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Dufour effect on free convection MHD flow past an impulsively started vertical oscillating plate through porous media with variable temperature and constant mass diffusion

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ABSTRACT

Dufour effect on free convection MHD flow past an impulsively started vertical oscillating plate through porous media with variable temperature and constant mass diffusion is studied here. The Laplace transform technique has been used to find the solutions for the velocity profile, temperature profile and concentration profile. The results obtained are discussed with the help of graphs drawn for different parameters like thermal Grashof number, mass Grashof number, Prandtl number, permeability parameter, the Hartmann number, Schmidt number, Dufour number, time, inclination of magnetic field and phase angle.

Introduction

The effect of magnetic field on viscous, incompressible and electrically conducting fluid plays important role in many applications such as glass manufacturing, control processing, paper industry, textile industry, magnetic materials processing and purification of crude oil etc. Nelson and Wood [2] have studied combined heat and mass transfer natural convection between vertical parallel plates with uniform heat flux boundary conditions. MHD flow between two parallel plates with heat transfer have been studied by Attia and Katb [3]. Rajput and Kumar [10] have studied radiation effects on MHD flow through porous media past an impulsively started vertical oscillating plate with variable mass diffusion. Kesavaiah and Satyanarayana [13] have discussed radiation absorption and Dufour effects to MHD flow in vertical surface. V M Soundalgekar [1] have studied free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction. Dufour effect on unsteady MHD flow past an impulsively started inclined oscillating plate with variable temperature and mass diffusion have been studied by Rajput and Kumar [14]. Das and Jana [8] have studied heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Rajesh [9] has studied MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Sandeep and Sugunamma [12] have discussed effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Postelnicu [5] has studied influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam et al. [4] have studied Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Reddy [7] have discussed Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Dipak et al. [11] have studied Soret and Dufour effects on steady MHD convective flow past a continuously moving porous vertical plate. Ibrahim [6] has discussed analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect.

In this paper we are analyzing Dufour effect on free convection MHD flow past an impulsively started vertical oscillating plate through porous media with variable temperature and constant mass diffusion.

Mathematical Analysis

We have considered an unsteady viscous incompressible electrically conducting fluid past an impulsively started vertical plate with velocity u_0. The plate is electrically non-conducting. A uniform inclined magnetic field B_0 is applied on the plate with angle α from vertical. Initially the fluid and plate are at the same temperature T_∞ and the concentration of the fluid is C_∞. At time $t > 0$, temperature of the plate is raised to T_w, the concentration of the fluid is raised to C_w and the plate starts oscillating in its own plane with frequency ω.

The governing equations under the usual Boussinesq's approximations are as follows:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}\sin^2(\alpha)u - \frac{\upsilon}{K}u, \quad (1)$$

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$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2}, \tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}. \tag{3}$$

The initial and boundary conditions are given as:

$$\left.\begin{aligned}
&t\le 0; u=0, T=T_\infty, C=C_\infty \text{ for each value of } y,\\
&t>0; u=u_0\cos\omega t, T=T_\infty+(T_w-T_\infty)\frac{u_0^2 t}{\upsilon}, C=C_w \text{ at } y=0,\\
&u\to 0, T\to T_\infty, C\to C_\infty \text{ as } y\to\infty.
\end{aligned}\right\} \tag{4}$$

Here u is the velocity of the fluid, g – the acceleration due to gravity, β – volumetric coefficient of thermal expansion, β^* – volumetric coefficient of concentration expansion, t – time, T – the temperature of the fluid, T_∞ – the temperature of the plate at $y\to\infty$, C – species concentration in the fluid, C_∞ – species concentration at $y\to\infty$, υ – the kinematic viscosity, ρ – the density, C_P – the specific heat at constant pressure, k – thermal conductivity of the fluid, K_T – thermal diffusion ratio, D – the mass diffusion constant, D_m – the effective mass diffusivity rate, T_w – the temperature of the plate at $y=0$, C_w – species concentration at the plate at $y=0$, C_S – Concentration susceptibility B_0 – the uniform magnetic field, σ – electrical conductivity, K – permeability of the porous medium and α – angle of inclination from vertical.

