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Optimization of an inventory model considering the cost of green research and green analysis of the product in multi dimensional aspect using Decagonal Fuzzy Number

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ABSTRACT

In recent times the production sectors (PS) have started to concentrate not only on product production but also on an additional perspective that is research and product analysis in multidimensional aspects. The PS before commencing the production process have now started to find the ways and means of enhancing the product's features and also they have began to analyze the propagation extent of the product that is the level of the reach of the product. The main notion of carrying out such activities is to produce novelty products. To achieve this PS seek the assistance of many consultancies or they establish research centre in their production environment. Conventional Inventory models do not satisfy the present needs of PS therefore inventive inventory model considering associated costs of research and product analysis have to be formulated. To fulfill this, inventory model is formulated with Decagonal Fuzzy Numbers to overcome the existing uncertainty. In this paper a new approach of Pascal's Triangle Graded mean Approach is used for Decagonal Fuzzy number. Also Decagonal fuzzy number gives a better optimal value than all other types of fuzzy number.

1.Introduction

The production sectors set up the business forum with the aim of succeeding its co producers by maximizing its profit. If PS attains its aim then sustaining at the same level becomes a tedious process for which they are in need of certain supportive strategies. These tactics can be coined together as function of success which comprise of research and analysis of product reach. In almost all the production sectors, a separate division or department consisting of experts working for promoting the novelty of the products. These experts first consider the need of inventive products, study the customer behavior and then estimate the reach extent of the product, therefore the experts initially sketches out the circle of groundwork. Before setting the frame work of this circle the PS must also be cautious of the circle of limits which includes the capital constraint and legal restrictions of the government. Therefore the effective progression of the circle of ground work depends on the circle of the limits of the PS.

The function of success has to be accelerated so as to create better products fulfilling the expectations of the customers. The benefits of consuming the product should also do well to the environment which has now led to the new era of green generation. All types of the product are labeled as eco- friendly, but what do it exactly means? The packaging of the product, the manual of the machines bares the words eco, green, a symbol of the tree, and even the corporate sectors are marching towards green. Is this all the bells of green revolution? If so the human community has to be alarmed of global disaster, yes, the entire world is getting deteriorated for every nano second. The present scenario is filled with agony and it has to be extinguished with the implementation of green ways. Since the production sectors are integrated with the life of people in almost all the ways, they can be modified as Green Production Sectors (GPS). Transformation of PS to GPS costs a little but it has more positive impacts. The production sectors have to allocate capital for the installment of research department and initialization of the product analysis for the stimulation of the function of success. The consideration of the cost parameters by the PS lead to the problem of uncertainty as the costs involved are subjectable to change over a period of time. Therefore it is the time for the emerge of inventory model taking into the account of green research and green analysis of the product with fuzzy parameters.

The origin of inventory models is traced back to a century; lakhs of models were formulated pertaining to many aspects such as shortages (Gardner,Harris,Taha), backlog (Biswajit, Tao Wau), trade credit, integrated vendor buyer (Yong.et.al), reverse logistics (Richter), remanufacture (Schrady) and disposal. These models are then modified by changing the nature (deterministic, probabilistic, fuzzy and stochastic) of demand but practically thinking do these models do actually reflect the real needs of PS. The answers to such questions are not satisfactory. Presently the PS aims at maximum reach of eco promoting novelty products. The PS is indeed feels tedious of accomplishing this herculean task. To make such jobs easily accessible classical inventory models are not sufficient as it ignores certain cost parameters, therefore trendy inventory models have to be formulated and this research work is a step for it.

If any new inventory model is framed it is done with the deterministic assumption then it has been shifted to

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probabilistic which is based on the randomness, later stochastic came into existence as time dependent factor is taken into account. Amidst all these representations, expressing in terms of fuzzy is more reliable to overcome the uncertainty, since the demand pattern and the cost parameters are subjected to change, representing them as fuzzy variables yield the optimum value after defuzzification. The Fuzzy set theory introduced by Lofti. A. Zadeh .In 1987, Park used fuzzy set concept to treat the inventory problem with fuzzy inventory cost under arithmetic operations of extension principle, followed by him, many inventory models were formulated by considering triangular fuzzy number, trapezoidal fuzzy number. But presently researchers are working with hexagonal, hepatagonal, octagonal , nonogonal, decagonal fuzzy numbers but they have not employed it in the inventory models. Therefore a fuzzy inventory model is formulated in this paper allowing for green research and green analysis. In this paper decagonal fuzzy number is used so as to break the restriction of a fuzzy number, also adding to it decagonal fuzzy number will help better in decision making by overcoming the impreciseness in a more effective manner. In this paper the Pascal's triangle graded mean approach is used to determine the graded mean representation for the decagonal fuzzy number. This approach was used only for triangular and trapezoidal fuzzy number (Chen) but here efforts were taken to extend it for the higher fuzzy number.

