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Study of the Approximate Solution of Fuzzy Volterra-Fredholm Integral Equations by using (ADM)

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ABSTRACT

A numerical method for solving fuzzy Volterra-Fredholm integral equation of the second kind will is introduced. We convert a nonlinear fuzzy Volterra-Fredholm integral equation to a nonlinear system of Volterra-Fredholm integral equation in crisp case. We use Adomian Decomposition Method (ADM) to find the approximate solution of this system and hence obtain an approximation for fuzzy solution of the nonlinear fuzzy Volterra-Fredholm integral equation. Also, some numerical examples are included to demonstrate the validity and applicability of the proposed technique.

Introduction

The topic of fuzzy integral equations which has attracted growing interest for some time, in particular in relation to fuzzy control, has been developed in recent years. Babolian et. al [1] proposed another numerical procedure for solving fuzzy linear Fredholm integral of the second kind using Adomian method. Moreover, Friedman et. al [7] and Seikkala in [2] defined the fuzzy derivative and then some generalizations of that have been investigated in [3, 4]. Consequently, the fuzzy integral which is the same as that of Dubois and Prade in [5]. However, there are several research papers about obtaining the numerical integration of fuzzy-valued functions and solving fuzzy Volterra and Fredholm integral equations [6, 7, 8, 10, 11, 12, 13, 14, 17]. As we know the fuzzy differential and integral equations are one of the important part of the fuzzy analysis theory that play major role in numerical analysis. The concept of fuzzy numbers and arithmetic operations on it was introduced by Bede [3] which was further enriched by Dubois and Prade [17]. Also they [16] was introduced the concept of integration of fuzzy functions. Chang and Zadeh [15] studied on fuzzy mapping and control. Dubois and Prade [16, 17] made a significant contribution by introducing the concept of fuzzy numbers and presented a computational formula for operations on fuzzy numbers. Shao and Zhang [21] chose to define the integral of fuzzy function, using the Lebesgue-type concept for integration. One of the first applications of fuzzy integration was given by Wu and Ma who investigated the fuzzy Fredholm integral equation of the second kind. Recently, Ghadle and Ahmed [9] solved Volterra-Fredholm integral equations by using Legendre and Chebyshev collocation methods. The properties of Chebyshev or Legendre polynomials are used to reduce the system of Fredholm integral equations to a system of nonlinear algebraic equations, some mathematician have studied solution of fuzzy integral equation by numerical method [2, 21, 22]. Bildik and Inc studied the approximate solution by using modified decomposition method for nonlinear Volterra-Fredholm Integral Equations [11]. Wazwaz studied new algorithm for calculationg adomian polynomials for nonlinear operators [23]. Gohary found an approximate solution for a system of linear fuzzy Fredholm integral equation of the second kind with two variables which exploit hybrid Legendre and block-pulse functions, and Legendre wavelets. In present, we try to employ Adomian decomposition method for solving fuzzy nonlinear Volterra-Fredholm integral equation. The structure of this paper is organized as follows: In Section 2, the basic concepts of fuzzy number and fuzzy set are discussed. In Section 3, we convert a fuzzy nonlinear Volterra-Fredholm integral equation to a nonlinear system of

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Volterra-Fredholm integral equation of second kind in crisp case and approximate (FVFIE) with ADM. We present and describe the basic formulation of the ADM required for our subsequent development, in Section 4. Finally, in Section 5, we will give report on our numerical findings and demonstrate the accuracy of the proposed scheme by considering numerical examples, and a brief conclusion is given in Section 6.

Basic Concepts

Fuzzy numbers generalize classical real numbers and we can say that a fuzzy number is a fuzzy subset of the real line which has some additional properties. The concept of fuzzy number is vital for fuzzy analysis, fuzzy differential equations and fuzzy integral equations, and a very useful tool in several applications of fuzzy sets. Basic definition of fuzzy numbers is given in [6, 17].

Definition 2.1. A fuzzy number is a fuzzy set like $\omega: R \to [0, 1]$ with the following properties:

(i) ω is upper semi-continuous function,

(ii) ω is fuzzy convex, i.e,
$\omega(\lambda x + (1-\lambda)y) \geq min\{\omega(x), \omega(y)\} \quad \forall \quad x, y \in R$, $\lambda \in [0,1]$,

(iii) ω is normal, i. e, $\exists\, x_0 \in R$ for which $\omega(x_0) = 1$,

(iv) $\text{Sup}\,\omega = \{x \in R \mid \omega(x) > 0\}$ is the support of the ω, and

its closure cl(sup ω) is compact.