By using the following dimensionless quantities, the above equations (1), (2), and (3) can be transformed into dimensionless form.

$$\left.\begin{aligned}
&\bar{y}=\frac{yu_0}{\upsilon}, \bar{t}=\frac{tu_0^2}{\upsilon}, \bar{u}=\frac{u}{u_0}, \theta=\frac{T-T_\infty}{T_w-T_\infty}, \bar{K}=\frac{Ku_0^2}{\upsilon^2}\\
&\bar{C}=\frac{C-C_\infty}{C_w-C_\infty}, Ha^2=\frac{\sigma B_0^2\upsilon}{\rho u_0^2}, D_f=\frac{D_m K_T(C_w-C_\infty)}{\upsilon C_S C_P(T_w-T_\infty)},\\
&Gr=\frac{g\beta\nu(T_w-T_\infty)}{u_0^3}, \Pr=\frac{\mu C_p}{k}, M=Ha^2*\sin^2(\alpha),\\
&\mu=\upsilon\rho, Sc=\frac{\upsilon}{D}, Gm=\frac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3}, \bar{\omega}=\frac{\omega\upsilon}{u_0^2}.
\end{aligned}\right\} \tag{5}$$

Here $\bar{u}$ is dimensionless velocity, $\bar{t}$ – dimensionless time, *Pr*- Prandtl number, *Sc*- Schmidt number, *Gr*- thermal Grashof number, *Gm*- mass Grashof number, θ- dimensionless temperature, $\bar{C}$ - dimensionless concentration, *Ha*- the Hartmann number, μ- the coefficient of viscosity, $\bar{K}$ – permeability parameter and D_f - Dufour number. Then model is transformed into the following non dimensional form of equations:

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+Gr\theta+Gm\bar{C}-A\bar{u}, \tag{6}$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\bar{y}^2}+D_f\frac{\partial^2\bar{C}}{\partial\bar{y}^2}, \tag{7}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{Sc}\frac{\partial^2\bar{C}}{\partial\bar{y}^2}. \tag{8}$$

Here
$$A=M+\frac{1}{\bar{K}}.$$

The initial and boundary conditions become:

$$\left.\begin{aligned}
&\bar{t}\le 0; \bar{u}=0, \theta=0, \bar{C}=0 \text{ for each value of } \bar{y},\\
&\bar{t}>0; \bar{u}=\cos\bar{\omega}\bar{t}, \theta=\bar{t}, \bar{C}=1 \text{ at } \bar{y}=0,\\
&\bar{u}\to 0, \theta\to 0, \bar{C}\to 0 \text{ as } \bar{y}\to\infty.
\end{aligned}\right\} \tag{9}$$

Dropping bars in the above equations, we get:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC-Au, \tag{10}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+D_f\frac{\partial^2 C}{\partial y^2}, \tag{11}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}. \tag{12}$$

Here $A=M+\frac{1}{K}.$

The initial and boundary conditions become:

$$\left.\begin{aligned}
&t\le 0; u=0, \theta=0, C=0 \text{ for each value of } y,\\
&t>0; u=\cos\omega t, \theta=t, C=1 \text{ at } y=0,\\
&u\to 0, \theta\to 0, C\to 0 \text{ as } y\to\infty.
\end{aligned}\right\} \tag{13}$$

Now the solution of equation (10), (11) and (12) under the boundary conditions (13) are obtained by the Laplace - transform technique. The exact solutions for species concentration *C*, fluid temperature θ and fluid velocity *u* are respectively:

$$C=Erfc\left[\frac{\sqrt{Sc}\,y}{2\sqrt{t}}\right]$$

$$\theta=-\frac{e^{\frac{-\Pr y^2}{4t}}\sqrt{\Pr t}\,y}{\sqrt{\pi}}-\frac{A_{25}}{2}\left(t+\frac{\Pr y^2}{2}+\frac{\Pr ScD_f}{Sc-\Pr}\right)+\frac{\Pr ScD_f}{2(Sc-\Pr)}A_{27}$$

The expressions for the constants involved in the above equations are given in the appendix.

Results and Discussion

The numerical values of velocity, concentration and temperature are computed for different parameters like thermal Grashof number *Gr*, mass Grashof number *Gm*, Hartmann number *Ha*, Prandtl number *Pr*, Schmidt number *Sc*, inclination α, permeability parameter *K*, phase angle ωt, Dufour number D_f and time *t*.

