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Comparative Study of Adaptive Filtering Algorithms and the Equalization of Channel

Said Elkassimi[1], Said Safi[1] and B. Manout[2]

[1]Said Elkassimi, Said Safi are with Equipe de Traitement de L'information et de Télécommunications, Facultés des Sciences et Techniques, USMS, Béni Mellal, Maroc.

[2]B.Manout is with Laboratoire Interdisciplinaire de Recherche en Science et Technique (LIRST), USMS Béni Mellal, Maroc.

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ABSTRACT

This paper presented a study of four algorithms, the equalization algorithm to equalize the transmission channel with ZF and MMSE criteria, application of channel Bran A, and adaptive filtering algorithms LMS, NMLS and RLS to estimate the parameters of the equalizer filter, i.e. move to the channel estimation and therefore reflect the temporal variations of the channel, and reduce the error in the transmitted signal. So far the performance of the algorithm equalizer with ZF and MMSE criteria both in the case without noise, a comparison of performance of the LMS, NLMS and RLS algorithm.

Introduction

In modern digital communications, it is well known that channel equalization plays an important role in compensating channel distortion. Unfortunately, various channels have time varying characteristic and their transfer functions change with time. Furthermore, time-varying multipath interference and multiuser interference are two major limitations for high speed digital communications. Usually, adaptive equalizers are applied in order to cope with these issues [15]. For adaptive channel equalization, we need a suitable filter structure and proper adaptive algorithms. High-speed digital transmissions mostly suffer from inter-symbol interference (ISI) and additive noise. The adaptive equalization algorithms recursively determine the filter coefficients in order to eliminate the effects of noise and ISI. Adaptive filtering [6] is based on finding optimal parameters by minimizing a performance criterion. Frequently, this minimization is done by seeking the least squares. The performances of digital transmission system [3][9] are expressed in terms of reliability. This may be achieved by:

- the coding of channel, or correct coding of error,
- equalization, which allows to make the most the pass band of the channel offsetting receipt [8] the distortions introduced by the transmission medium, electronic equipment, etc...

There are two approaches:

- the adaptive approach to switch to the channel estimation [11] and therefore take into account the temporal variations of the channel,
- A suboptimal approach called LEVELS.

In this paper, we study the adaptive filtering algorithms such as LMS, NLMS and RLS algorithms to estimate of the FIR filter h_E in the noisy cases [4], and the comparison between the LMS, NLMS and RLS. LMS algorithm is significantly have slow convergence and poor tracking as compare to the normalized least-mean-square (NLMS), and the Recursive Least Square (RLS) algorithms, and even with perfect knowledge of the channel and noise power would be susceptible to mis convergence. And we will make the equalization algorithm comparison based with ZF and MMSE criteria [7].

Adaptive Equalization

The equalization approach has some drawbacks related to the need for accurate channel estimation and calculation of the correlation matrix of the received data and its inverse [5]. On the other hand, if the channel varies in time, this approach does not allow adjusting the coefficients of the equalizer [1].

In fact, the transversal equalizer on the MSE criterion is based on minimizing the function:

$$J(h_E) = E[(a_k - z_k)^2] \quad (1)$$

It is therefore necessary to calculate the gradient as:

$$\nabla J(h_E) = 2(R_y H_E - R_{ay}) = 0 \quad (2)$$

This leads to a complexity in costly analytical solution:

$$H_E = R_y^{-1}.R_{ay} \quad (3)$$

In the adaptive approach, one can dispense with the channel estimation and therefore take into account the temporal variations of the channel [10]:

LMS (Least Mean Square) Algorithm

In the implementation of the MSE criterion, an alternative to avoid reverse of R_y is to apply an iterative method to calculate the coefficients that minimize the cost function: $J(h_E)$.

From the values of $h_E(k-1)$ the values can be calculated from $h_E(k)$ using the algorithm of the gradient

Tele:
E-mail address: saidelkassimi@gmail.com

$$h_E(k) = h_E(k-1) + \mu(R_{ay} - R_y h_E(k-1)) \tag{4}$$

With μ positive constant called the coefficient adaptation (replacing R_y^{-1}) for controlling the convergence. However, the calculation of $\nabla J(h_E^{(n)})$ always requires knowledge R_y and R_{ay} by using a training sequence.

It then modifies the algorithm by replacing the gradient by its estimated (LMS is a gradient algorithm called "stochastic" and not deterministic). Is replaced at each step R_y and R_{ay} estimated by $y_k \cdot y_k^T$ and $a_k . Y_k$. The equation becomes:

$$h_E(k) = h_E(k-1) + \mu\left(a_k - y_k^T h_E(k-1)\right) y_k$$
$$= h_E(k-1) + \mu(a_k - z_k) y_k = h_E(k-1)\mu e_k y_k \tag{5}$$

The error signal e_k represents the desired difference between the data at time k and the actual output $z(kT)$.