Let E be the set of all fuzzy numbers on R. The α-level set of a fuzzy number ω ∈ E, 0 ≤ α ≤1, denoted by $[\omega]_\alpha$, is defined as

$$[\omega]_\alpha = \begin{cases} \{x \in R : \omega(x) \geq \alpha\}, & 0 < \alpha \leq 1 \\ cl(\sup \omega), & \alpha = 0. \end{cases}$$

It is clear that the α-level set of a fuzzy number is a closed and bounded interval $[\underline{\omega}(\alpha), \overline{\omega}(\alpha)]$, where $\underline{\omega}(\alpha)$ denotes the left-hand end point of $[\omega]_\alpha$ and $\overline{\omega}(\alpha)$ denotes the right hand end point of $[\omega]_\alpha$. Since each $u \in R$ can be regarded as a fuzzy number $\tilde{u}$ defined by:

$$\tilde{u}(t) = \begin{cases} 1, & t = u \\ 0, & t \neq u. \end{cases}$$

An equivalent parametric definition is also given in [6] as:

Definition 2.2. A fuzzy number ω in parametric form is a pair $(\underline{\omega}, \overline{\omega})$ of functions $\underline{\omega}(\alpha)$ *and* $\overline{\omega}(\alpha)$, 0 ≤ α ≤ 1, which satisfy the following requirements:

i. $\underline{\omega}(\alpha)$ is a bounded non-decreasing left continuous function in (0,1], and right continuous at 0,

ii. $\overline{\omega}(\alpha)$ is a bounded non-increasing left continuous function in (0,1], and right continuous at 0,

iii. $\underline{\omega}(\alpha) \leq \overline{\omega}(\alpha)$, 0 ≤ α ≤ 1. A crisp number α is simply represented by $\underline{\omega}(\alpha) = \overline{\omega}(\alpha) = \alpha$, 0 ≤ α ≤ 1. We recall that for a < b < c which a, b, c ∈ R, the triangular fuzzy number $\omega = (a, b, c)$ determined by a, b, c are given such that $\underline{\omega}(\alpha) = a + (b - a)\alpha$ and $\overline{\omega}(\alpha) = c - (c - b)\alpha$ are the end points of the α-level sets, for all α ∈ [0,1]. The Hausdorff distance between fuzzy numbers given by
$d : E \times E \to R_+ \cup \{0\}$.

$$d(\omega, v) = \sup_{\alpha\in[0,1]} \max\{|\underline{\omega}(\alpha) - \underline{v}(\alpha)|, |\overline{\omega}(\alpha) - \overline{v}(\alpha)|\}$$

where $\omega = (\underline{\omega}(\alpha), \overline{\omega}(\alpha))$, $v = (\underline{v}(\alpha), \overline{v}(\alpha)) \subset R$ are utilized in [15]. Then, it is easy to see that d is a metric in E and has the following properties (see [3]):

(i) $d(\omega + \rho, v + \rho) = d(\omega, v), \forall \omega, v, \rho \in E$,

(ii) $d(k\omega, kv) = |k|\, d(\omega, v), \forall\, k \in R;\ \omega, v \in E$,

(iii) $d(\omega + v, \rho + e) \leq d(\omega, \rho) + d(v, e), \forall\, \omega, v, \rho, e \in E$,

(iv) (d, E) is a complete metric space.

Definition 2.3 [8]. Let $f : R \to E$, be a fuzzy valued function. If for arbitrary fixed $t_0 \in R$ and $\epsilon \geq 0, \delta \geq 0$ such that $|t - t_0| < \delta \Rightarrow (f(t), f(t_0)) < \epsilon$, f is said to be continuous.

