[20] Pitz. G. (1980):*Bayesian analysis of randomized response models*. J. Psychological Bull, 87, 209-212.

[21] Singh et al (1998):*Estimation of stigmatized characteristics of a hidden gang in finite population*. Austral.& New ZealandJ. Statist, 40(3), 291-297.

[22] Spurrier, J., Padgett, W. (1980): *The application of Bayesian techniques in randomized response*. Sociological Methodology, 11, 533-544.

[23] Unnikrishnan, N., Kunte, S. (1999): *Bayesian analysis for randomized response models*. Sankhya, B, 61, 422-432.

[24] Warner, S.L. (1965): *Randomized Response: a survey technique for eliminatingevasive answer bias*. J. Amer. Statist. Assoc., 60, 63-69.

[25] Winkler, R., Franklin, L. (1979): *Warner's randomized response model: A Bayesian approach*. Journal of the American Statistical Association, 74, 207-214.