Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 98 (2016) 42701-42705

ISSN: 2229-712X

Soft Rough Sets in approximation spaces

I. Arockiarani and C. Antony Crispin Sweety
Nirmala College for women, Coimbatore,TamilNadu, India-18.

ARTICLE INFO

Article history:
Received: 29 March 2016;
Received in revised form:
13 September 2016;
Accepted: 23 September 2016;

Keywords
Rough sets,
Neutrosophic sets,
Relations.

ABSTRACT

A rough set, first described by Zdzisław I. Pawlak, is a formal approximation of a crisp set in terms of a pair of sets which give the lower and the upper approximation of the original set. Soft set theory is a generalization of fuzzy set theory, was proposed by Molodtsov to deal with uncertainties. In this paper, we present concepts of soft rough neutrosophic Sets and neutrosophic soft rough Sets, and investigate some of its properties. Furthermore, we develop a decision making approach to neutrosophic Soft Rough Sets and a numerical example is illustrated.

Introduction

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approach is based on the fuzzy set notion proposed by L. Zadeh [12]. Rough set theory proposed by Z. Pawlak in [10] presents still another attempt to this problem. Rough sets have been proposed for a very wide variety of applications. In particular, the rough set approach seems to be important for Artificial Intelligence and cognitive sciences, especially in machine learning, knowledge discovery, data mining, expert systems, approximate reasoning and pattern recognition. Inspite, all these theories have their inherit difficulties as pointed out by Molodsov [6].

To overcome these difficulties, in 1999 Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties.This so-called soft set theory seems to be free from the difficulties affecting the existing methods. The study of hybrid models combining soft sets with other mathematical structures is emerging as an active research topic of soft set theory [2, 4, 5, 8]. But these sets fail when we talk about indeterminate state which exits in the belief system.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [11]. In order to give a new approach to decision making problems, we combine a fuzzy soft relation with neutrosophic rough sets and propose the concept of neutrosophic soft rough sets. Then we can define the upper and lower approximations of any neutrosophic set on parameter set E. Like the traditional neutrosophic rough set models, neutrosophic soft rough sets can also be exploited to extend many practical applications in reality. Therefore, we propose a novel approach to decision making based on neutrosophic soft rough set theory.

Prelimnaries

Definition 2.1: [11]

A neutrosophic set A on the universe of discourse X is defined as

A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to\,]^-0, 1^+[$ and $^-0 \le T_A(x)+I_A(x)+F_A(x) \le 3^+$.

Definition 2.2: [1]

Let U be the initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a non-empty set A , A $\subset$ E .A pair (F,A) is called a soft set over U, where F is a mapping given by F: A $\to$ P(U)

Definition 2.3. [5]. Let (F, E) be a soft set over U. Then a subset of $U \times E$ called a crisp soft relation from U to E is uniquely defined by $= \{\langle (u, x), T_A(u, x)\rangle \mid (u, x) \in U \times E\}$, where T_A: $U \times E \to [0, 1]$,

$$T_A(u,x) = \begin{cases} 1, & (u,x) \in R \\ 0, & (u,x) \notin R \end{cases}$$

Definition 2.4. [4]. Let U be an initial universe set and let E be a universe set of parameters. A pair (F, E) is called a fuzzy soft set over U if F: $E \to F(U)$, where $F(U)$ is the set of all fuzzy subsets of U.

Definition 2.5.[1] Let U be a nonempty and finite universe of discourse and$R \subseteq U \times U$an arbitrary crisp relation on U. We define a set-valued function R_s: $U \to (U)$ by

$R_s(x) = \{y \in U \mid (x,y) \in R\}, x \in U$.

The pair (U, R) is called a crisp approximation space. For any $A \subseteq U$, the upper and lower approximations of A with respect to (U, R), denoted by $\overline{R}(A)$ and $\underline{R}(A)$, are defined, respectively, as follows:

$\overline{R}(A) = \{x \in U \mid R_s(x) \cap A \neq \phi\}$

$\underline{R}(A) = \{x \in U \mid R_s(x) \subseteq A\}$.

The pair $((A), R(A))$ is referred to as a crisp rough set, and $R, R : P(U) \to P(U)$ are, respectively, referred to as upper and lower crisp approximation operators induced from (U, R).

Example 2.6. Let U be a universal set, which is denoted by $U = \{u_1, u_2, u_3, u_4, u_5\}$. Let E be a set of parameters, where $= \{e_1, e_2, e_3, e_4\}$. Suppose that a soft set over U is defined as Follows

$(e_1) = \{u_1, u_3, u_4\}$, $F(e_2) = \{u_2, u_4\}$, $F(e_3) = \Phi$, $F(e_4) = U$.