The LMS allows every moment to "update" the equalizer filter coefficients in proportion to the estimation error e_k.

In case of variations of the channel, the equalizer will be able to adapt more rapidly than the constant μ is greater. It can be assumed that an adequate value μ to ensure convergence in the case of channels with slow variations is: $\mu = \frac{0.2}{(P_s+P_n)(2N+1)}$ avec (2N+1) number of coefficients of the equalizer, P_s signal power and P_n noise power. We can summarize the LMS algorithm in the following diagram:

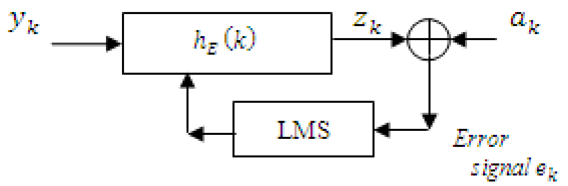

Figure 1. LMS Algorithm diagram

NLMS (Normalized Least Mean Square) Algorithm

As the step size parameter is chosen based on the current input values, the NLMS algorithm shows far greater stability with unknown signals [13]. This combined with good convergence speed and relative computational simplicity make the NLMS algorithm ideal for the real time adaptive echo cancellation system. As the NLMS is an extension of the standard LMS algorithm, the NLMS algorithms practical implementation is very similar to that of the LMS algorithm. Each iteration of the NLMS algorithm requires these steps in the following order [14].

a) The output of the adaptive filter is calculated

$$y(k) = \sum_{i=1}^{N-1} h_E(k)\, a(k-i) = h^T(k)\, a(k) \tag{6}$$

b) An error signal is calculated as the difference between the desired signal and the filter output

$e(k) = d(k) - y(k)$

c) The step size value for the input vector is calculated

$$\mu(k) = \frac{1}{a^T(k)\, a(k)} \tag{7}$$

d) The filter tap weights are updated in preparation for the next iteration.

$$h_E(k) = h_E(k-1) + \mu e(k)\, a(k) \tag{8}$$

Each iteration of the NLMS algorithm requires 3N+1 multiplications, this is only N more than the standard LMS algorithm. This is an acceptable increase considering the gains in stability and echo attenuation achieve.

RLS (Recursive Least Square) Algorithm

The basic algorithm of the stochastic gradient is LMS wherein the vector is approximated by a gradient from the estimation data. However, when the channel has a very even spread impulse response; the LMS converges very slowly due to a single parameter control (no adaptation). Can implement algorithms that are faster at the cost of some complexity. This is the case of the RLS algorithm.

It comprises:

a) Calculating the error signal at time kT dependent coefficients at instant $(k-1)T$ previous:

$$e_k = a_k - z_k = a_k - y_k^T h_E(k-1) \tag{9}$$

b) Update the coefficients:

$$h_E(k) = h_E(k-1) + \mu P(k) y_k (a_k - z_k) \tag{10}$$

The difference from the LMS is within the term $P(k)$; is an estimate of R_y^{-1} obtained recursively:

$$P(k) = \frac{1}{1-\mu}\left(P(k-1) - \frac{\mu P(k-1)\, y_k\, y_k^T P(k-1)}{1-\mu+\mu\, y_k^T\, P(k-1)\, y_k}\right) \tag{11}$$

The term $P(k)$ makes optimum use of the various coefficients which explains the superiority of the RLS algorithm in terms of speed of convergence.

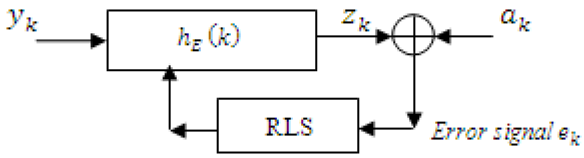

Figure 2. RLS Algorithm diagram

Equalization Algorithm

Samples received are written by:

$$y_k = \sum_n a_n g_{k-n} + \tilde{w}_k = a_k g_0 \sum_{n \neq k} a_n g_{k-n} + \tilde{w}_k$$

Or $\tilde{W}_k$ is a sample of additive Gaussian noise centered (AWGN) of variance $\sigma^2 = E(|\tilde{w}_k|^2)$.

The general idea is to apply an equalizer filter $H_E(k)$ to the samples y_k compensate for the equivalent channel $G(k)$.