The values of the parameters considered are *Gr* = 5, 10, 15, *Ha* = 2, 4, 6, *Gm* = 50, 60, 70, $\alpha=15°, 30°, 60°$, *Pr* = 7, 10, *Sc* = 2.01, 2.10, 2.20, *K* = 0.2, 0.4, 0.6, $\omega t=\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, D_f = 0.15, 0.23, 0.5 and *t* = 0.15, 0.18, 0.2. Figures 3, 6, 7 and 9 show that velocity increases when *Gm, K, Pr* and *t* are increased. Figures 1, 2, 4, 5, 8 and 10 show that velocity decreases when α, D_f, *Gr, Ha, Sc* and ωt are increased.

$$u=\frac{1}{4A^{2}}Gr\left[\begin{array}{l}2A_{9}B_{16}B_{13}+y\sqrt{A}A_{9}B_{17}+A_{13}B_{18}(1-\Pr)\\ +\dfrac{A\Pr ScD_{f}}{Sc-\Pr}(-2A_{9}B_{19}+A_{13}B_{20})\end{array}\right]+$$

$$\frac{(A_{18}B_{11}-A_{9}B_{12})}{2A}\left[\frac{Gr\Pr ScD_{f}}{Sc-\Pr}+Gm\right]-Gr$$

$$\left[\begin{array}{l}\dfrac{1}{2A^{2}}\{-2A_{22}B_{13}+\dfrac{A\sqrt{\Pr}y}{\sqrt{\pi}}(2e^{\frac{-\Pr y^{2}}{4t}}\sqrt{t}+\sqrt{\pi}\Pr yA_{22})\\ +\dfrac{1}{2}A_{13}B_{21}(\Pr-1)\}+\dfrac{\Pr Sc}{2A(Sc-\Pr)}\\ (A_{25}+\dfrac{A_{13}}{2}(1+A_{23}+A_{14}A_{26}))D_{f}\end{array}\right]-$$

$$\frac{Gr\Pr ScD_{f}B_{14}}{2A(Sc-\Pr)}-\frac{1}{2A}GmB_{15}+\frac{1}{4}C_{1}[C_{2}^{-1}(1-C_{4})$$

$$+C_{2}(1-C_{5})+C_{2}^{-2}(C_{3}^{-1}+C_{3}-C_{3}^{-1}C_{6})]$$

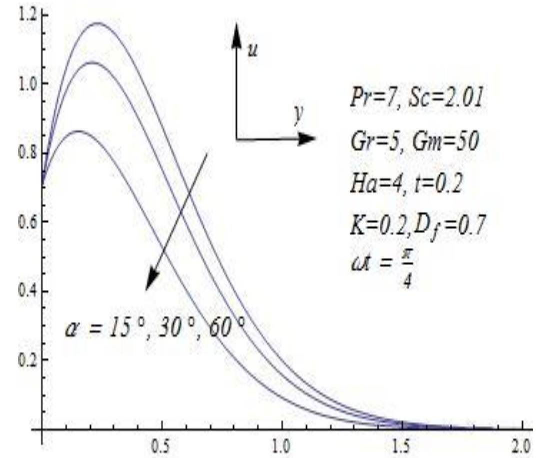

Figure1.Velocity profile for different values of α **.**

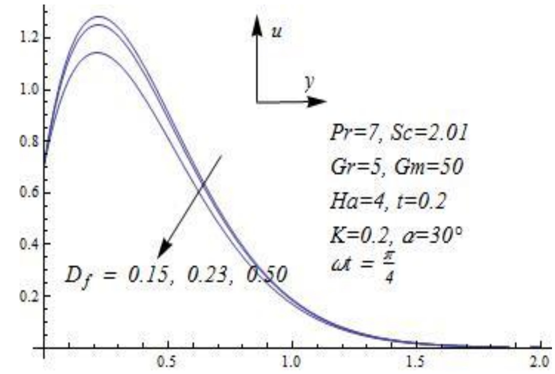

Figure 2. Velocity profile for different values of D_f

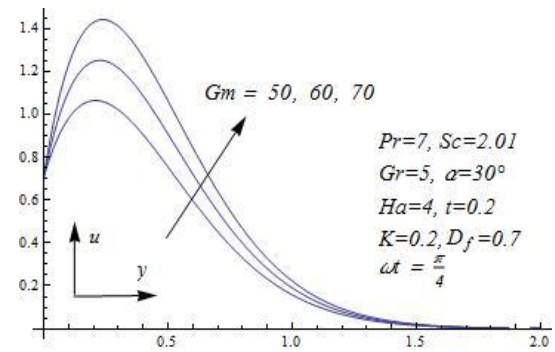

Figure 3.Velocity profile for different values of *Gm*.