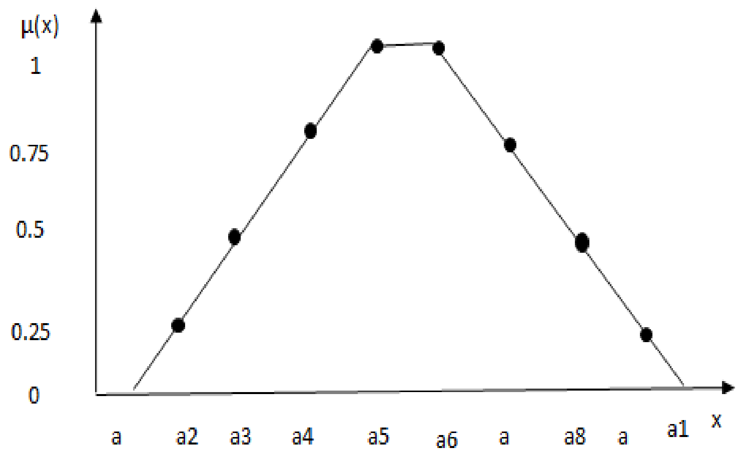
The following paper is organized as follows: section 2 presents the preliminaries of decagonal fuzzy number, section 3 discusses the crisp model, section 4 consists of fuzzy model, section 5 comprises of numerical example to validate the model and section 6 concludes the paper.

2. Decagonal fuzzy number

A Decagonal Fuzzy number is defined as (a1, a2, a3, a4, a5,a6, a7, a8, a9, a10) and the membership function is defined as

$$\mu D(x) = \begin{cases} \frac{1}{4}\frac{x-a1}{a2-a1}, & a1 \le x \le a2 \\ \frac{1}{4}+\frac{1}{4}\frac{x-a2}{a3-a2}, & a2 \le x \le a3 \\ \frac{1}{2}+\frac{1}{4}\frac{x-a3}{a4-a3}, & a3 \le x \le a4 \\ \frac{3}{4}+\frac{1}{4}\frac{x-a4}{a5-a4}, & a4 \le x \le a5 \\ & a5 \le x \le a6 \\ 1 - & a6 \le x \le a7 \\ \frac{3}{4}-\frac{1}{4}\frac{x-a7}{a8-a7}, & a7 \le x \le a8 \\ \frac{1}{2}-\frac{1}{4}\frac{x-a8}{a9-a8}, & a8 \le x \le a9 \\ \frac{1}{4}\frac{a10-x}{a10-a9}, & a9 \le x \le a10 \\ 0 & \text{Otherwise} \end{cases}$$

The membership function of the decagonal number is presented in the figure



3. Crisp Inventory Model

This section encompasses the development of an inventory model which includes the costs of green research and green analysis. This model enables the promotion of Green Production Sectors.

3.1 Notations

The following notations will be used throughout the paper when developing the mathematical model.

D demand per unit of time

sP production per unit of time

x $\frac{D}{P}$

1-x the fraction of time the production process spends actually idling

A procurement costs/ set up cost per production run

h holding cost per unit per unit of time.

R installation costs of research facilities in the production sectors g Associated functioning costs of green research activities

G Related costs of analysis of product towards green approach

The Production Inventory Model cost per unit of time

$C(Q) = A\frac{D}{Q} + \frac{hQ(1-x)}{2}$

The processing cost per cycle

$C_V(Q) = V$

The costs of function of success is

$C_S Q) = R + g + G$

The total cost per unit of time including the costs of creating green atmosphere is

$\Psi(Q) = C(Q) + \frac{C_V(Q)+CS(Q)}{T}$ where $T = \frac{Q}{D}$

$$= A\frac{D}{Q} + \frac{hQ(1-x)}{2} + \frac{D}{Q}(V+R+g+G)$$

$$\frac{\partial\Psi(Q)}{\partial Q} = \frac{\partial}{\partial Q}(A\frac{D}{Q} + \frac{h(1-x)Q}{2} + \frac{D}{Q}(V+R+g+G)$$

The objective is to determine the optimal quantity. The necessary condition is $\frac{\partial\Psi(Q)}{\partial Q} = 0$

The optimal solution is

$$Q = \sqrt{\frac{2D[A+V+R+g+G]}{h(1-x)}}$$

4. Fuzzification of the crisp model

Throughout this paper, we use the following variables in order to simplify the treatment of the green inventory model .Let $\tilde{A}, \tilde{R}, \tilde{h}, \tilde{g}, \tilde{V}, \tilde{G}, \tilde{D}$ be fuzzy parameters. We introduce a green Inventory Model with fuzzy parameters for crisp production quantity $\psi(Q)$