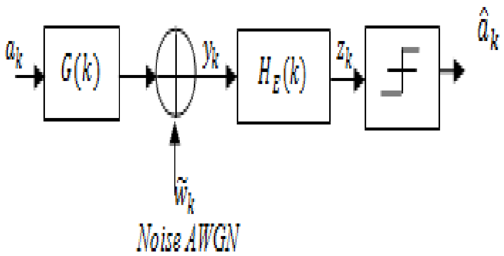

Figure 3. Equalization Algorithm diagram

The problem is: what criteria to choose $H_E(k)$?

Consider a transverse filter for $(2N+1)$ coefficients, transverse equalizers [12] are the easiest to implement. Indeed, this is simply to use a digital finite impulse response filter [9] for which the methods of calculation and implementation are well known.

$$z(k) = \sum_{n=-N}^{+N} y(-n) h_{E,n} \quad (12)$$

k represents the time flowing from -2N à 2N for (2N+1) input samples. We can write the relation of convolution matrix form:

$Z = Y.H_E$. With:

$$Z = \begin{pmatrix} z(-2N) \\ z(-2N+1) \\ \vdots \\ z(0) \\ \vdots \\ z(2N) \end{pmatrix} \quad (13)$$

column vector of dimension (4N+1)

$$H_E = \begin{pmatrix} h_{E,-N} \\ h_{E,-N+1} \\ \vdots \\ h_{E,0} \\ \vdots \\ h_{E,N} \end{pmatrix} \quad (14)$$

column vector of dimension (2N+1)

$$Y = \begin{pmatrix} y(-N) & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ y(-N+1) & y(-N) & 0 & 0 & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y(N) & y(N-1) & y(N-2) & \square & \square & y(-N+1) & y(-N) \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & 0 & y(N-1) \\ 0 & 0 & \cdots & \cdots & \cdots & & y(N) \end{pmatrix} \quad (15)$$

Is a matrix of dimension $(4N+1) \times (2N+1)$ the purpose of the equalization algorithm [7] is to determine the coefficients $\{h_{E,n}\}$ to minimize the error probability Pe, and remove the IES; this algorithm is based on the criteria «Zéro-Forcing» (ZF) and «Minimum Mean Square Error» (MMSE).

Simulation and Comparison

Performance of the Algorithm of the Equalizer

The Equalization Algorithm Based on ZF Criterion

ZF criterion is applied with equalization, for comparing the output of the equalizer, with the channel Bran A [3] response, in the environment noise [10 and 30 dB]

