Available online at www.elixirpublishers.com (Elixir International Journal)

Chemical Physics Letters

Elixir Chem. Phys. Letters 97 (2016) 42214-42220

ISSN: 2229-712X

Determination of Mean free Path

Jasim Mohammed Salih Ali, Mohammed Mohsin Shneter, Saadallah Taha Idan, Ahmed Ibrahim Alwan and Ibrahim Kaittan Fayyadh

Ministry of Science and Technology.

ARTICLE INFO

Article history:

Received: 23 May 2016;
Received in revised form: 14 August 2016;
Accepted: 19 August 2016;

Keywords

Scattering theory,
Vacuum,
Optics,
Mean free path ,
Gas phase kinetics,
Physical chemistry,
Molecular collisions.

ABSTRACT

The mean free path, in edition, gas density ratio, molecule radius average time between collisions average velocity of molecule, and the diameter of the molecule for both Maxwell and Druyvestyn Velocities distribution law are performed by numerically solving the Boltzmann transport equation. This achieved for helium and nitrogen gases under influence the applied electric field to the gas pressure ratio ,E/P, between ($3.9131x10^{-3}$-0.9767) and (1.611-16.115) (V cm^{-1} $Torr^{-1}$) respectively at 300^0 K. The obtained results are agreement with the experimental data.

1. Introduction

When the imagine gas diffuses and spread into the environment, because gas molecules collide with each other, causing them to change in speed and direction. Therefore, they can never move in a straight path without interruptions. Between every two consecutive collisions, a gas molecule travels a straight path. The average distance of all the paths of a molecule is the mean free path. For Example ; a ball traveling in a box where the ball represents a moving molecule. Every time it hits the wall, a collision occurs and the direction of the ball changes.

In figure (1), the ball hits the wall four times, causing four collisions. Between every two consecutive collisions, the ball travels an individual path.It travels a total of three paths between the four collisions;each path has a specific distance,d.The mean free path of this ball is the average distance of all three paths, which is [1,2,3].

$$\lambda = \frac{d1+d2+d3}{3} \qquad (1)$$

where λ refers the mean free path

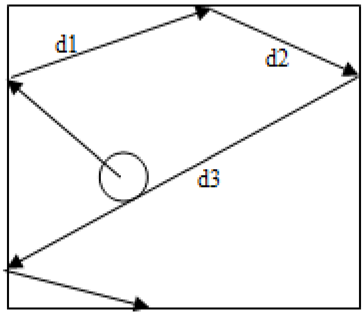

Figure 1. The diagram of mean free path.

The concept of the mean free path for the particles is the common used in the physics, such as, in kinetic theory, in acoustics, in optics, in radiography, in particle physics, and in nuclear physics [4,5].

2. Derivation

Consider a beam of particles being shot though a target, and assume an infinitesimally thin slab of the target Figure (2).

The atoms (or particles) that might stop a beam particle.The value of mean free path depends on the characteristic of the system the particle is [6]:

Tele:
E-mail address: Ikaittan@yahoo.com

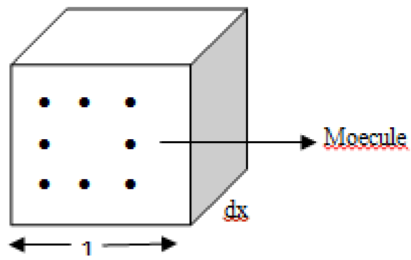

Figure 2. Target slab.

$$\ell = \frac{1}{\sigma n} \tag{2}$$

whereas ℓ refers to the mean free path, n refers to the number of the target particles per unit volume, and σ refers to the effective cross sectional area for collision. The area of the slab is L^2 and its volume is L^2dx. The number of stopping atoms in the slab is the concentration n times the volume. i.e. nL^2dx. The probability that beam particle will be stopped in that slab is the net area of the stopping atoms divided by the total area of the slab.

$$P = \frac{Area_{atoms}}{Area_{slab}} \quad (stopping\ within\ dx)$$

$$\frac{\sigma n L^2 dx}{L^2} = n\sigma dx \tag{3}$$

where σ refers to the area (or, more formally, the "scattering cross section") of one atom.