The annual total inventory cost for the green inventory model

$$\begin{aligned}\Psi(Q) = &A1\frac{D1}{Q} + \frac{h1Q(1-x)}{2} + \frac{D1}{Q}(V1+R1+g1+G1), A2\frac{D2}{Q} + \frac{h2Q(1-x)}{2} + \frac{D2}{Q}(V2+R2+g2+G2),\\ &A3\frac{D3}{Q} + \frac{h3Q(1-x)}{2} + \frac{D3}{Q}(V3+R3+g3+G3), A4\frac{D4}{Q} + \frac{h4Q(1-x)}{2} + \frac{D4}{Q}(V4+R4+g4+G4),\\ &A5\frac{D5}{Q} + \frac{h5Q(1-x)}{2} + \frac{D5}{Q}(V5+R5+g5+G5), A6\frac{D6}{Q} + \frac{h6Q(1-x)}{2} + \frac{D6}{Q}(V6+R6+g6+G6),\\ &A7\frac{D7}{Q} + \frac{h7Q(1-x)}{2} + \frac{D7}{Q}(V7+R7+g7+G7), A8\frac{D8}{Q} + \frac{h8Q(1-x)}{2} + \frac{D8}{Q}(V8+R8+g8+G8),\\ &A9\frac{D9}{Q} + \frac{h9Q(1-x)}{2} + \frac{D9}{Q}(V9+R9+g9+G9), A10\frac{D10}{Q} + \frac{h10Q(1-x)}{2} + \frac{D10}{Q}(V10+R10+g10+G10)\end{aligned}$$

Suppose $\tilde{A} = (A1, A2A3, A4, A5, A6, A7, A8, A9, A10)$ $\tilde{R} = (R1, R2, R3, R4, R5, R6, R7, R8, R9, R10)$

$\tilde{h} = (h1, h2, h3, h4, h5, h6, h7, h8, h9, h10)$ $\tilde{g} = (g1, g2, g3, g4, g5, g6, g7, g8, g9, g10)$

$\tilde{V} = (V1, V2, V3, V4, V5, V6, V7, V8, V9, V10)$ $G = (G1, G2, G3, G4, G5, G6, G7, G8, G9, G10)$

$\tilde{D} = (D1, D2, D3, D4, D5, D6, D7D, D8, D9, D10)$

are nonnegative decagonal fuzzy numbers. Then we solve the optimal order quantity by using the following steps.

Second by defuzzification, we get

$$\widetilde{P(\psi(Q))} =$$

$$\begin{aligned}&\frac{1}{512}\Big\{A1\frac{D1}{Q} + \frac{h1Q(1-x)}{2} + \frac{D1}{Q}(V1+R1+g1+G1) + 9\left(A2\frac{D2}{Q} + \frac{h2Q(1-x)}{2} + \frac{D2}{Q}(V2+R2+g2+G2)\right) +\\ &36\left(A3\frac{D3}{Q} + \frac{h3Q(1-x)}{2} + \frac{D3}{Q}(V3+R3+g3+G3)\right) + 84\left(A4\frac{D4}{Q} + \frac{h4Q(1-x)}{2} + \frac{D4}{Q}(V4+R4+g4+G4)\right) +\\ &126\left(A5\frac{D5}{Q} + \frac{h5Q(1-x)}{2} + \frac{D5}{Q}(V5+R5+g5+G5)\right) + 126\left(A6\frac{D6}{Q} + \frac{h6Q(1-x)}{2} + \frac{D6}{Q}(V6+R6+g6+\right.\\ &\left.G6)\right) + 84\left(A7\frac{D7}{Q} + \frac{h7Q(1-x)}{2} + \frac{D7}{Q}(V7+R7+g7+G7)\right) + 36\left(A8\frac{D8}{Q} + \frac{h8Q(1-x)}{2} + \frac{D8}{Q}(V8+R8+g8+\right.\\ &\left.G8)\right) + 9\left(A9\frac{D9}{Q} + \frac{h9Q(1-x)}{2} + \frac{D9}{Q}(V9+R9+g9+G9)\right) + A10\frac{D10}{Q} + \frac{h10Q(1-x)}{2} + \frac{D10}{Q}(V10+R10+\\ &g10+G10)\Big\}\end{aligned}$$

Third, we can get the optimal production quantity Q* when $\widetilde{P}(\psi(Q))$ is minimization. In order to find the minimization of $\widetilde{P}(\psi(Q)$ the derivative of $\widetilde{P}(\psi(Q))$with Q is $\frac{\partial(\widetilde{P}(\psi(Q))}{\partial Q} = 0$

We find the optimal production quantity Q = Q* =

$$\sqrt{\frac{2(D1[A1+V1+R1+g1+G1]+9D2[A2+V2+R2+g2+G2]+36D3[A3+V3+R3+g3+G3+84D4[A4+V4+R4+g4+G4]+126D5[A5+V5+R5+g5+G5]+126D6[A6+V6+R6+g6+G6]+84D7[A7+V7+R7+g7+G7]+36D8[A8+V8+R8+g8+G8]+9D9[A9+V9+R9+g9+G9]+D10[A10+V10+R10+g10+G10])}{(h1+9h2+36h3+84h4+126h5+126h6+84h7+36h8+9h9+h10)(1-x)}}$$