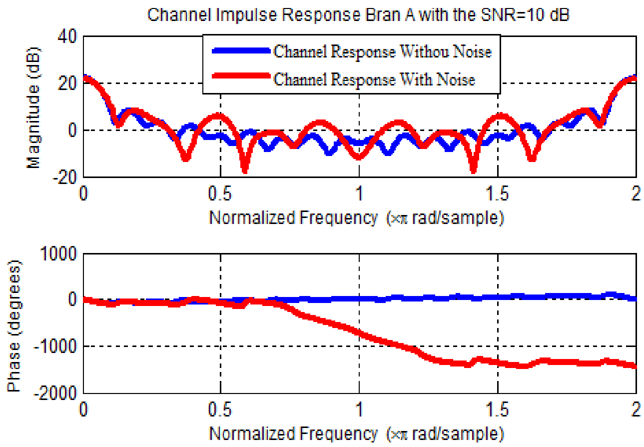

Figure 4. Channel impulse response Bran A, with the SNR=10 dB

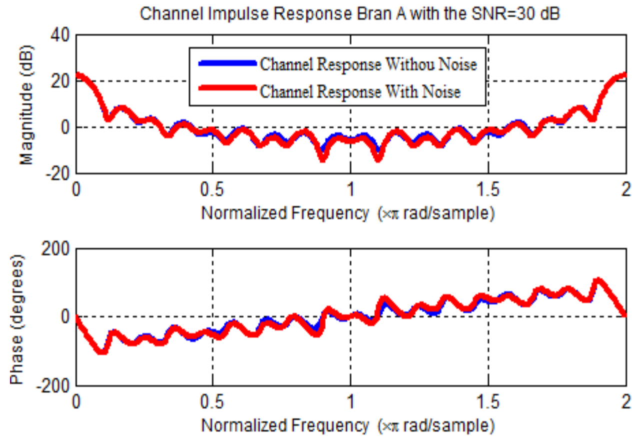

Figure 5. Channel impulse response Bran A, with the SNR=30 dB

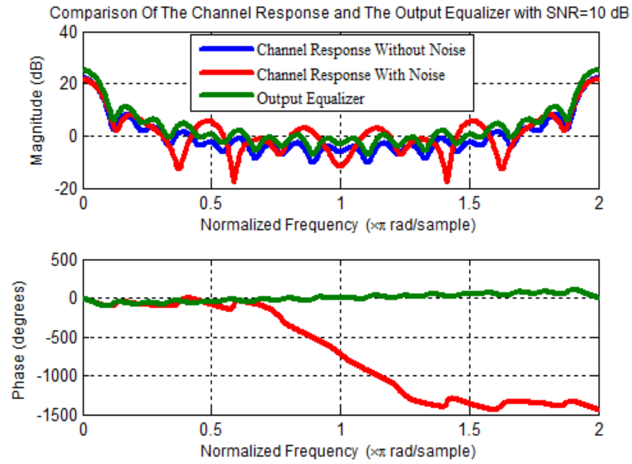

Figure 6. Comparison of the channel response Bran A; and the sortie equalizer with the ZF criterion in the SNR=10 dB cases

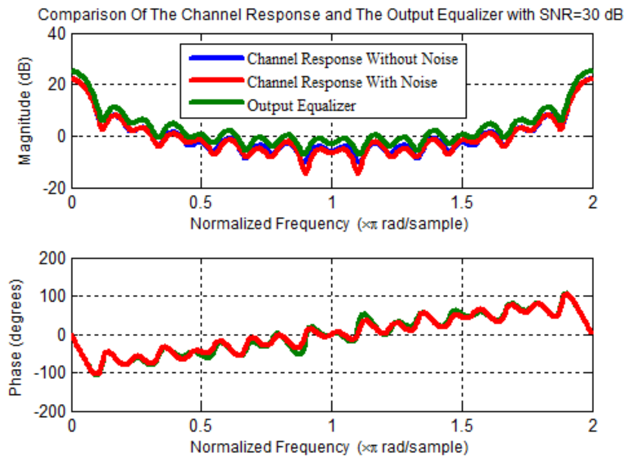

Figure 7. Comparison of the channel response Bran A; and the sortie equalizer with the ZF criterion in the SNR=30 dB cases

From results obtained it can be seen that the algorithm of the equalizer with ZF criterion gives a satisfactory equalization Bran A channel, consequently, it reduces the effect of noise.

a. The Equalization Algorithm Based on MMSE Criterion

We test the performance of the algorithm equalizer with MMSE criterion, with and without noise, to the Bran A channel; values of the SNR by 10 and 30 dB.

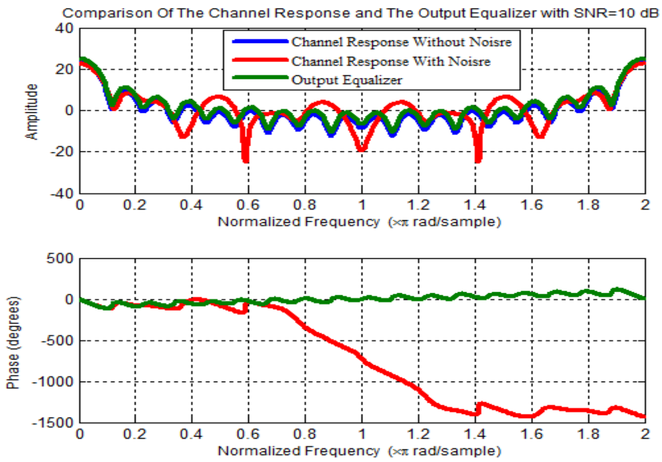

Figure 8. Comparison of the channel response Bran A; and the sortie equalizer with the MMSE criterion in the SNR=10 dB cases

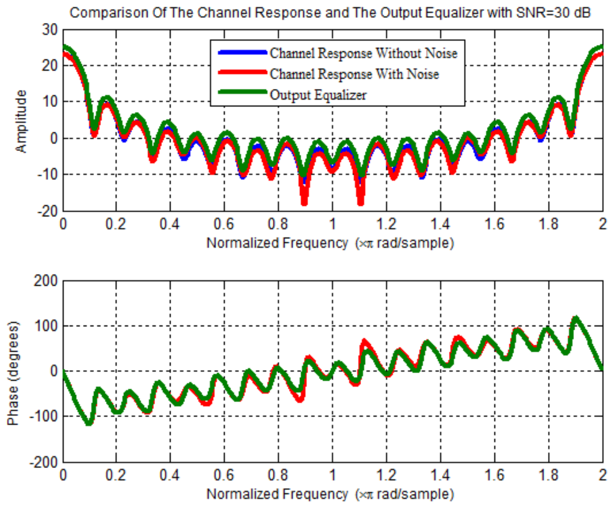

Figure 9. Comparison of the channel response Bran A and the sortie equalizer with the MMSE criterion in the SNR=30 dB cases

The algorithm equalization with the MMSE criteria; gives a good equalization of Bran A channel; then the criterion of Mean Square Error (MMSE) criterion is a more robust with respect to noise. It enables a compromise between reducing noise and the interference between the symbols (IES) (Fig. 8).