The drop in beam intensity equals the incoming beam intensity multiplied by the probability of being stopped within the slab:

$$dI = -In\sigma\, dx \tag{4}$$

this equation is called ordinary differential equation. By simplified the Eq.(4) yields:

$$\frac{dI}{dx} = -In\sigma \tag{5}$$

substitute Eq.(2) into Eq. (5) yields:

$$\frac{dI}{dx} = -\frac{I}{\ell} \tag{6}$$

the solution of the Eq. (6) is known as:

$$I = I_o e^{-\frac{x}{\ell}} \tag{7}$$

where x is the distance traveled by the beam through the target and I_o is the beam intensity before it entered the target, ℓ is called the mean free path because it equals the mean distance traveled by a particle beam before being stopped. The probability that a particle is absorbed between x and x+dx is given by:

$$dp(x) = \frac{I(x) - I(x+dx)}{I_o} = \frac{I}{\ell} e^{-x/\ell} dx \tag{8}$$

the value average x is :

$$\langle X \rangle = \int_0^\infty x\,dp(x) \tag{9}$$

substitute Eq.(8)into Eq.(9) yields:

$$\langle X \rangle = \int_0^\infty x \frac{I}{\ell} e^{-x/\ell} dx$$

$$\langle X \rangle = \ell \tag{10}$$

where x is equal to the thickness of the slab x=dx, which is the fraction of particles that were not stopped (attenuated)by the slab called transmission

T= $\frac{I}{I_o} = \exp(-x/\ell)$.

3. Theoretical background

Consider the steady-state f^o distribution for the electric swarm in an applied uniform electric field strength as[7,8]:

$$\frac{1}{2\nu^2}\frac{1}{\partial\nu}\left\{G\nu_m\nu^3\left[f^o+\left\{\frac{kT_g}{m}+\frac{2}{3G}\left(\frac{eE}{m\nu_m}\right)^2\frac{1}{\nu}\frac{\partial f^o}{\partial\nu}\right\}\right]\right\}+\frac{1}{3}\frac{\partial}{\partial z}\left[\frac{eE}{m\nu_m}\nu\frac{\partial f^o}{\partial\nu}+\frac{1}{\nu^2}\frac{1}{\partial\nu}\left(\frac{eE}{m\nu_m}\nu^3 f^o\right)\right]$$

$$+\frac{\nu^2}{3\nu_m}\nabla_r^2 f^o=0 \qquad (11)$$

where ν is the electron velocity, G is the energy loss factor, ν_m is the momentum transfer collision frequency, k is the Boltzmann factor, T_g is the gas temperature, e is the columbic charge, and the electric field E is a long the negative z-axis direction .

4. Calculation of the transport coefficients

When solved the Boltzmann transport equation numerically (11) could be obtained electron average energy, $\langle u\rangle$ and the diffusion coefficient to the mobility ratio, D/μ.

5. Formulation of the problem

From equation (1) we cannot be calculate the mean free path by taking the average of all the paths because it is impossible to know the distance of each path traveled by a molecule. However we can calculate it according to the Maxwell-Boltzmann distribution law for molecular velocities by the following relations [9,10].

$$\lambda=\frac{\bar{\nu}}{\nu}$$

$$where$$

$$\bar{\nu}=\left(\frac{8KT}{\pi m}\right)^{\frac{1}{2}}$$

$$\nu=2.39\times10^{-9}nK_1 \qquad (Maxwell) \qquad (14)$$

$$=\ 2.22\times10^{-9}nK_T \qquad (Druyvesteyn) \qquad (15)$$

$$\langle u\rangle=\frac{KT}{e} \qquad \Rightarrow \qquad KT=e\langle u\rangle$$

$$K_1=\frac{e}{KT_g}\frac{D}{\mu} \qquad (16)$$

$$K_T=0.877K_1 \qquad (17)$$

substitute Eqs. (13-17) into Eq(12 gives:

$$\lambda=\frac{\bar{\nu}}{2.39\times10^{-9}nK_1} \qquad (Maxwell) \qquad (18)$$

$$\lambda=\frac{\bar{\nu}}{2.22\times10^{-9}nK_T} \qquad (Druyvesteyn) \qquad (19)$$

In edition, the collision frequency ,ν, can be find from the relation

$$\nu=\sqrt{2}\pi D^2\bar{\nu}\frac{N}{V} \qquad (20)$$

Eqs. (18) and (19) are represent the mean free path or average distance between collisions in terms of Maxwell and Druyvesteyn distribution law respectively.