4.1 Fuzzy Green Inventory Model for fuzzy order quantity

In this section, we introduce a green inventory model by changing the crisp production quantity into fuzzy production quantity. Suppose fuzzy production quantity $\tilde{Q}$ be a decagonal fuzzy number $\tilde{Q}$ = (Q1, Q2, Q3, Q4,Q5,Q6,Q7,Q8,Q9,Q10) with 0 < Q1 ≤ Q2 ≤ Q3 ≤ Q4≤Q5, ≤Q6, ≤Q7≤Q8≤Q9≤Q10. Thus we can get the fuzzy total inventory cost

$$\widetilde{P}(\psi(Q)) = A1\frac{D1}{Q10}+\frac{h1Q1(1-x)}{2}+\frac{D1}{Q10}(V1+R1+g1+G1), A2\frac{D2}{Q9}+\frac{h2Q2(1-x)}{2}+\frac{D2}{Q9}(V2+R2+g2+G2),$$
$$A3\frac{D3}{Q8}+\frac{h3Q3(1-x)}{2}+\frac{D3}{Q8}(V3+R3+g3+G3), A4\frac{D4}{Q7}+\frac{h4Q4(1-x)}{2}+\frac{D4}{Q7}(V4+R4+g4+G4),$$
$$A5\frac{D5}{Q6}+\frac{h5Q5(1-x)}{2}+\frac{D5}{Q6}(V5+R5+g5+G5), A6\frac{D6}{Q5}+\frac{h6Q6(1-x)}{2}+\frac{D6}{Q5}(V6+R6+g6+G6),$$
$$A7\frac{D7}{Q4}+\frac{h7Q7(1-x)}{2}+\frac{D7}{Q4}(V7+R7+g7+G7), A8\frac{D8}{Q3}+\frac{h8Q8(1-x)}{2}+\frac{D8}{Q3}(V8+R8+g8+G8),$$
$$A9\frac{D9}{Q2}+\frac{h9Q9(1-x)}{2}+\frac{D9}{Q2}(V9+R9+g9+G9), A10\frac{D10}{Q1}+\frac{h10Q10(1-x)}{2}+\frac{D10}{Q1}(V10+R10+g10+G10)$$

The Graded Mean Integration Representation of $\widetilde{P}(\psi(Q))$ is obtained by using the Pascal's Triangle approach which is as follows:

$$\widetilde{P}(\psi(Q)) = \frac{1}{512}\left\{A1\frac{D1}{Q10}+\frac{h1Q1(1-x)}{2}+\frac{D1}{Q10}(V1+R1+g1+G1)+9\left(A2\frac{D2}{Q9}+\frac{h2Q2(1-x)}{2}+\frac{D2}{Q9}(V2+R2+g2+G2)\right)+36\left(A3\frac{D3}{Q8}+\frac{h3Q3(1-x)}{2}+\frac{D3}{Q8}(V3+R3+g3+G3)\right)+84\left(A4\frac{D4}{Q7}+\frac{h4Q4(1-x)}{2}+\frac{D4}{Q7}(V4+R4+g4+G4)\right)+126\left(A5\frac{D5}{Q6}+\frac{h5Q5(1-x)}{2}+\frac{D5}{Q6}(V5+R5+g5+G5)\right)+126\left(A6\frac{D6}{Q5}+\frac{h6Q6(1-x)}{2}+\frac{D6}{Q5}(V6+R6+g6+G6)\right)+84\left(A7\frac{D7}{Q4}+\frac{h7Q7(1-x)}{2}+\frac{D7}{Q4}(V7+R7+g7+G7)\right)+36\left(A8\frac{D8}{Q3}+\frac{h8Q8(1-x)}{2}+\frac{D8}{Q3}(V8+R8+g8+G8)\right)+9\left(A9\frac{D9}{Q2}+\frac{h9Q9(1-x)}{2}+\frac{D9}{Q2}(V9+R9+g9+G9)\right)+A10\frac{D10}{Q1}+\frac{h10Q10(1-x)}{2}+\frac{D10}{Q1}(V10+R10+g10+G10)\right\}$$

….. 4.1

with 0 < *Q1* ≤ *Q2* ≤ *Q3* ≤ *Q4*≤Q5, ≤Q6, ≤Q7≤Q8≤Q9≤Q10. It will not change the meaning of formula(4.1) if we replace inequality conditions 0 < *Q1* ≤ *Q2* ≤ *Q3* ≤ *Q4* ≤Q5, ≤Q6, ≤Q7≤Q8≤Q9≤Q10 into the following inequality

Q2 − *Q1* ≥ 0, *Q3* − *Q2* ≥ 0, *Q4* − *Q3* ≥ 0, *Q5* − *Q4* ≥ 0, *Q6* − *Q5* ≥ 0,
Q7 − *Q6* ≥ 0, *Q8* − *Q7* ≥ 0, *Q9* − *Q8* ≥ 0, *Q10* − *Q9* ≥ 0 *Q1* > 0. In the following steps, extension of the Lagrangean method is used to find the solutions of *Q1*, *Q2* , *Q3*, *Q4* to minimize $\widetilde{P(\psi(Q))}$ in formula (4.1).