Comparison of Results

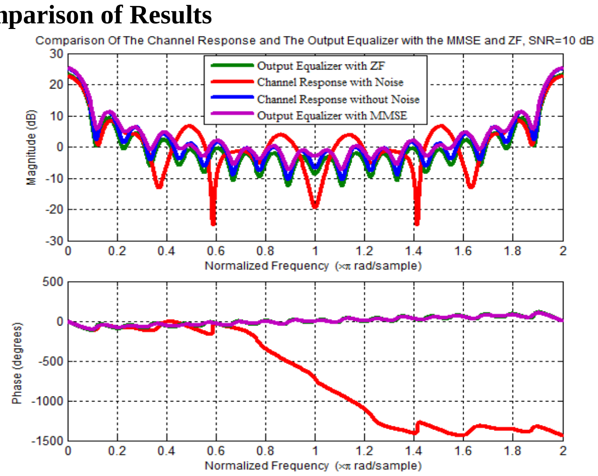

Figure 10. Comparison of the channel response Bran A, and the sortie Equalizer with the MMSE and ZF criterion in the SNR=10 dB cases

From the simulation results, we see that the equalizer obtained by the criterion of MMSE is better than that provided by the criterion ZF, due to the effective inclusion of noise.

Performance of the LMS, NLMS and RLS Algorithms

In this section we will make a comparison between the two algorithms of the LMS adaptive equalization and RLS are studied previously for that. Consider the channel Proakis (B) [2], and a modulation amplitude states 4 (4-ASK), with equalization coefficients 9.

It was found by applying the algorithm of the equalizer coefficient values for SNR=50 dB and the two ZF and MMSE criteria:

Table I. Coefficients Calculated by the Algorithm of the Equalizer with the MMSE Criterion

Coefficients EQM : h_E				
0.0652	-0.1480	0.2814	-0.4215	1.4793
-0.4228	0.2832	-0.1524	0.0705	

Table II. Coefficients Calculated by the Algorithm of the Equalizer with ZF Criterion

Coefficients ZF : h_E				
0.0816	-0.1640	0.2958	-0.4346	1.4921
-0.4365	0.2986	-0.1698	0.0883	

Performance of the LMS Algorithm

The values of the coefficients h_E calculated by the LMS algorithm at the last iteration are:

Table III. The Coefficients Calculated by the LMS Adaptation Algorithm with $u = 0.0053$

0.0606	-0.1416	0.2679	0.2679	1.4707
-0.4099	0.2667	-0.1381	0.0603	

From Fig. 11 we see that the error signal e_k is low when the number of iterations is important (M=7000). And from Fig. 12 and Fig. 13 we notice that for a low pitch results in slow convergence.

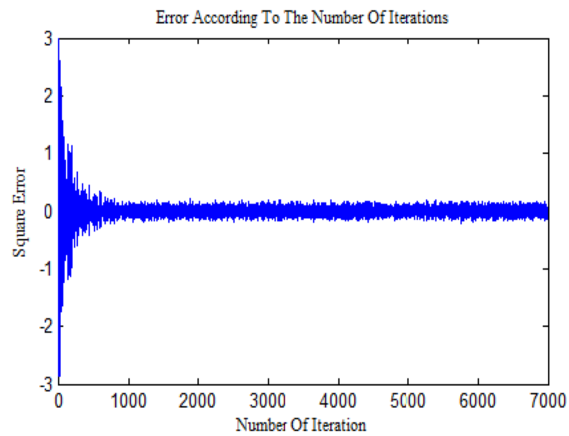

Figure 11. The variation of the error e_k against the number of iterations M=7000 with u= 0.0053

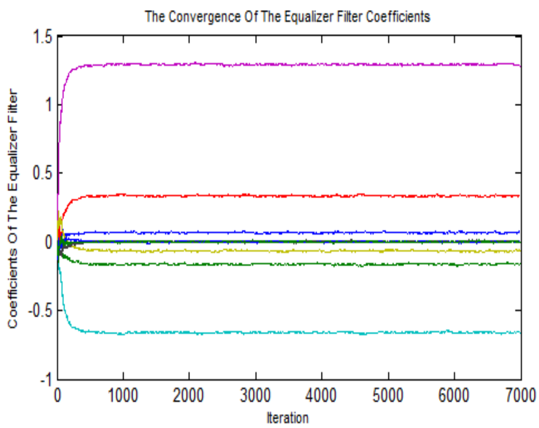

Figure 12. The convergence of the equalizer filter coefficients with no convergence of the LMS, u= 0.0053

