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Paradoxes in Electromagnetic Induction

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ABSTRACT

As a frequent phenomenon, paradoxes often occur in physics. In electromagnetic, paradox arises mainly due to the existence of bulk conductor in the loop. The paradoxes are analyzed by two experiments. The experimental results demonstrated that the law of flux should not be used to solve some problems, otherwise paradox may arise.

1.Introduction

A paradox is a statement that, despite apparently sound reasoning from true premises, leads to a self-contradictory or a logically unacceptable conclusion. Some logical paradoxes are known to be invalid arguments but are still valuable in promoting critical thinking [1]. A paradox is often used to make a reader think over an idea in innovative way.

Paradox is a frequent phenomenon present in physics [2-5]. There are many paradoxes in electromagnetic. On August 29th of 1831, Faraday first observed the phenomenon of electromagnetic induction, which marked the beginning of electricity linking with magnetism. As we know, electromagnetic induction can be expressed as[6]

$$\varepsilon = \varepsilon_M + \varepsilon_I = \oint_L (\vec{V} \times \vec{B}) \cdot d\vec{l} - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \tag{1}$$

or

$$\varepsilon = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S} \tag{2}$$

where ε_I is the induction electromotive force, ε_M is the motional electromotive force, $\vec{B}$ is the magnetic field, $\vec{V}$ is the velocity of a nonmagnetic conducting bar.

While Eq. (1) is the generally accepted expression of electromagnetic induction in physics, its application may be subject to some limitations. First, speed is the first derivative of displacement to time, which requires the displacement of conductor to be continuously differentiable throughout the process; second, the integral path L in the right first item is required to be the boundary of the integral surface S in the second item, integrated along the conductor loop, if the conductor loop is closed, then directly integrated, if the conductor loop is not closed, the wires should be added to form a closed loop then subtract the electromotive force as generated by the added wires; finally, the right second item requires the magnetic field $\vec{B}$ is the continuously differentiable function of time, and with first derivative.

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2. The Paradox Phenomenon in Electromagnetic Induction

The phenomenon of electromagnetic induction as discovered by Faraday links electricity with magnetism, the familiarity and application of the Law of Electromagnetic Induction appears to quite important in this society full of electricity and magnetism. Thus, electromagnetic induction is always the key point throughout our learning process, but the research on electromagnetic induction is a difficulty and the elusive part, especially some problems arising from the physics may yield different results, if Eqs. (1) and (2) as described above are applied to the same problem, also referred to as the paradox phenomenon of electromagnetic induction.

2.1 Two expressions for the Law of Electromagnetic Induction

The paradox phenomenon arises when Eqs. (1) and (2) are applied to solve the same problem. We carried two experiments to analyze the paradoxes in electromagnetic.

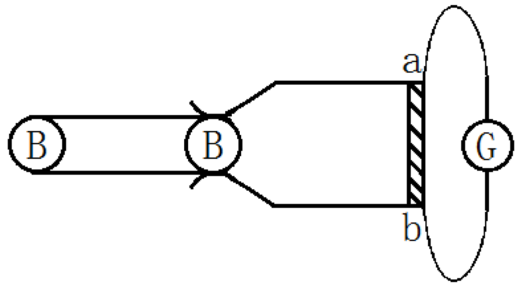

Fig1. Toroidal magnet.

In experiment 1, as shown in Fig.1, a metal ring is permanently magnetized along the axis of circular cross-section, and the brass spring clamp impacted by inrush current is connected to an insulating block, until it is encased with the magnetic ring. Then the brass spring clamp is pulled rightward or the toroidal magnet is pulled leftward, until a toroidal magnet is pulled out. Throughout this process, keep the path while removing the clamp out of the toroidal magnet, if by Eq. (2), the magnetic flux through the clamp loop changes from ϕ_m to zero, thereby generating electromotive force in the loop.

But by Eq. (1), because $\partial\vec{B}/\partial t=0$, and there is no magnetic field in the moving part of loop, then $\vec{v}\times\vec{B}=0$, so $\varepsilon=0$.