Step 1 : Solve the unconstraint problem. Consider min $\widetilde{P(\psi(Q))}$. To find the min $\widetilde{P(\psi(Q))}$ we have to find the derivative of $\widetilde{P(\psi(Q))}$ with respect to *Q1*, *Q2*, *Q3*, *Q4*, Q5,Q6,Q7,Q8,Q9,Q10

$$\frac{\partial\psi}{\partial Q_1} = \frac{1}{512}\left\{\frac{h1}{2}(1-x) - \frac{A10D10}{Q_1^2} - \frac{D10}{Q_1^2}\,[V10+R10+g10+G10]\right\}.$$
$$\frac{\partial\psi}{\partial Q_2} = \frac{1}{512}\left\{\frac{9h2}{2}(1-x) - 9\{\frac{A9D9}{Q_2^2} - \frac{D9}{Q_2^2}\,[V9+R9+g9+G9]\}\right\}.$$
$$\frac{\partial\psi}{\partial Q_3} = \frac{1}{512}\left\{\frac{36h3}{2}(1-x) - 36\{\frac{A8D8}{Q_3^2} - \frac{D8}{Q_3^2}\,[V8+R8+g8+G8]\}\right\}$$
$$\frac{\partial\psi}{\partial Q_4} = \frac{1}{512}\left\{\frac{84h4}{2}(1-x) - 84\{\frac{A7D7}{Q_4^2} - \frac{D7}{Q_4^2}\,[V7+R7+g7+G7]\}\right\}$$

$$\frac{\partial\psi}{\partial Q_5}=\frac{1}{512}\left\{126\frac{h5}{2}(1-x)-126\{\frac{A6D6}{Q_5^2}-\frac{D6}{Q_5^2}\ [V6+R6+g6+G6]\}\right\}$$
$$\frac{\partial\psi}{\partial Q_6}=\frac{1}{512}\left\{126\frac{h6}{2}(1-x)-126\{\frac{A5D5}{Q_6^2}-\frac{D5}{Q_6^2}\ [V5+R5+g5+G5]\}\right\}$$
$$\frac{\partial\psi}{\partial Q_7}=\frac{1}{512}\left\{84\frac{h7}{2}(1-x)-84\{\frac{A4D4}{Q_7^2}-\frac{D4}{Q_7^2}\ [V4+R4+g4+G4]\}\right\}$$
$$\frac{\partial\psi}{\partial Q_8}=\frac{1}{512}\left\{36\frac{h8}{2}(1-x)-36\{\frac{A3D3}{Q_8^2}-\frac{D3}{Q_8^2}\ [V3+R3+g3+G3]\}\right\}$$
$$\frac{\partial\psi}{\partial Q_9}=\frac{1}{512}\left\{9\frac{h9}{2}(1-x)-9\{\frac{A2D2}{Q_9^2}-\frac{D2}{Q_9^2}\ [V2+R2+g2+G2]\}\right\}$$
$$\frac{\partial\psi}{\partial Q_{10}}=\frac{1}{512}\left\{\frac{h10}{2}(1-x)-\{\frac{A1D1}{Q_{10}^2}-\frac{D1}{Q_{10}^2}\ [V1+R1+g1+G1]\}\right\}$$

Let all the above results partial derivatives equal to zero and solve $Q1$, $Q2$, $Q3$,$Q4$, Q5,Q6,Q7,Q8,Q9,Q10.

Let $\frac{\partial\psi}{\partial Q_1}=0$. Then

$$Q_1=\sqrt{\frac{2D10[A10+V10+R10+g10+G10]}{h1(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_2}=0$. Then

$$Q_2=\sqrt{\frac{2D[A9+V9+R9+g9+G9]}{h2(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_3}=0$. Then

$$Q_3=\sqrt{\frac{2D8[A8+V8+R8+g8+G8]}{h3(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_4}=0$. Then

$$Q_4=\sqrt{\frac{2D7[A7+V7+R7+g7+G7]}{h4(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_5}=0$. Then

$$Q_5=\sqrt{\frac{2D6[A6+V6+R6+g6+G6]}{h5(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_6}=0$. Then

$$Q_6=\sqrt{\frac{2D5[A5+V5+R5+g5+G5]}{h6(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_7}=0$. Then