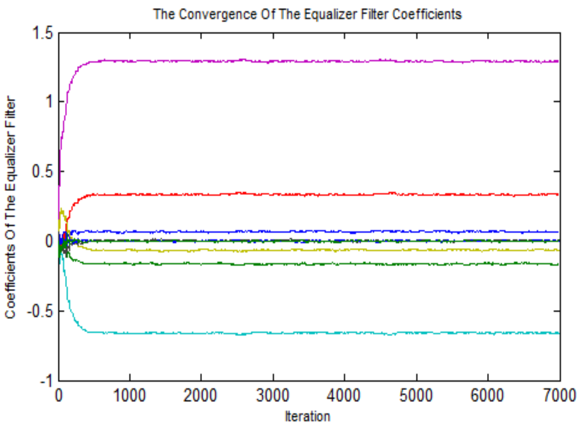

Figure 13. The convergence of the equalizer filter coefficients with no convergence of the LMS, u= 0.002

A strong will not lead to closer than results obtained by the algorithm equalization with criterion MMSE (Tables I and III). The LMS allows every moment to "update" the equalizer filter coefficients in proportion to the estimation error e_k. In case of variations of the channel, the equalizer will be able to adapt more rapidly than the constant u is large.

Performance of the NLMS Algorithm

The values of the coefficients h_E calculated by the LMS algorithm at the last iteration are:

Table IV. The Coefficients Calculated by the LMS Adaptation Algorithm with u = 0.0053

0.0634	-0.1536	0.3318	-0.6546	1.2883
-0.0626	0.0028	-0.0042	0.0055	

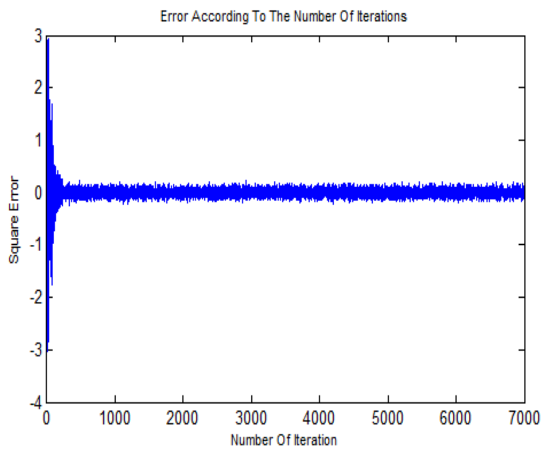

Figure 14. The variation of the error e_k against the number of iterations M=7000 with u= 0.0053

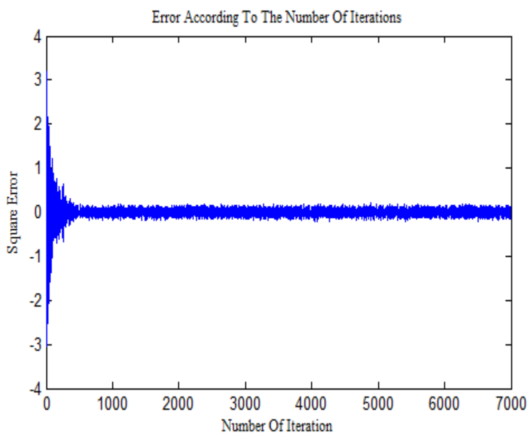

Figure 15. The variation of the error e_k against the number of iterations M=7000 with u= 0.002

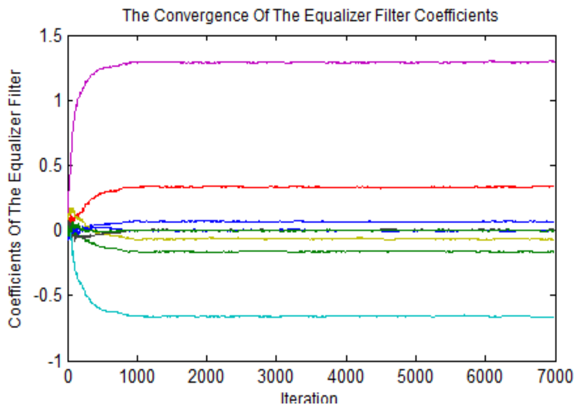

Figure 16. The convergence of the equalizer filter coefficients with no convergence of the LMS, u= 0.0053