Tele:
E-mail address: riosweety@gmail.com

Then the crisp soft relation on $U \times E$ is written by

$R = \{(u_1, e_1), (u_3, e_1), (u_4, e_1), (u_2, e_2), (u_4, e_2), (u_1, e_4), (u_2, e_4), (u_3, e_4), u_4, e_4), (u_5, e_4)\}$

From Definition 2.5, we can obtain

$(u_1) = \{e_1, e_4\}$, $(u_2) = \{e_2, e_4\}$, $R_s(u_3) = \{e_1, e_4\}$, $R_s(u_4) = \{e_1, e_2, e_4\}$, and $R_s(u_5) = \{e_4\}$.

If the set of parameter $A = \{e_2, e_3, e_4\}$, we have $\overline{R}(A) = \{u_2, u_5\}$ and $\underline{R}(A) = U$.

Rough Neutrosophic Soft Set

Definition 3.1. Let (U, E, R) be a crisp soft approximation space. For any A= $\{\langle x, T_A(x), I_A(x), F_A(x)\rangle /x \in E\} \in N(E)$, the lower upper approximatios of A with respect to (U, E, R), denoted by $\overline{R}(A)$ and $\underline{R}(A)$ are respectively defined as follows:

$\overline{R}(A) = \{\langle u, T_{\overline{R}(A)}(u), I_{\overline{R}(A)}(u), F_{\overline{R}(A)}(u)\rangle / u \in U\}$

$\underline{R}(A) = \{\langle u, T_{\underline{R}(A)}(u), I_{\underline{R}(A)}(u), F_{\underline{R}(A)}(u)\rangle / u \in U\}$ Where

$T_{\overline{R}(A)}(u) = \bigvee_{x\in R_s(u)} T_A(x)$, $\quad I_{\overline{R}(A)}(u) = \bigvee_{x\in R_s(u)} I_A(x)$,

$F_{\overline{R}(A)}(u) = \bigwedge_{x\in R_s(u)} F_A(x)$

$T_{\underline{R}(A)}(u) = \bigwedge_{x\in R_s(u)} T_A(x)$, $\quad I_{\underline{R}(A)}(u) = \bigwedge_{x\in R_s(u)} I_A(x)$,

$F_{\underline{R}(A)}(u) = \bigvee_{x\in R_s(u)} F_A(x)$

Remark 3.2. $\overline{R}(A)$ and $\underline{R}(A)$ are two neutrosophic sets on U and the pair $(\overline{R}(A), \underline{R}(A))$ is referred as a soft rough neutrosophic set of A with respect to (U, E, R) and $\overline{R}$ and $\underline{R}$: N(E) $\rightarrow$ N(U) are referred to as upper and lower soft rough neutrosophic approximation operator.

Example 3.3.

Let U be the universal set, which is denoted by $U = \{u_1, u_2, u_3, u_4, u_5\}$. Let be the set of parameters , where $E = \{e_1, e_2, e_3, e_{4,} e_5\}$. Suppose that a soft set over U is defined as

F $(e_1) = \{u_1, u_2\}$ F $(e_2) = \{u_1, u_2, u_3\}$ F $(e_3) = \{u_3, u_4\}$

F $(e_4) = \varphi$ F $(e_4) =$ U

The crisp soft relation on $U \times E$ is given by

$R=$

$\{(u_1,e_1),(u_2,e_1),(u_1,e_2),(u_2,e_2),(u_3,e_2),(u_3,e_3),(u_4,e_3),(u_1,e_5),(u_2,e_5),(u_3,e_5),(u_4,e_5),(u_5,e_5)\}$

$R_s(u_1) = \{e_1, e_2, e_5\}$ $R_s(u_2) = \{e_1, e_2, e_5\}$ $R_s(u_3) = \{e_2, e_3, e_5\}$ $R_s(u_4) = \{e_3, e_5\}$ $R_s(u_5) = \{e_5\}$

We can define a neutrosophic set $A \in$ N(E) as follows:

Let A = { e_1, e_2, e_5}

Then $\overline{R}(A) = \{ _1, u_2, u_5\}$ $\underline{R}(A) = U$

We can define an neutrosophic set $A \in$ N(E) as follows:

A= $\{\langle e_1, 0.5, 0.6, 0.2\rangle, \langle e_2, 0.8, 0.3, 0.16\rangle, \langle e_3, 0.4, 0.7, 0.3\rangle, \langle e_4, 0.6, 0.7, 0.1\rangle, \langle e_5, 0.5, 0.4, 0.4\rangle\}$.

we have

$T_{\overline{R}(A)}(u_1) = 0.8$, $I_{\overline{R}(A)}(u_1) = 0.6$, $F_{\overline{R}(A)}(u_1) = 0.2$