$$Q_7=\sqrt{\frac{2D4[A4+V4+R4+g4+G4]}{h7(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_8}=0$. Then

$$Q_8=\sqrt{\frac{2D3[A3+V3+R3+g3+G3]}{h8(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_9} = 0$. Then

$$Q_9 = \sqrt{\frac{2D2[A2+V2+R2+g2+G2]}{h9(1-x)}}$$

Let $\frac{\partial\psi}{\partial Q_{10}} = 0$. Then

$$Q_{10} = \sqrt{\frac{2D1[A1+V1+R1+g1+G1]}{h10(1-x)}}$$

Because the above show that *Q*1 >*Q*2 >*Q*3 > *Q*4, it does not satisfy the constraint 0 < *Q*1 ≤ *Q*2 ≤ *Q*3≤ *Q*4≤ *Q5*≤ *Q6*≤ *Q7*≤ *Q8*≤ *Q9*≤ *Q10*. Therefore set *K* = 1 and go to

Step 2 : Convert the inequality constraint *Q*2 − *Q*1 ≥ 0 into equality constraint *Q*2 −*Q*1 = 0 and optimize $\widetilde{P(\psi(Q))}$ subject to *Q*2 −*Q*1 = 0 by the Lagrangean Method. We have Lagrangean function as *L*(*Q*1 , *Q*2 , *Q*3 , *Q*4,*Q5,Q6,Q7,Q8,Q9,Q10 λ*) = $\widetilde{P(\psi(Q))}$ − *λ*(Q2 − Q1).

Taking the partial derivatives of *L*(*Q*1, *Q*2, *Q*3, *Q*4 *Q5,Q6,Q7,Q8,Q9,Q10, λ*) with respect to *Q*1 , *Q*2, *Q*3, *Q*4 , *Q*6,*Q*7,*Q*8,*Q*9,*Q*10 and *λ* to find the minimization of *L*(*Q*1, *Q*2, *Q*3, *Q*4*, Q6,Q7,Q8,Q9,Q10, λ*). Let all the partial derivatives equal to zero and solve *Q*1, *Q*2, *Q*3 ,*Q*4 *Q*6,*Q*7,*Q*8,*Q*9,*Q*10. Then we get,

$$\frac{\partial L}{\partial Q_1} = \frac{1}{512}\left\{\frac{h1}{2}(1-x) - \frac{A10D10}{Q_1^2} - \frac{D10}{Q_1^2}\,[V10+R10+g10+G10]\right\} + \lambda = 0$$
$$\frac{\partial L}{\partial Q_2} = \frac{1}{512}\left\{9\frac{h2}{2}(1-x) - 9\{\frac{A9D9}{Q_2^2} - \frac{D9}{Q_2^2}\,[V9+R9+g9+G9]\}\right\} - \lambda = 0$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{512}\left\{36\frac{h3}{2}(1-x) - 36\{\frac{A8D8}{Q_3^2} - \frac{D8}{Q_3^2}\,[V8+R8+g8+G8]\}\right\} = 0$$
$$\frac{\partial L}{\partial Q_4} = \frac{1}{512}\left\{84\frac{h4}{2}(1-x) - 84\{\frac{A7D7}{Q_4^2} - \frac{D7}{Q_4^2}\,[V7+R7+g7+G7]\}\right\} = 0$$
$$\frac{\partial L}{\partial Q_5} = \frac{1}{512}\left\{\frac{126h5}{2}(1-x) - 126\{\frac{A6D6}{Q_5^2} - \frac{D6}{Q_5^2}\,[V6+R6+g6+G6]\}\right\} = 0$$
$$\frac{\partial L}{\partial Q_6} = \frac{1}{512}\left\{126\frac{h6}{2}(1-x) - 126\{\frac{A5D5}{Q_6^2} - \frac{D5}{Q_6^2}\,[V5+R5+g5+G5]\}\right\} = 0$$
$$\frac{\partial L}{\partial Q_7} = \frac{1}{512}\left\{\frac{84.h7}{2}(1-x) - 84\{\frac{A4D4}{Q_7^2} - \frac{D4}{Q_7^2}\,[V4+R4+g4+G4]\}\right\} = 0$$
$$\frac{\partial L}{\partial Q_8} = \frac{1}{512}\left\{36\frac{h8}{2}(1-x) - 36\{\frac{A3D3}{Q_8^2} - \frac{D3}{Q_8^2}\,[V3+R3+g3+G3]\}\right\} = 0$$
$$\frac{\partial L}{\partial Q_9} = \frac{1}{512}\left\{9\frac{h9}{2}(1-x) - 9\{\frac{A2D2}{Q_9^2} - \frac{D2}{Q_9^2}\,[V2+R2+g2+G2]\}\right\} = 0$$
$$\frac{\partial L}{\partial Q_{10}} = \frac{1}{512}\left\{\frac{h10}{2}(1-x) - \{\frac{A1D1}{Q_{10}^2} - \frac{D1}{Q_{10}^2}\,[V1+R1+g1+G1]\}\right\} = 0$$
$$\frac{\partial L}{\partial \lambda} = -(Q_2 - Q_1) = 0$$