Substitute Equation (20) into Equation (12) gives:

$$\lambda=\frac{\bar{\nu}}{4.4447D\bar{\nu}\left(\frac{N}{V}\right)}$$

$$\lambda=\frac{1}{4.4447D^2\left(\frac{N}{V}\right)} \qquad (21)$$

where

$\bar{\nu}$: average speed of the molecule, (cm/sec).

v : collision frequency, (sec^{-1}).

K : Boltzmann constant, (1.3805×10^{-23} J/°K).

T : gas temperature. Kelvin, (k°)
m : electron mass, (9.109×10^{-28} gm).
e : electronic charge, (1.602×10^{-19} coulomb).
n : concentration of the particles, (cm^{-3}).
K_1 : Townsend's energy factor, (eV).
K_T : loss energy factor, (eV).
D/μ : The ratio of the diffusion coefficient to the particle mobility, (eV).
D : diameter of the molecule, (cm).
d : The traveled distance between two consecutive collisions, (cm).
N/V : particle density, (gm/cm^3).
n/N : density ratio.
E/P : The ratio of the applied electric field to the gas pressure, ($V.cm^{-1}.torr^{-1}$).

$$\lambda = \frac{1}{4.447 D^2 (\frac{n}{N})} \qquad (22)$$

where

$$\frac{n}{N} = 10^{-7}\left(\frac{E}{P}\right)^2\left(\frac{kT}{e}\right)^2 \qquad (23)$$

since the factor (n/N) replace the factor (n/V),
from Eq.(22) we can find:

$$D^2 = \frac{1}{4.4447\lambda\left(\frac{n}{N}\right)} \qquad (24)$$

Substitute Eqs. (18,19) into Eq. (24) gives:

$$D^2 = \frac{1}{4.4447\left(\frac{n}{N}\right)\frac{\bar{v}}{2.39\times10^{-9}nK_1}} \qquad Maxwell \qquad (25)$$

$$D = \sqrt{\frac{1}{4.4447\left(\frac{n}{N}\right)\frac{\bar{v}}{2.39\times10^{-9}nK_1}}} \qquad Maxwell \qquad (26)$$

$$R = \frac{D}{2} \qquad Maxwell \qquad (27)$$

$$D^2 = \frac{1}{4.4447\left(\frac{n}{N}\right)\frac{\bar{v}}{2.22\times10^{-9}nK_T}} \qquad Druyvesteyn \qquad (28)$$

$$D = \sqrt{\frac{1}{4.4447\left(\frac{n}{N}\right)\frac{\bar{v}}{2.22\times10^{-9}nK_T}}} \qquad Druyvesteyn \qquad (29)$$

$$R = \frac{D}{2} \qquad Druyvesteyn \qquad (30)$$

where R refer to the radius of the molecule .

Equations (25) and (28) were represent the diameter of the molecule in terms of Maxwell and Druyvesteyn distribution law for velocities respectively and Equations (27) and (30) were represent the radius of the molecule in terms of Maxwell and Druyvesteyn distribution respectively.

We consider that the molecules interact like hard spheres, according to the kinetic theory of gases, it follows that[11]:

$$\tau = \frac{\lambda}{\bar{v}} \qquad (31)$$

where τ refers to the mean time between the collisions .
substitute Equations (18) and (19) into Equation (31) respectively, yields:

$$\tau = \frac{1}{2.39\times10^{-9} nK_1} \qquad Maxwell \qquad (32)$$

$$\tau = \frac{1}{2.22\times10^{-9} nK_T} \qquad Druyvesteyn \qquad (33)$$

Equations (32) and (33) were represent the average time between the collisions in terms of the Maxwell and Druvesteyn distribution respectively.

6. Results and Discussion

The molecules can never move in a straight path without interruptions because they collide with each other, causing them to change in speed and direction as showing in the figures. In kinetic energy mean free path or average distance between collisions for a gas molecule may be verified with the literature [6,12].