$T_{\overline{R}(A)}(u_2) = 0.8$, $I_{\overline{R}(A)}(u_2) = 0.6$, $F_{\overline{R}(A)}(u_2) = 0.2$

$T_{\overline{R}(A)}(u_3) = 0.8$, $I_{\overline{R}(A)}(u_3) = 0.7$, $F_{\overline{R}(A)}(u_3) = 0.3$

$T_{\overline{R}(A)}(u_4) = 0.5$, $I_{\overline{R}(A)}(u_4) = 0.7$, $F_{\overline{R}(A)}(u_4) = 0.3$

$T_{\overline{R}(A)}(u_5) = 0.5$, $I_{\overline{R}(A)}(u_5) = 0.4$, $F_{\overline{R}(A)}(u_5) = 0.4$

$T_{\underline{R}(A)}(u_1) = 0.5$, $I_{\underline{R}(A)}(u_1) = 0.3$, $F_{\underline{R}(A)}(u_1) = 0.4$

$T_{\underline{R}(A)}(u_2) = 0.5$, $I_{\underline{R}(A)}(u_2) = 0.3$, $F_{\underline{R}(A)}(u_2) = 0.6$

$T_{\underline{R}(A)}(u_3) = 0.4$, $I_{\underline{R}(A)}(u_3) = 0.3$, $F_{\underline{R}(A)}(u_3) = 0.6$

$T_{\underline{R}(A)}(u_4) = 0.4$, $I_{\underline{R}(A)}(u_4) = 0.4$, $F_{\underline{R}(A)}(u_4) = 0.4$

$T_{\underline{R}(A)}(u_5) = 0.5$, $I_{\underline{R}(A)}(u_5) = 0.4$, $F_{\underline{R}(A)}(u_5) = 0.4$

$\overline{R}(A) = \{\langle u_1 0.8, 0.6, 0.2\rangle, \langle u_2, 0.8, 0.6, 0.2\rangle, \langle u_3, 0.8, 0.7, 0.3\rangle, \langle u_4, 0.5, 0.7, 0.3\rangle, \langle u_5, 0.5, 0.4, 0.4\rangle\}$

$\underline{R}(A) = \{\langle u_1, 0.5, 0.3, 0.6\rangle, \langle u_2, 0.5, 0.3, 0.6\rangle, \langle u_3, 0.4, 0.3, 0.6\rangle, \langle u_4, 0.4, 0.4, 0.4\rangle, \langle u_5, 0.5, 0.4, 0.4\rangle\}$

Theorem 3.4. Let $(U,,)$ be a crisp soft approximation space. Then the upper and lower soft rough neutrosophic approximation operators $\overline{R}(A)$ and $\underline{R}(A)$ satisfies the following properties, $\forall A, B \in$ N(E), $\forall \alpha, \beta, \gamma \in [0, 1]$ with $\alpha + \beta + \gamma \le 3$.

(SRN1) $\underline{R}(A) = \sim\overline{R}(\sim A)$,

(SRN2) $\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$,

(SRN3) $A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B)$,

(SRN4) $\underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$,

(SRN5) $\overline{R}(A) = \sim \underline{R}(\sim A)$

(SRN6) $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$,

(SRN7) A $\subseteq B \Rightarrow \overline{R}(A) \subseteq \overline{R}(B)$

(SRN8) $\overline{R}(A \cap B) \subseteq \overline{R}(A) \cap \overline{R}(B)$,

Where $\sim A$is the complement of A.

Proof.

We only prove properties of the lower soft rough neutrosophic approximation operator $\overline{R}$ (A). The upper soft rough neutrosophic approximation operator $\underline{R}$ (A) can be proved similarly.

Then we have

$\sim\underline{R}\ (\sim A) = \{\langle u, F_{\overline{R}(\sim A)}(u), 1 - I_{\overline{R}(\sim A)}(u), T_{\overline{R}(\sim A)}(u)\rangle / u \in U\rangle\}$

$= \{\langle u, \bigvee_{x\in R_s(u)} T_{(\sim A)}(x), 1- \bigvee_{x\in R_s(u)} I_{(\sim A)}(x), \bigwedge_{x\in R_s(u)} F_{(\sim A)}(x)\rangle\}$

$= \{ u, \bigvee_{x\in R_s(u)} T_A(x), \bigvee_{x\in R_s(u)} I_A(x), \bigwedge_{x\in R_s(u)} F_A(x) / u \in U\rangle\}$

$= \{\langle u, T_{\overline{R}(A)}(u), I_{\overline{R}(A)}(u), F_{\overline{R}(A)}(u)\rangle / u \in U\} = \overline{R}(A)$