In experiment 2, a conductor loop is placed in the magnetic field of a long cylindrical magnetic body, as shown in Fig.2, G is a galvanometer. At first, K_2 is disconnected, K_1 is connected, $\phi=0$ in the loop composed of K_1 and galvanometer, and the switch in action is assumed to be done vertically. By Eq. (2), $\Delta\phi\neq 0$, so $\varepsilon\neq 0$. However, if by Eq. (1), $\partial\vec{B}/\partial t=0$ and each part of wires has no movement, i.e.: $v=0$, at the switch, $\vec{v}//\vec{B}$, so $\vec{v}\times\vec{B}=0$, then the electromotive force of the entire loop $\varepsilon=0$.

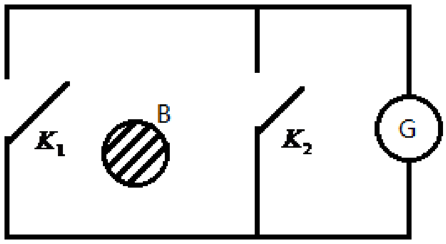

Fig 2. Conductor loop.

These two experiments are typical paradox of electromagnetic induction and it is always a difficulty in physics to examine the exact cause to the error of analysis as described above.

2.2 Discussion on Two Expressions for the Law of Electromagnetic Induction

It can be seen from the above two cases, Eqs. (1) and (2) are used to calculate the induced electromotive force for the above-mentioned two cases respectively, which yield conflicting results. From the angle of experiment, Eq. (1) is the superposition of electromotive force generated from these two experiments, or directly derived according to $\nabla\times\vec{E}=-\partial\vec{B}/\partial t=0$ in Maxwell's equations and $\vec{F}=e(\vec{E}+\vec{v}\times\vec{B})$ in Lorentz force, so Eq. (1) is constructed upon the strict theoretical basis. By the calculation using Eq. (1), the electromotive force can be divided into two items, the first item considers that the loop does not move, the induced electromotive force in the loop as $\vec{B}$ changes with the time should be a unique value, so long as $\vec{B}$ is the continuously differentiable second-order function. The second item is the motional electromotive force as induced by the partial or whole motion of conductor loop cutting off the magnetic flux, when the electromotive force is calculated only by integrating the moving part of conductor loop, namely $\varepsilon=\oint_{l}(\vec{v}\times\vec{B})\cdot d\vec{l}-\oint_{l'}(\vec{v}\times\vec{B})\cdot d\vec{l}'$, where l' is the effective part of conductor loop cutting off the magnetic flux. In performing this calculation, no uncertainty may appear whether one part of the loop includes a bulk conductor or not, or regardless of which loop the moving conductor belongs to, so the result is a unique one.

By the calculation of electromotive force in the loop using Eq. (2), some limitations must be added before obtaining a correct result.

First, the conductor loop as observed here must be linear, namely the cross-sectional area of conductors composing the loop is negligible, only in this way can the loop itself and the magnetic flux running through it have a precise meaning, so that the change of magnetic flux and its corresponding induced electromagnetic force can be uniquely determined. For example, as described in the above Paradox 1, when the brass clamp contacts the magnetic ring of conductor (see Fig.3), if the loop *abcdea* is selected, then the magnetic flux running through the loop is zero, if the loop *abcfea* is selected, then the magnetic flux running through the loop is φ_m, and there are many other loops between *abcdea* and *abcfea*, which leads to the uncertainty of loops and the magnetic fluxes running through them. Therefore, the change of magnetic flux becomes uncertain as soon as the brass clamp just contacts or detaches from the magnetic ring of conductor, the voltage between *ab* cannot be determined.

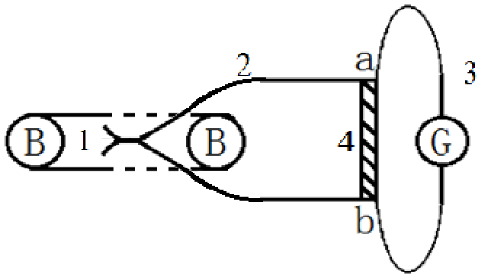

Fig 3. Toroidal magnet after change.