Figure(1) represents the mean free path of the molecule as a function of the average velocity, and according to the Maxwell-Bolzmann distribution velocities law as in equations(18) and (19) are showing the average distance traveled by a moving molecule between collisions could be increased at average velocities between (8.8×10^{4}-1.6×10^{5}) cm/sec for Helium gas and (1.089×10^{6}-5.752×10^{6}) cm/sec for Nitrogen gas, but reduces between (1.66×10^{5}-8.45×10^{5}) cm/sec for Helium gas.

Figure (2) was show the mean free path increase with the density ratio between (2.571×10^{-13}-3.616×10^{-12}) and (4.176×10^{-8}-6.679×10^{-7}) for Helium and Nitrogen respectively but after the point 2.444×10^{11} for Helium gas, the mean free path was decreased with increasing of the density ratio, this mean, the molecules were become closer to each other, when a gas density increases, for both Maxwell and Druyvestyn distribution law Helium(He) and Nitrogen(N_2) gases.

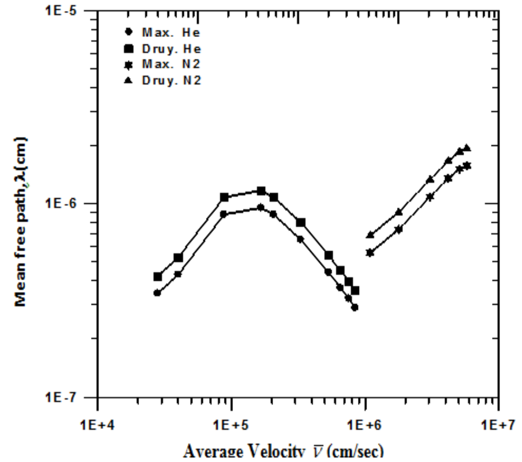

Fig 3. The mean free path, λ , as a function of the molecule average velocity, $\bar{v}$, for both Maxwell and Druyvestyn velocities distribution law in He and N_2 gases.

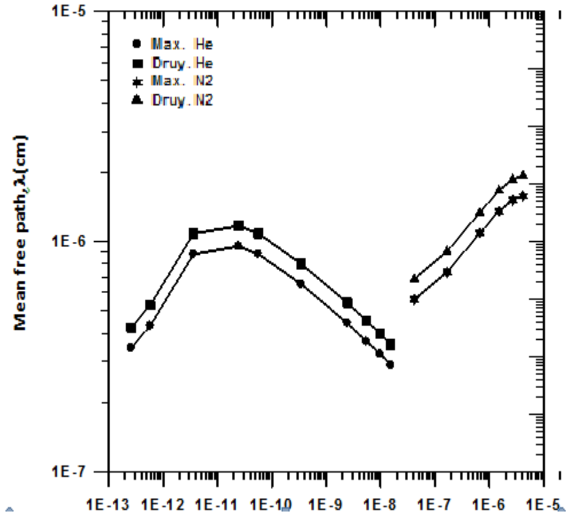

Fig 4. The mean free path ,λ, as a function of the density ratio , n/N,for both Maxwell and Druyvestyn velocities distribution lawin He and N_2 gases.

Figure (3) shows decreasing the radius of the molecules will increase the space between them, causing to run far each other more, wherefore mean free path increase.

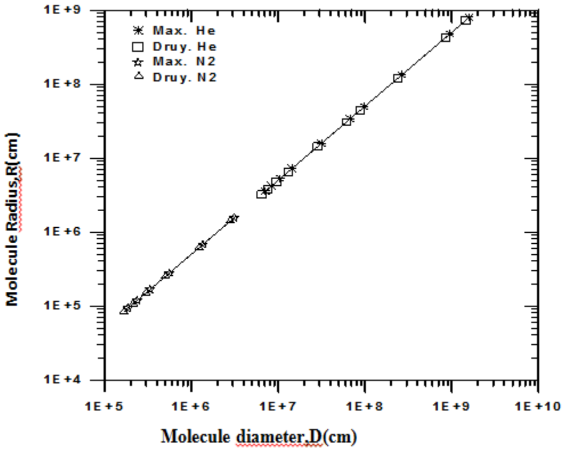

Fig 5. The radius of the molecule, R, as a function of molecule diameter ,D, for both Maxwell and Druyvestyn velocities distribution law in He and N_2 gases.