$\underline{R}(A \cap B) = \{\langle u, T_{\underline{R}(A\cap B)}(u), I_{\underline{R}(A\cap B)}(u), F_{\underline{R}(A\cap B)}(u) | u \in U\}$

$= \{\langle u, \bigwedge_{x\in R_s(u)} T_{A\cap B}(x), \bigwedge_{x\in R_s(u)} T_{A\cap B}(x), \bigvee_{x\in R_s(u)} F_{A\cap B}(x) | u \in U\}$

$= \{\langle u, \bigwedge_{x\in R_s(u)} (T_A(x) \wedge T_B(x)), \bigwedge_{x\in R_s(u)} (I_A(x) \wedge I_B(x)), \bigvee_{x\in R_s(u)} (F_A(x) \vee F_B(x)) | u \in U\}$

$= \{\langle u, T_{\underline{R}(A)}(u) \wedge T_{\underline{R}(B)}(u), I_{\underline{R}(A)}(u) \wedge I_{\underline{R}(B)}(u), F_{\underline{R}(A)}(u) \wedge F_{\underline{R}(B)}(u) | u \in U\}$

$= \underline{R}\ (A) \cap \underline{R}\ (B)$

Remark.3.5. The properties (1) and (5) shows that the upper and lower approximation operators $\overline{R}$ and $\underline{R}$ are dual to each other.

Example

A = $\{\langle e_1, 0.5, 0.6, 0.2\rangle, \langle e_2, 0.8, 0.3, 0.16\rangle, \langle e_3, 0.4, 0.7, 0.3\rangle, \langle e_4, 0.6, 0.7, 0.1\rangle, \langle e_5, 0.5, 0.4, 0.4\rangle\}$.

$\sim A = \{\langle e_1, 0.2, 0.4, 0.5\rangle, \langle e_2, 0.6, 0.7, 0.8\rangle, \langle e_3, 0.3, 0.3, 0.4\rangle, \langle e_4, 0.1, 0.3, 0.6\rangle, \langle e_5, 0.4, 0.6\ \ 0.5\rangle\}$.

$\overline{R}$ (~ A) = {⟨e_1, 0.6, 0.7, 0.5⟩ , ⟨e_2, 0.6,0.7, 0.5⟩ ,⟨e_3, 0.6,0.7, 0.4⟩ ,
⟨e_4, 0.3, 0.6, 0.4⟩, ⟨e_5, 0.1, 0.6, 0.5⟩}

~$\underline{R}$ (~A) = {⟨e_1, 0.5, 0.3, 0.6⟩ , ⟨e_2, 0.5,0.3, 0.6⟩ ,⟨e_3, 0.4,0.3, 0.6⟩ ,
⟨e_4, 0.4, 0.4, 0.4⟩, ⟨e_5, 0.5, 0.4, 0.4⟩} = $\underline{R}$(A)

(SRN) $\underline{R}$(A) = ~$\overline{R}$ (~ A), holds.

Example 3.6:

A = {⟨e_1, 0.5, 0.6, 0.2⟩ , ⟨e_2, 0.8,0.3, 0.16⟩ ,⟨e_3, 0.4,0.7, 0.3⟩ ,
⟨e_4, 0.6, 0.7, 0.1⟩, ⟨e_5, 0.5, 0.4, 0.4⟩} .

B = {⟨e_1, 0.7, 0.3, 0.2⟩ , ⟨e_2, 0.7,0.6, 0.3⟩ ,⟨e_3, 0.4,0.4, 0.5⟩ ,
⟨e_4, 0.5, 0.6, 0.1⟩, ⟨e_5, 0.4, 0.6, 0.7⟩} .

$A \cap B$ = {⟨e_1, 0.5, 0.3, 0.2⟩ , ⟨e_2, 0.4,0.3, 0.7⟩ ,⟨e_3, 0.4,0.4, 0.5⟩ ,
⟨e_4, 0.5, 0.6, 0.1⟩, ⟨e_5, 0.4, 0.4, 0.7⟩} .