$$Q_2 = Q_1 = \sqrt{\frac{2\left(D10(A10+V10+R10+g10+G10)+9D9(A9+V9+R9+G9+g9)\right)}{h1+9h2(1-x)}}$$ similarly we get

$$Q_3 = \sqrt{\frac{2D8[A8+V8+R8+g8+G8]}{h3(1-x)}} \quad Q_4 = \sqrt{\frac{24D7[A7+V7+R7+g7+G7]}{h4(1-x)}}$$

$$Q_5 = \sqrt{\frac{2D6[A6+V6+R6+g6+G6]}{h5(1-x)}} \quad Q_6 = \sqrt{\frac{2D5[A5+V5+R5+g5+G5]}{h6(1-x)}}$$

$$Q_7 = \sqrt{\frac{2D4[A4+V4+R4+g4+G4]}{h7(1-x)}} \quad Q_8 = \sqrt{\frac{2D3[A3+V3+R3+g3+G3]}{h8(1-x)}}$$
$$Q_9 = \sqrt{\frac{2D2[A2+V2+R2+g2+G2]}{h9(1-x)}} \quad Q_{10} = \sqrt{\frac{2D1[A1+V1+R1+g1+G1]}{h10(1-x)}}$$

Because the above results show that *Q*i > *Q*i+1,*I = 3,4…..9*

it does not satisfy the constraint 0 < *Q*1 ≤ *Q*2 ≤ *Q*3 ≤ *Q*4≤ *Q5*≤ *Q6*≤ *Q7*≤ *Q8*≤ *Q9*≤ *Q10*. Therefore it is not a local optimum. Similarly we can get the same result if we select any other one inequality constraint to be equality constraint, therefore set *K* = 2 and go to Step 3.

Step 3 : Convert the inequality constraints $Q2 - Q1 \geq 0$, $Q3 - Q2 \geq 0$, into equality constraints $Q2 - Q1 = 0$ and $Q3 - Q2 = 0$. We optimize $\widetilde{P(\psi(Q))}$. Subject to $Q2 - Q1 = 0$ and $Q3 - Q2 = 0$ by the Lagrangean Method. Then the Lagrangean method is

$L(Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, \lambda 1, \lambda 2) = \widetilde{P(\psi(Q))} - \lambda 1(Q2 - Q1) - \lambda 2(Q3 - Q2)$.

In order to find the minimization of $L(Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, \lambda 1, \lambda 2)$. Let all the partial derivatives equal to zero and solve $Q1, Q2, Q3, Q4\ Q6, Q7, Q8, Q9, Q10$. Then we get,

$$\frac{\partial L}{\partial Q_1} = \frac{1}{512}\left\{\frac{h1}{2}(1-x) - \frac{A10D10}{Q_1^2} - \frac{D10}{Q_1^2}\,[V10 + R10 + g10 + G10]\right\} + \lambda 1 = 0$$

$$\frac{\partial L}{\partial Q_2} = \frac{1}{512}\left\{\frac{h2}{2}(1-x) - 9\{\frac{A9D9}{Q_2^2} - \frac{D9}{Q_2^2}\,[V9 + R9 + g9 + G9]\}\right\} - \lambda 1 + \lambda 2 = 0$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{512}\left\{\frac{h3}{2}(1-x) - 36\{\frac{A8D8}{Q_3^2} - \frac{D8}{Q_3^2}\,[V8 + R8 + g8 + G8]\}\right\} - \lambda 2 = 0$$

$$\frac{\partial L}{\partial Q_4} = \frac{1}{512}\left\{\frac{h4}{2}(1-x) - 84\{\frac{A7D7}{Q_4^2} - \frac{D7}{Q_4^2}\,[V7 + R7 + g7 + G7]\}\right\} = 0$$

$$\frac{\partial L}{\partial Q_5} = \frac{1}{512}\left\{\frac{h5}{2}(1-x) - 126\{\frac{A6D6}{Q_5^2} - \frac{D6}{Q_5^2}\,[V6 + R6 + g6 + G6]\}\right\} = 0$$

$$\frac{\partial L}{\partial Q_6} = \frac{1}{512}\left\{\frac{h6}{2}(1-x) - 126\{\frac{A5D5}{Q_6^2} - \frac{D5}{Q_6^2}\,[V5 + R5 + g5 + G5]\}\right\} = 0$$

$$\frac{\partial L}{\partial Q_7} = \frac{1}{512}\left\{\frac{h7}{2}(1-x) - 84\{\frac{A4D4}{Q_7^2} - \frac{D4}{Q_7^2}\,[V4 + R4 + g4 + G4]\}\right\} = 0$$