Figure (4) were appear the mean time between collisions as a function of the Townsend's energy factor, K_1 and the loss energy factor, K_T, therefore, according to the equation (32) and (33), the mean time decreases with increasing of K_1 and K_T .

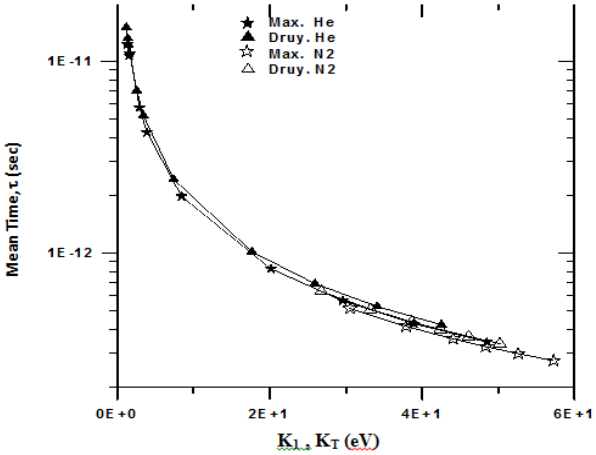

Fig 6. The mean time between the collision as a function of Townsend'senergy factor, K_1, and the loss energy factor , K_T, for both Maxwelland Druyvestyn velocities distribution law in He and N_2 gases.

7. Conculusion

I- Radius of molecule :increasing the radius of the molecules will decrease the space between them. Causing them to run into each other more, Therefore, mean free path decrease.

II- The molecules become closer to each other ,when a gas density increases. Therefore, They are more likely to run into each other, so the mean free decreases.

III- Another factors that affect density can indirectly affect mean free path as, temperature, pressure.

8. References

1. Atkins Perter and Julio de paulu (2006) Physics Chemistry for the life Sciences. New York, Oxford University press.

2. Cook, Norman D.(2010),Modells of the Atomic Nuclear, The mean free path of Nucleons in Nuclei, Heidel Berg:Springer, ed.2,Chap.5,pp.3243.

3. Change Raymond (2005) Physical Chemistry for the Biosciences. California, University Science Books.

4. Mengual O. (1999) TURBISCAN MA2000.multiple light scattering measurement For concentrated emulsion and suspension instability analysis, Talanta,Vol.50. No.445, doi: 10.016/S0039-9140(99).

5. Hubbell, J. H. Seltzer,S. M.," Tables of X-Ray Mass Attenuation Coefficients an Mass Energy-Absorption Coefficient. National Institute of Standards and Technology (NIST).Hu:/physicsnistgov/physRefDate/XrayMassCoef/coverhtm /Retrieved Septemper2007

6 .S Chapman and T.G. Cowling (1990) mathematical theory of non-unformed gases "3rd Edition- Cambridge University Press, pp. 88.

7. Ibrahim G. Faiadh. Mohammed I. Ismaeel, Faisal G. Harudy, saad S. Dawood(2011) Magnetic Deflection Coefficient Investigation For low Energy Particles, Baghdad Science journal, Vol.8, No.2, pp.302-317.

8 .S, D. Rockwood, A. E. Greene. (1980) Numerical Solutions the Boltzmann Transport Equation ".Computer Physics Communication Vol, 19, pp, 377-393.

9 . I. G. Faiadh, M. L. Ismael, F. G. Hamid, and S. S. Dawood, (2011) Applicab Studies of the Slow Electron Motion in Air with application in the Ionosphere, Eng. and Tech. Journal, Vol.29, No.3, 44.

10 . H. Dreicer (1961) Electron velocity Distribution in a partially ionized Gas, phys. Rev., Vol. 117, No.2, pp. 343-354.

11 . F. T. Bagnall and S. C. Haypon (1965) Pre –Breakdown Ionization in Molecula Nitrogen in EXB Field, Aust. J. phys.,Vol. 18,pp. 227-236.

12 .Jenning S. (1988) The mean free path in Air, J. Aerosol Science vol. 19, No.159, pp. 9021-90224.
13 . Atkins,peter and Jutio de paula.(2006) Physical Chemistry for the Life Sciences. New pork,Oxford University Press .
14 . R. schlickeiser,M. LaZar., and M. Vukcevic,(2010) The influence of Dissipation Range Scattering Mean Free Path Astronomical,J.Vol.719,No.2.