$\underline{R}(A)$ ={⟨u_1, 0.5,0.3, 0.6⟩ , ⟨u_2, 0.5,0.3,0.6⟩ , ⟨u_3, 0.4, 0.3,0.6⟩,
⟨u_4, 0.4,0.4, 0.4⟩ , ⟨u_5, 0.5,0.4, 0.4⟩}

$\underline{R}(B)$ ={⟨u_1, 0.4,0.3, 0.7⟩ , ⟨u_2, 0.4,0.3,0.7⟩ , ⟨u_3, 0.4, 0.4,0.7⟩,
⟨u_4, 0.4,0.4, 0.7⟩ , ⟨u_5, 0.4,0.6, 0.7⟩}

$\underline{R}$(A) ∩ $\underline{R}$(B) = {⟨u_1, 0.4,0.3, 0.7⟩ , ⟨u_2, 0.4,0.3,0.7⟩ , ⟨u_3, 0.4, 0.3,0.7⟩,
⟨u_4, 0.4,0.4, 0.7⟩ , ⟨u_5, 0.4,0.4, 0.7}

$\underline{R}$ (A∩B) ={⟨u_1, 0.4,0.3, 0.7⟩ , ⟨u_2, 0.4,0.3,0.7⟩ , ⟨u_3, 0.4, 0.3,0.7⟩,
⟨u_4, 0.4,0.4, 0.7⟩ , ⟨u_5, 0.4,0.4, 0.7⟩}

Neutrosophic Soft Rough Sets

Definition 4.1. Let U be an initial universe set and let E be a universe set of parameters. For an arbitrary fuzzy soft relation R over U×E, the pair (U, R) is called a fuzzy soft approximation space. For any$A$∈N(E), we define the upper and lower soft approximations of A with respect to (U, E, R), denoted by $\overline{R}(A)$ and $\underline{R}(A)$, respectively, as follows

$$\overline{R}(A) = \{\langle u, T_{\overline{R}(A)}(u), I_{\overline{R}(A)}(u), F_{\overline{R}(A)}(u) \rangle / u \in U\}$$

$$\underline{R}(A) = \{\langle u, T_{\underline{R}(A)}(u), I_{\underline{R}(A)}(u), F_{\underline{R}(A)}(u) \rangle / u \in U\}$$

Where

$$T_{\overline{R}(A)}(u) = \bigvee_{x \in E} [T_R(u,x) \wedge T_A(x)],$$

$$I_{\overline{R}(A)}(u) = \bigvee_{x \in E} T_R(u,x) \wedge I_A(x)$$

$$F_{\overline{R}(A)}(u) = \bigwedge_{x \in E} [(1-T_R(u,x) \wedge F_A(x)]$$

$$T_{\underline{R}(A)}(u) = \bigwedge_{x \in E} [(1-T_R(u,x) \wedge T_A(x)]$$

$$I_{\underline{R}(A)}(u) = \bigwedge_{x \in E} [(1-T_R(u,x) \wedge T_A(x)]$$

$$F_{\underline{R}(A)}(u) = \bigvee_{x \in E} [I_R(u,x) \wedge I_A(x)]$$

Then the pair $(\overline{R}(A), \underline{R}(A))$ is called neutrosophic soft rough set and $\overline{R}(A)$ and $\underline{R}(A)$ ∈ N(U).

In fact, $T_{\overline{R}(A)}(u) + I_{\overline{R}(A)}(u) + F_{\overline{R}(A)}(u) = \bigwedge_{x \in E}$

$$T_R(u,x) \wedge T_A(x) + \bigvee_{x \in E} T_R(u,x) \wedge I_A(x) + \bigwedge_{x \in E} [(1-T_R(u,x)) \wedge F_A(x)] \leq 3$$

Hence, ($\overline{R}(A)$ ∈N(U). Similarly, we can obtain $\overline{R}(A)$ ∈N(U). So we call R,

R:N(E) →N(U) the upper and lower neutrosophic soft rough approximation operators, respectively.

Remark 4.2. Let (U, E, R) be a fuzzy soft approximation space. If A ∈ (E), then neutrosophic soft rough approximation operators $\overline{R}(A)$ and $\underline{R}(A)$ degenerate to the following forms:

$$\overline{R}(A) = \{\langle u, T_{\overline{R}(A)}(u) \rangle / u \in U\}$$

$$\underline{R}(A) = \{\langle u, T_{\underline{R}(A)}(u) \rangle / u \in U\}$$

$$T_{\overline{R}(A)}(u) = \bigvee_{x \in E} [T_R(u,x) \wedge T_A(x)]$$

$$T_{\underline{R}(A)}(u) = \bigwedge_{x \in E} [(1-T_R(u,x)) \wedge T_A(x)]$$

In this case, neutrosophic soft rough approximation operators $\overline{R}(A)$ and $\underline{R}(A)$ are identical with the soft fuzzy rough approximation operators. That is, neutrosophic soft rough approximation operators are an extension of the soft fuzzy rough approximation operators.

Example 4.3: Suppose that U = {u_1, u_2, u_3, u_4, u_5} is the set of five SMART PHONES under consideration of a decision maker to purchase. Let E be a parameter set, where

E = {e_1, e_2, e_3, e_4} = {operating system, apps, web access, messaging}.