Second, in order to make $d\Phi/dt$ meaningful, it is required that:

(1) $\Phi=\iint_S\vec{B}\cdot d\vec{S}$ is a continuous function of the time t with first derivative, when the loop is fixed, $\vec{B}$ is the continuously derivative first-order function of time.

(2) $\vec{B}$ does not change with the time, but the partial or whole motion of loop in the magnetic field is with respect to the same whole loop or the same partially moving conductor, which cannot suddenly "switching path" in the motion of loop, only in this way can the change of magnetic flux continuously change with a precise meaning.

For example, as described in paradox 1, a magnetic ring of bulk conductor is inserted when the brass clamp is pulled out of the magnetic ring, but the loop cannot be uniquely determined, although the loop is selected, e.g.: *abcfea*, when the brass clamp slips through the magnetic ring, the *cfe* of the loop is suddenly removed, as if the loop *abcfea* suddenly becomes the loop only composed of brass clamps. Therefore, when the brass clamp is pulled out of the magnetic ring, the loop composed of brass clamps does not continuously change, which fails to meet the required condition by $\varepsilon=-d\Phi/dt$, so it causes the analysis error. Paradox 2 also arises from the magnetic flux running through the loop of K_1 after the switch change, instead of the change of original loop, so it is meaningless to examine the change of magnetic flux running through two different loops.

The above analysis suggests that paradox arises from two cases. One is that bulk conductor in the loop makes the change and its flux as well as the change of flux uncertain, the other is that required condition by Eq. (1) is not satisfied, so Eq. (2) in physics cannot be used as the general expression of electromagnetic induction. Because Eq. (2) is generally adopted in textbook, it is beneficial to point out the equivalent condition for Eqs. (1) and (2), by the foregoing analysis, the equivalent condition can be summarized as:

(1) The loop is linear, which means that it does not contain the bulk conductor;

(2) The magnetic flux ϕ_m is the continuous derivative function of time, which requires that "switching path" cannot

be suddenly done in the motional electromotive force, when the conductor loop partially or wholly moves;

(3) If the conductor loop is more than one, it is required that the calculation using Eqs. (1) and (2) should yield the same result, which further requires the boundary of the surface integral S to overlap with the periodic boundary of the line integral *L* in Eqs. (1) and (2). For example, in a fixed linear conductor loop, as long as $\vec{B}$ is the continuously differentiable function of time, Eqs. (1) and (2) turn to the same form $\varepsilon = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S}$. In such a case, Eqs. (1) and (2) are equivalent for solving the electromotive force of the same loop, when the loop is not limited to the conductor loop. For another example, when the conductor loop is a simple linear loop, whether this loop is translational or rotational in the magnetic field, the calculation using Eqs. (1) and (2) are also equivalent.

3 Conclusions

Paradox arises mainly due to the existence of bulk conductor in the loop, so the magnetic flux running through the loop and the change of flux are uncertain, the relevant problems of paradox could be reasonably explained only by selecting an appropriate loop and applying the law of magnetic flux to the analysis of cross-sectional conductor. Therefore, the problem of paradox arising from the electromagnetic induction can be always solved, and various circumstances of generating the induced electromotive force can be always attributed to the induced electromotive force caused by the change of magnetic flux, so long as an appropriate integral path is selected, whether for the loop composed of thin wires or the loop composed of cross-sectional conductors.

In conclusion, it is deemed as inappropriate to treat $\varepsilon_I = -\iint_S \frac{\partial\vec{B}}{\partial t}\cdot\overrightarrow{dS} = -\frac{d\Phi}{dt}$ as the basic law, although it comes from the fundamental law and fundamental equation of electromagnetic theory, only under certain condition it could be equivalent to the universal equation $\varepsilon_M = -\iint_S -\frac{\partial\vec{B}}{\partial t}\cdot\overrightarrow{dS} + \oint_L (\vec{v}\times\vec{B})\cdot\overrightarrow{dL}$. It is justifiably impossible to expect such law to solve all problems, since the law of flux is not a fundamental law. In other words, the law of flux should not be used to solve some difficulties, otherwise paradox may arise.

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