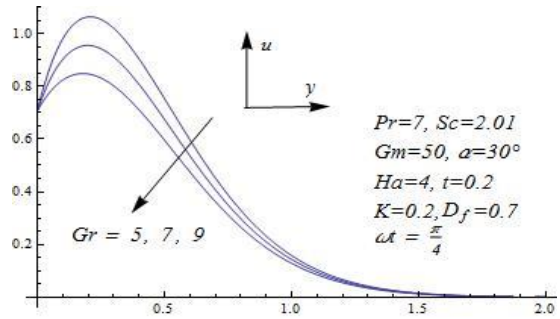

Figure 4. Velocity profile for different values of *Gr*

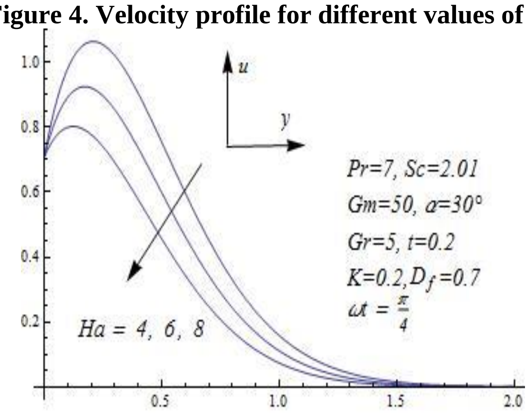

Figure 5. Velocity profile for different values of *Ha*.

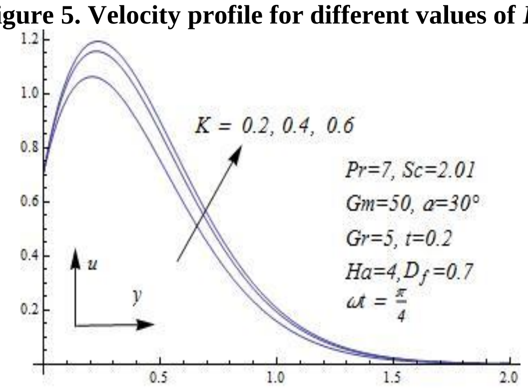

Figure 6. Velocity profile for different values of *K*.

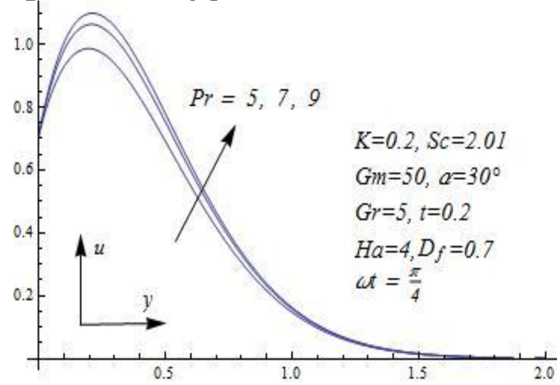

Figure 7.Velocity profile for different values of *Pr*.

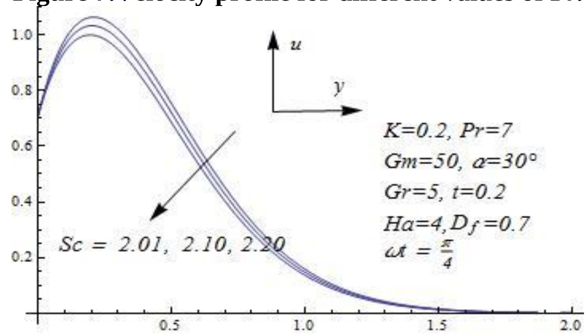

Figure 8. Velocity profile for different values of Sc.

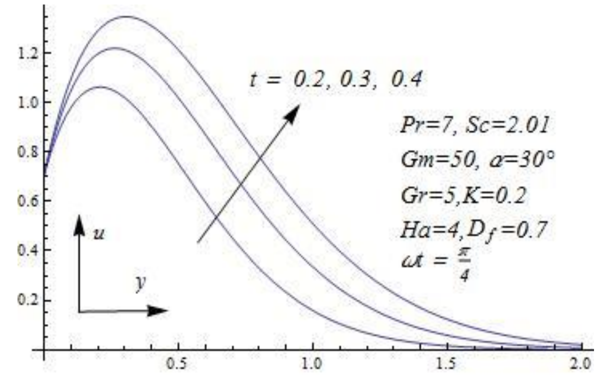

Figure 9. Velocity profile for different values of *t*.