Mr. X wants to purchase a phone which qualifies with the parameters of . Assume that Mr. X describes the phones by constructing a fuzzy soft set (F, E) which is a fuzzy soft relation R from U to E. And it is presented by a table as in the following form:

Table representing T_R(u,x), I_R(u,x), F_R(u,x)

R	e_1	e_2	e_3	e_4
u_1	0.7,0.8,0.28	0.9,0.8,0.1	0.7,0.8,0.3	0.6,0.7,0.2
u_2	0.8,0.5,0.7	0.6,0.6,0.2	0.5,0.4,0.7	0.5,0.6,0.7
u_3	0.5,0.6,0.6	0.5,0.4,0.6	0.6,0.5,0.5	0.4,0.5,0.3
u_4	0.9,0.3,0.3	0.4,0.2,0.7	0.4,0.5,0.4	0.3,0.4,0.6
u_5	0.5,0.7,0.1	0.3,0.1,0.7	0.5,0.6,0.7	0.7,0.6,0.2

Now suppose that Mr. X gives the optimum normal decision object A which is an neutrosophic subset defined as follows:

A= {⟨e_1, 0.7, 0.7, 0.1⟩ , ⟨e_2, 0.7,0.6, 0.5⟩ ,⟨e_3, 0.8,0.9, 0.1⟩ , ⟨e_4, 0.7, 0.5, 0.4⟩}

The Truth, Indeterminacy and False values are given as

$T_{\overline{R}(A)}(u_1) = 0.7$, $I_{\overline{R}(A)}(u_1) = 0.8$, $F_{\overline{R}(A)}(u_1) = 0.2$

$T_{\overline{R}(A)}(u_2) = 0.7$, $I_{\overline{R}(A)}(u_2) = 0.6$, $F_{\overline{R}(A)}(u_2) = 0.5$.

$T_{\overline{R}(A)}(u_3) = 0.6$, $I_{\overline{R}(A)}(u_3) = 0.6$, $F_{\overline{R}(A)}(u_3) = 0.4$

$T_{\overline{R}(A)}(u_4) = 0.7$, $I_{\overline{R}(A)}(u_4) = 0.5$, $F_{\overline{R}(A)}(u_4) = 0.3$

$T_{\overline{R}(A)}(u_5) = 0.7$, $I_{\overline{R}(A)}(u_5) = 0.7$, $F_{\overline{R}(A)}(u_5) = 0.1$

$T_{\underline{R}(A)}(u_1) = 0.7$, $I_{\underline{R}(A)}(u_1) = 0.5$, $F_{\underline{R}(A)}(u_1) = 0.5$

$T_{\underline{R}(A)}(u_2) = 0.7$, $I_{\underline{R}(A)}(u_2) = 0.5$, $F_{\underline{R}(A)}(u_2) = 0.5$

$T_{\underline{R}(A)}(u_3) = 0.6$, $I_{\underline{R}(A)}(u_3) = 0.5$, $F_{\underline{R}(A)}(u_3) = 0.5$

$T_{\underline{R}(A)}(u_4) = 0.6$, $I_{\underline{R}(A)}(u_4) = 0.5$, $F_{\underline{R}(A)}(u_4) = 0.4$

$T_{\underline{R}(A)}(u_5) = 0.6$, $I_{\underline{R}(A)}(u_5) = 0.5$, $F_{\underline{R}(A)}(u_5) = 0.4$

$\overline{R}(A)$) = {⟨u_1, 0.7, 0.8, 0.2⟩ ,⟨u_2, 0.7,0.6, 0.5⟩ ,⟨u_3, 0.6, 0.6, 0.4⟩,
⟨u_4, 0.7, 0.5, 0.3⟩ ,⟨u_5, 0.7, 0.7, 0.1⟩} ,

$\underline{R}(A)$ ={⟨u_1, 0.7,0.5, 0.5⟩ , ⟨u_2, 0.7,0.5,0.5⟩ , ⟨u_3, 0.6, 0.5,0.5⟩,
⟨u_4, 0.6,0.6, 0.4⟩ , ⟨u_5, 0.6,0.5, 0.4⟩}

Theorem 4.4. Let (U, E, R) be a fuzzy soft approximation space. Then the upper and lower neutrosophic soft rough approximation operators $\overline{R}(A)$ and $\underline{R}(A)$ satisfy the following properties, ∀ A, B ∈ N(E),

(NSR1) $\underline{R}$(A) =~$\overline{R}$ (~ A), (NSR2) $\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$,

(NSR3) $A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B)$,

(NSR4) $\underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$,

(NSR5) $\overline{R}(A) = \sim \underline{R}(\sim A)$

(NSR6) $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$,

(NSR7) $A \subseteq B \Rightarrow \overline{R}(A) \subseteq \overline{R}(B)$

(NSR8) $\overline{R}(A \cap B) \subseteq \overline{R}(A) \cap \overline{R}(B)$,

Where $\sim A$ is the complement of A.