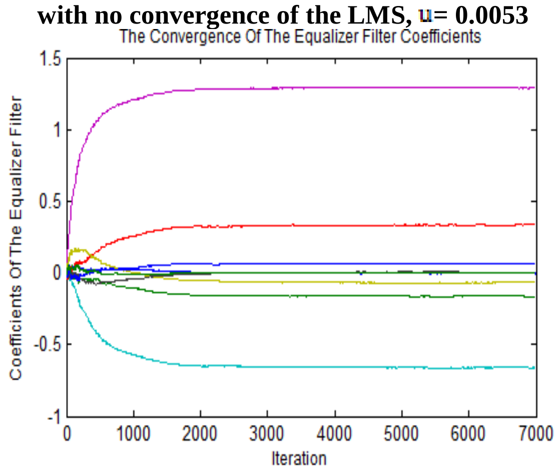

Figure 17. The convergence of the equalizer filter coefficients with no convergence of the LMS, u= 0.002

From Fig. 14 and Fig.15 we see that the error signal e_k is low when the number of iterations is important (M=7000). And from Fig. 12 and Fig. 13 we notice that for a low pitch results in slow convergence. A strong will not lead to closer than results obtained by the algorithm equalization with criterion MMSE (Tables I and IV). The NLMS allows every moment to "update" the equalizer filter coefficients in proportion to the estimation error e_k. In case of variations of the channel, the equalizer will be able to adapt more rapidly than the constant u is large.

Performance of the RLS Algorithm

The values of the coefficients h_E calculated by the RLS algorithm adaptation at the last iteration are:

Table V. The Coefficients Calculated by the Adaptation Algorithm with RLS, u= 0.0053.

0.0666	-0.1510	0.3214	0.4350	1.4689
-0.4149	0.2797	-0.1491	0.0686	

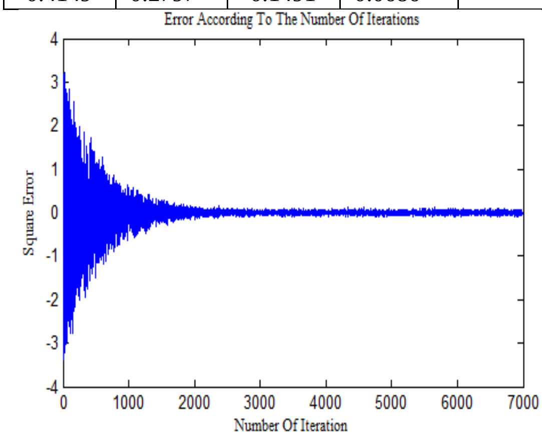

Figure 18. The variation of the error e_k against the number of iterations M=7000 with u= 0.0053

Figs. 14 and 15 show different results with u= 0.0053 and u= 0.002, we note that the estimate of the error e_k is tends to rapidly to low values when the number of iterations M and u are stronger. Then filter the RLS algorithm is performed correctly, it means that all influences of the noise was suppressed.

The curves in Figs. 16 and 17 shows the variation of the filter coefficients depending on numbers of iterations, we find that for a low pitch, slow convergence is obtained.

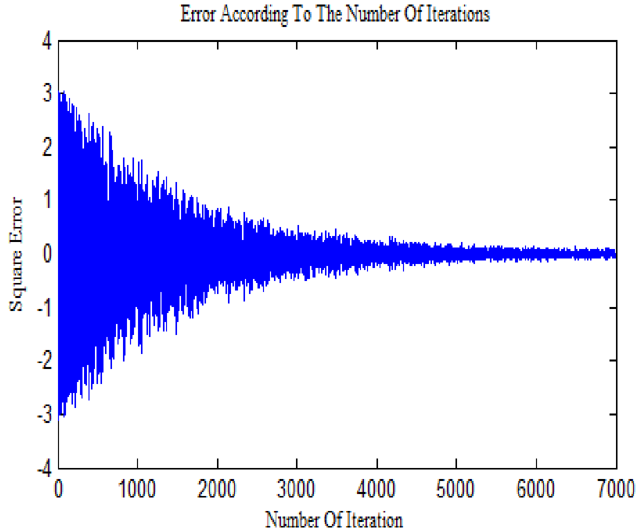

Figure 19, The variation of the error e_k against the number of iterations M=7000 with u= 0.002