$$\frac{\partial L}{\partial Q_8} = \frac{1}{512}\left\{\frac{h8}{2}(1-x) - 36\{\frac{A3D3}{Q_8^2} - \frac{D3}{Q_8^2}\,[V3 + R3 + g3 + G3]\}\right\} = 0$$

$$\frac{\partial L}{\partial Q_9} = \frac{1}{512}\left\{\frac{h9}{2}(1-x) - 9\{\frac{A2D2}{Q_9^2} - \frac{D2}{Q_9^2}\,[V2 + R2 + g2 + G2]\}\right\} = 0$$

$$\frac{\partial L}{\partial Q_{10}} = \frac{1}{512}\left\{\frac{h10}{2}(1-x) - \{\frac{A1D1}{Q_{10}^2} - \frac{D1}{Q_{10}^2}\,[V1 + R1 + g1 + G1]\}\right\} = 0$$

$$\frac{\partial L}{\partial \lambda 1} = -(Q_2 - Q_1) = 0 \quad \frac{\partial L}{\partial \lambda 2} = -(Q3 - Q2) = 0$$

$$Q_3 = Q_2 = Q_1 = \sqrt{\frac{2(D10(A10+V10+R10+g10+G10)+9D9(A9+V9+R9+G9+g9)+(36D3[A3+V3+R3+g3+G3)}{(1-x)(h1+9h2+36h3)}}$$

$$Q_4 = \sqrt{\frac{24D7\,[A7+V7+R7+g7+G7\,]}{h4(1-x)}}$$

$$Q_5 = \sqrt{\frac{2D6\,[A6+V6+R6+g6+G6\,]}{h5(1-x)}} \qquad Q_6 = \sqrt{\frac{2D5\,[A5+V5+R5+g5+G5\,]}{h6(1-x)}}$$

$$Q_7 = \sqrt{\frac{2D4[A4+V4+R4+g4+G4\,]}{h7(1-x)}} \quad Q_8 = \sqrt{\frac{2D3[A3+V3+R3+g3+G3\,]}{h8(1-x)}}$$

$$Q_9 = \sqrt{\frac{2D2[A2+V2+R2+g2+G2]}{h9(1-x)}} \quad Q_{10} = \sqrt{\frac{2D1[A1+V1+R1+g1+G1]}{h10\,(1-x)}}$$

The local optima is not obtained for K = 3,4,5,6,7,8 as some of the inequalities are not satisfied, therefore the value of K is set to be 9 and we get the local optimum which is as follows

Q1=Q2=Q3=Q4=Q5=Q6=Q7=Q8=Q9=Q10=

$$\sqrt{\frac{\begin{array}{c}2(D1[A1+V1+R1+g1+G1]+9D2\,[A2+V2+R2+g2+G2]+36D3\,[A3+V3+R3+g3+G3+84D4[A4+V4+R4+g4+G4]+\\ 126D5\,[A5+V5+R5+g5+G5]+126D6\,[A6+V6+R6+g6+G6]+84D7\,[A7+V7+R7+g7+G7]+36D8\,[A8+V8+R8+g8+G8]\\ +9D9[A9+V9+R9+g9+G9]+D10[A10+V10+R10+g10+G10])\end{array}}{(h1+9h2+36h3+84h4+126h5+126h6+84h7+36h8+9h9+h10\,)(1-x)}}$$

5. Numerical Example

Consider an inventory system with the following data

A = \$100 / cycle, h = \$5/ unit/ cycle, D = 50,000 units/ year, P = 75,000 units / year,

V= \$ 800,R = \$ 600, g = \$ 650, G = \$500, The optimal crisp order quantity is 12673 units.

Suppose if we represent these parameters as linguistic variables then they must be expressed in terms of decagonal fuzzy number such as

A=(94,96,98,100,102,104,106,108,110,112) h=(4.4,4.6,4.8,5,5.2,5.4,5.6,5.8,6,6.2) ,

V=(794,796,798,800,802,804,806,808,900,902) R=(594,596,598,600,602,604,606,608,610,612)

g =(644,646,648,650,652,654,656,658,700,702) G=(494,496,498,500,502,504,506,508,510,512)

D=(49994,49996,49998,50000,50002,50004,50006,50008,50010,50012) in this case 1-x = 0.33 is taken irrespective of the change in demand pattern.

The optimal fuzzy order quantity is 12672.9 units.

Conclusion

In this paper a step towards green approach is taken by devising an inventory model considering the activities of green research and product analysis. This work also encloses the need of fuzzy inventory model. The decagonal fuzzy number used has given better optimal solution than all other fuzzy numbers. This research work has laid a platform for the extension of Pascal's triangle graded mean approach. This work can be further developed by comparative analysis of other fuzzy numbers. This model benefits the PS to become GPS.

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