Proof is similar to Theorem 3.4.

Example 4.5:

Consider A= {⟨e_1, 0.7, 0.7, 0.1⟩ , ⟨e_2, 0.7,0.6, 0.5⟩ ,⟨e_3, 0.8,0.9, 0.1⟩ , ⟨e_4, 0.6, 0.5, 0.4⟩}

$\sim A$= {⟨e_1, 0.1, 0.3, 0.7⟩, ⟨e_2, 0.5, 0.4, 0.7⟩ ,⟨e_3, 0.1,0.1, 0.8⟩ , ⟨e_4, 0.4, 0.5, 0.7⟩}

($\sim$ A) = {⟨u_1,0.5,0.5,0.7⟩ , ⟨u_2,0.5,0.5,0.7⟩,⟨u_3,0.5,0.5,0.6⟩,⟨u_4,0.4,0.4,0.6⟩,⟨u_5,0.4,0.5,0.6⟩},

$\overline{R}(\sim A)$= {⟨u_1,0.5,0.5,0.7⟩,⟨u_2,0.5,0.5,0.7⟩,⟨u_3,0.5,0.5,0.6⟩,⟨u_4,0.4,0.4,0.6⟩,⟨u_5,0.4,0.5,0.6⟩},

$\sim \overline{R}(\sim A)$= {⟨$u_1$,0.7,0.5,0.5⟩,⟨$u_2$,0.7,0.5,0.5⟩,⟨$u_3$,0.6,0.5,0.5⟩,⟨$u_4$,0.6,0.6,0.4⟩,⟨$u_5$,0.6,0.5,0.4⟩}

= $\underline{R}(A)$

Consider another set B = {⟨u_1,0.7,0.6,0.5⟩,⟨u_2,0.6,0.4,0.2⟩,⟨u_3,0.3,0.8,0.3⟩,⟨u_4,0.4,0.7,0.1⟩}

$\underline{R}(B)$= {⟨u_1,0.3,0.4,0.5⟩,⟨u_2,0.6,0.4,0.5⟩,⟨u_3,0.4,0.6,0.5⟩,⟨u_4,0.4,0.7,0.5⟩,⟨u_5,0.4,0.6,0.5⟩}

A ∩ B = {⟨e_1,0.7,0.6,0.5⟩,⟨e_2,0.6,0.4,0.5⟩,⟨e_3,0.3,0.8,0.1⟩,⟨e_4,0.4,0.5,0.4⟩}.

$\underline{R}$(A ∩ B) = A = {⟨e_1,0.7,0.7,0.1⟩,⟨e_2,0.7,0.6,0.5⟩,⟨e_3,0.8,0.9,0.1⟩,⟨e_4,0.6,0.5,0.4⟩}

We know

$\underline{R}(A)$= {⟨u_1,0.6,0.6,0.4⟩,⟨u_2,0.6,0.5,0.4⟩,⟨u_3,0.7,0.6,0.4⟩,⟨u_4,0.6,0.6,0.4⟩, ⟨u_5,0.6,0.5,0.4⟩}

$\underline{R}$(A)∩ $\underline{R}$(B) = ⟨u_1,0.3,0.4,0.5⟩,⟨u_2,0.6,0.4,0.5⟩,⟨u_3,0.4,0.6,0.5⟩,⟨u_4,0.4,0.6,0.5⟩,⟨u_5,0.4,0.5,0.5⟩}

It follows that (NSR2) holds. Similarly, we can verify that (NSR6) also holds.

In Example 4.4, we can note that $(A) \not\subseteq (A)$. But if R is referred to as a serial fuzzy soft relation from U to parameter set E, that is, for each $u \in U$, there exists $e \in E$ such that $R(u, x) = 1$, we have $R(A) \subseteq R(A)$.

Theorem 4.6. Let (U, E, R) be a fuzzy soft approximation space. If R is serial, then the upper and lower IF soft rough approximation operators $\overline{R}(A)$ and $\underline{R}(A)$ satisfy the following properties:

(1) $\underline{R}(\Phi) = \Phi$, $\overline{R}(E) = U$.

(2) $\underline{R}(A) \subseteq \overline{R}(A)$, $\forall A \in N(E)$.

Proof. It is straightforward.

Application Of Neutrosophic Soft Rough Sets In Decision Making

In the above-mentioned sections, to demonstrate the validity of that new models properties, several examples are carried out. For example, by data validation all upper and lower approximation operators $\overline{R}$ and $\underline{R}$ in the above examples are dual to each other. By those examples, the models are further understood, laying a good foundation for further study and application.