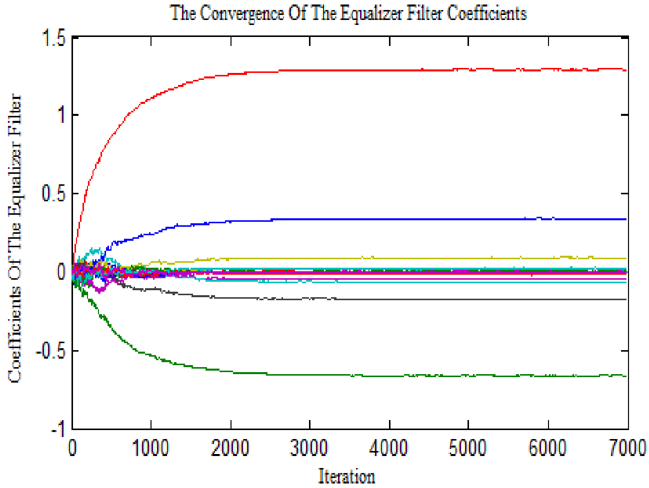

Figure 20. The convergence of the equalizer filter coefficients with u= 0.0053

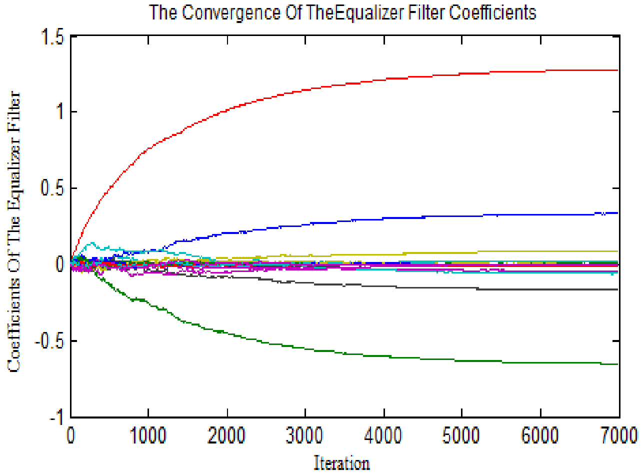

Figure 21. The convergence of the equalizer filter coefficients with u= 0.002

Comparison between the LMS, NLMS and RLS

From simulation results, we see that the RLS converges quickly compared to the LMS and NLMS algorithm because only one control parameter (the u adaptation) and will lead to results closer to that obtained by the algorithm of the equalizer with the MMSE criterion (Tables I, III, IV and V). There is another difference between the LMS, NLMS and RLS is in the term $P(k)$, which allows you to update various coefficients and gives the superiority of the RLS algorithm in terms of speed of convergence but time is running slower.

Conclusion

In this paper we presented four algorithms, the first algorithm to equalize the channel Bran A; with the two criteria ZF and MMSE, and the other three algorithms for estimating the parameters of the equalizer filter adjust the channel and reduce the error signal.

Simulation results show that the algorithm of the equalizer is able to equalize the channel Bran A with the MMSE criterion, due to the effective inclusion of noise. Thus the RLS adaptive filter algorithm converges quickly with respect to the LMS and the NLMS algorithm because of the adaptation step, another difference between the LMS, NMLS and RLS is within the term $P(k)$, which gives a superiority of RLS algorithm in terms of speed of convergence but time is running slower.

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Authors

Said Elkassimi obtained the B.Sc. degree in Physics and M.Sc. degree in computer science from polydisciplinary faculty, in 2009 and from Faculty of Science and Technics Beni Mellal, Morocco, in 2012, respectively. Now he is Ph.D student and his research interests include communications, wide-band wireless communication systems, signal processing, and system identification.

Said Safi received the B.Sc.degree in Physics (option Electronics) from Cadi Ayyad University, Marrakech, Morocco in 1995, M.Sc. and Doctorate degrees from Chouaib Doukkali University and Cadi Ayyad Univerrsity, in 1997 and 2002, respectively. He has been a Professor of information theory and telecommunication systems at the National School for applied Sciences, Tangier, Morocco, from 2003 to 2005. Since 2006, he is a Professor of applied mathematics and programming at the Faculty of Science and Technics, Beni Mellal, Morocco. In 2008 he received the Ph.D. degree in Telecommunication and Informatics from the Cadi Ayyad University. His general interests span the areas of communications and signal processing, estimation, time-series analysis, and system identification – subjects on which he has published 14 journal papers and more than 60 conference papers. Current research topics focus on transmitter and receiver diversity techniques for single- and multi-user fading communication channels, and wide-band wireless communication systems.

Bouzid Manaut received the B.Sc.degree in Physics (Theoritical Physics) from Cadi Ayyad University, Marrakech, Morocco in 1999, M.Sc. and Doctorate degrees from Cadi Ayyad Univerrsity and Moulay Ismail University, in 2002 and 2005, respectively. Professor of theoretical Physics from 2006 to now at the Polydisciplinary Faculty , Beni Mellal, Morocco. Ph.D. degree in Theoritical Physics, in 2011, from the Faculty of Science and Technics, USMS, Beni Mellal, Morocco. Current research topics focus on Theoritical Physics.