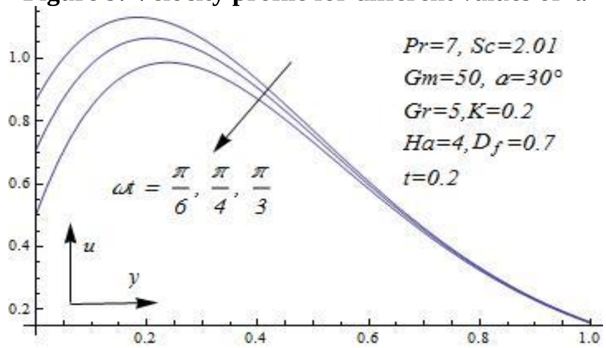

Figure 10. Velocity profile for different values of *ωt*.

Conclusion

In this paper we have studied Dufour effect on free convection MHD flow past an impulsively started vertical oscillating plate through porous media with variable temperature and constant mass diffusion. Some conclusions of study are as below:

- The velocity of the fluid increases with increasing the values of *K*, *Gm*, *t* and *Pr*.
- The velocity of the fluid decreases with increasing the values of *Ha*, *Gr*, *ωt*, *Sc*, α and D_f.

Appendix

$$A_9 = e^{-\sqrt{A}y},$$

$$A_{10} = Erfc\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right],$$

$$A_{11} = Erf\left[\frac{2\sqrt{A}t - y}{2\sqrt{t}}\right],$$

$$A_{12} = Erf\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right]$$

$$A_{13} = 2e^{\frac{At}{\Pr-1} - \sqrt{\frac{A\Pr}{\Pr-1}}y},$$

$$A_{14} = e^{2\sqrt{\frac{A\Pr}{\Pr-1}}y}$$

$$A_{15} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t - y}{2\sqrt{t}}\right],$$

$$A_{16} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t + y}{2\sqrt{t}}\right]$$

$$A_{17} = Erfc\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t + y}{2\sqrt{t}}\right],$$

$$A_{18} = e^{\frac{At}{Sc-1} - \sqrt{\frac{ASc}{Sc-1}}y},$$

$$A_{19} = e^{2\sqrt{\frac{ASc}{Sc-1}}y},$$

$$A_{20} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t - y}{2\sqrt{t}}\right]$$

$$A_{21} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t + y}{2\sqrt{t}}\right],$$

$$A_{22} = -1 + Erf\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{23} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t - \sqrt{\Pr}y}{2\sqrt{t}}\right]$$

$$A_{24} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{25} = -2Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{26} = Erfc\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right]$$

$$A_{27} = -2Erfc\left[\frac{\sqrt{Sc}y}{2\sqrt{t}}\right],$$

$$A_{28} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t - \sqrt{Sc}y}{2\sqrt{t}}\right],$$

$$A_{29} = Erfc\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right]$$

$$A_{30} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right],$$

$$B_{11} = 1 + A_{19} + A_{20} - A_{19}A_{21},$$

$$B_{12} = 1 + A_{11} + A_9^{-2}A_{10}$$

$$B_{13} = 1 - \Pr - At,$$

$$B_{14} = A_{27} + A_{18}(1 + A_{28} + A_{19}A_{29}),$$

$$B_{15} = A_{18}(1 + A_{19} + A_{28} - A_{19}A_{30}) + A_{27}$$

$$B_{16} = 1 + A_9^{-2} + A_{11} - A_9^{-2}A_{12},\ B_{17} = 1 - A_9^{-2} + A_{11} + A_9^{-2}A_{12},$$

$$B_{18} = -1 - A_{14} - A_{15} + A_{14}A_{17}$$

$$B_{19} = 1 + A_{11} + A_9^{-2}A_{10},$$

$$B_{20} = 1 + A_{15} + A_{14}A_{17},$$

$$B_{21} = 1 + A_{14} + A_{23} - A_{14}A_{24}$$

$$C_1 = e^{-i\omega t}$$

$$C_2 = e^{y\sqrt{A-i\omega}},$$

$$C_3 = e^{y\sqrt{A+i\omega}},$$

$$C_4 = Erf\left[\frac{y - 2t\sqrt{A-i\omega}}{2\sqrt{t}}\right],$$

$$C_5 = Erf\left[\frac{y+2t\sqrt{A-i\omega}}{2\sqrt{t}}\right]$$

$$C_6 = Erf\left[\frac{y-2t\sqrt{A+i\omega}}{2\sqrt{t}}\right],$$

$$C_7 = Erf\left[\frac{y+2t\sqrt{A+i\omega}}{2\sqrt{t}}\right]$$

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