In this section, we present an approach to the decision making based on neutrosophic soft rough sets.

Let (U, E, R) be a fuzzy soft approximation space, where U is the universe of the discourse, E is the parameter set, and R is a fuzzy soft relation on $\times E$. Then we can give an algorithm based on IF soft rough sets with five steps.

First, according to their own needs, the decision makers can construct a fuzzy soft relation R from U to E, or fuzzy soft set (F, E) over U.

Second, for a certain decision evaluation problem, each person has various opinions on the attributes of the same parameter.

we can compute then neutrosophic soft rough approximation operators $\overline{R}(A)$ and $\underline{R}(A)$ of the optimum normal decision object A. Thus, we obtain two most close values $\overline{R}(A)$ and $\underline{R}(A)$ to the decision alternative uj of the universe set U.

Fourth two operations on two neutrosophic sests, shown as follows, for all A, $B \in N(U)$.

(i) Union operation:

$$A \tilde{\cup} B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$$

(ii) Intersection operation:

$$A \tilde{\cap} B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$$

(iii) Ring sum operation:

$A \oplus B = \{\langle x, T_A(x) + T_B(x) - I_A(x) + I_B(x), F_A(x) F_B(x) \rangle \mid x \in U\}$.

(iv) Ring product operation:

$A \otimes B = \{\langle x, T_A(x) \; T_B(x) \;, I_A(x) I_B(x), F_A(x) + F_B(x) - F_A(x) F_B(x) \rangle \mid x \in U\}$.

$\overline{R}(A)$ ={⟨$u_1$0.80,0.7, 0.5⟩ ,⟨u_2, 0.7,0.7, 0.3⟩ , ⟨u_3, 0.4, 0.5, 0.5⟩ , ⟨u_4, 0.4, 0.4, 0.6⟩ , ⟨u_5, 0.6, 0.5, 0.4,⟩} ,

$\underline{R}(A)$ ={⟨u_1, 0.4,0.5, 0.4⟩ , ⟨u_2, 0.5,0.5,0.4⟩ , ⟨u_3, 0.5, 0.6,0.4⟩ , ⟨u_4, 0.6,0.5, 0.4⟩ , ⟨u_5, 0.5,0.5, 0.4⟩}

$\overline{R}(A) \oplus \underline{R}(A) \Rightarrow$ H

={⟨u_1, 0.88,0.875, 0.02⟩ , ⟨u_2, 0.88,0.85,0.12⟩ , ⟨u_3, 0.7, 0.36,0.02⟩ ,

⟨u_4, 0.76, 0.7, 0.24⟩ , ⟨u_5, 0.8,0.75, 0.16⟩}

It is noted that the optimal decision is still u_1. Hence, Mr. X will purchase the phone u_1.

References

[1] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, "On some new operations in soft set theory," Computers and Mathematics with Applications, vol. 57, no. 9, 2009, pp. 1547–1553.

[2]C.Antony Crispin Sweety and I.Arockiarani,"Rough sets in neutrosophic approximation space." [accepted].

[3]K.T. Atanassov, Intuitionistic Fuzzy sets- Theory and Applications, Physica – Verlag, Springer – Verlag Company,neutrosophicew York (1999).

[4]P. K.Maji, R. Biswas, and A. R. Roy, "Fuzzy soft set," Journal of Fuzzy Mathematics, vol. 9, no. 3, 2001, pp. 589–602.

[5]P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," Computers and Mathematics with Applications, vol. 45,no. 4-5 ,2003 , pp. 55–562.

[6] D. Molodtsov, "Soft set theory—first results," Computers and Mathematics with Applications, vol. 37, no. 4-5, 1999, pp. 19–31.
[7] D. Molodtsov, The Theory of Soft Sets, URSS Publishers, Moscow, Russia, 2004 (Russian).
[8] J. H. Park, O. H. Kim and Y. C. Kwun, "Some properties of quivalence soft set relations", Computers and Mathematics with Applications, vol. 63, no. 6, 2012, pp. 1079–1088.
[9]Z. Pawlak, "Rough sets,"International Journal of Computer & Information Sciences, vol. 11, no. 5, 1982, pp. 145– 172.
[10] Z. Pawlak, Rough Sets-Theoretical Aspects to Reasoning about Data, Kluwer Academic Publisher, Boston, Mass, USA, 1991.
[11] F. Smarandache, Neutrosophic set, a generialization of the intuituionistics fuzzy sets, Inter.J.Pure Appl.Math.,24 (2005),287 -297.
[12]L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338-353 .