Theorem 2.1 [17]. Let $f(x)$ be a fuzzy valued function on $[a, \infty)$ and it is represented by $(\underline{f}(x,\alpha), \overline{f}(x,\alpha))$. For any fixed $\alpha \in [a, b]$ assume $\underline{f}(x,\alpha)$ and $\overline{f}(x,\alpha)$ are Riemann-integrable on $[a, b]$ for every $b \geq a$, and assume there are two positive $\underline{M}(\alpha)$ and $\overline{M}(\alpha)$ such that $\int_a^b |\underline{f}(x,\alpha)|\, dx \leq \underline{M}(\alpha)$ and $\int_a^b |\overline{f}(x,\alpha)|\, dx \leq \overline{M}(\alpha)$ for every $b \geq a$. Then $f(x)$ is improper fuzzy Riemann integrable on $[a, \infty)$ and the improper fuzzy Riemann-integral is a fuzzy number. Furthermore, we have:

$$\int_a^\infty f(x)\, dx = (\int_a^\infty \underline{f}(x,\alpha)\, dx, \int_a^\infty \overline{f}(x,\alpha)\, dx)$$

Proposition 2.1 [18], If each of $f(x)$ and $g(x)$ is fuzzy-valued function and fuzzy Riemann integrable on $\Omega = [a, \infty)$ then $f(x) + g(x)$ is fuzzy Riemann integrable on Ω. Moreover, we have:

$$\int_\Omega (f(x) + g(x))dx = \int_\Omega f(x)dx + \int_\Omega g(x)dx$$

Definition 2.4. The integral of a fuzzy function was define in [1] by using the Riemann integral concept. Let $f : [a, b] \to E$, for each partition $P = t_0, t_1, \ldots, t_n$ of $[a, b]$ and for arbitrary $\xi_i \in [t_{i-1}, t_i], 1 \leq i \leq n$, suppose

$$R_p = \sum_{i=1}^{n} f(\xi_i)(t_i - t_{i-1})$$

$$\Delta := \max|t_i - t_{i-1}|, 1 \leq i \leq n$$

the definite integral of $f(t)$ over $[a, b]$ is

$$\int_a^b f(t)dt = \lim_{\Delta\to 0} R_p$$

Provided that this limit exists in the metric d. If the fuzzy function $f(t)$ is continuous in the metric d, its definite integral exists [7], and also

$$\underline{\int_a^b f(t,r)dt} = \int_a^b \underline{f}(t,r)dt$$

$$\overline{\int_a^b f(t,r)dt} = \int_a^b \overline{f}(t,r)dt$$

It should be noted that the fuzzy integral can be also defined

using the Lebesgue-type approach [22]. However, if $f(t)$ is continuous, both approaches yield the same value. More details about the properties of the fuzzy integral are given in [16, 18].

Fuzzy Volterra-Fredholm Integral Equation

Integral equations which are used in this section are fuzzy Volterra-Fredholm integral equations. Consider the following types fuzzy Volterra-Fredholm integral equations:

$$\widetilde{\omega}(x,y) = \tilde{f}(x,y) + \int_c^y \int_a^b \varphi(x,y,s,t)\widetilde{\omega}(s,t)\,dsdt \qquad (1)$$

and

$$\widetilde{\omega}(x) - \lambda_1 \int_c^y \varphi_1(x,r)\xi\big(r,\widetilde{\omega}(r)\big)\,dr - \lambda_2 \int_a^b \varphi_2(x,r)\mu\big(r,\widetilde{\omega}(r)\big)\,dr = \tilde{f}(x) \qquad (2)$$

where $\varphi, \varphi_1, \varphi_2$ are a known functions and $\widetilde{\omega}, \xi, \mu$ are unknown and known fuzzy valued functions, respectively. Now, parametric form of a linear fuzzy Volterra-Fredholm integral equation can be written as the following:

$$\underline{\omega}(x,y,r) = \underline{f}(x,y,r) + \int_c^y \int_a^b \Psi_1(x,y,s,t,\underline{\omega}(s,t,r),\overline{\omega}(s,t,r))dsdt \quad (3)$$

$$\overline{\omega}(x,y,r) = \overline{f}(x,y,r) + \int_c^y \int_a^b \Psi_2(x,y,s,t,\underline{\omega}(s,t,r),\overline{\omega}(s,t,r))dsdt \quad (4)$$

where

$$\tilde{f}(x,y) = \left(\underline{f}(x,y,r), \overline{f}(x,y,r)\right)$$

$$\widetilde{\omega}(x,y) = \left(\underline{\omega}(x,y,r), \overline{\omega}(x,y,r)\right)$$

and

$$\Psi_1(x,y,s,t,\underline{\omega}(s,t,r),\overline{\omega}(s,t,r)) = \begin{cases} \varphi(x,y,s,t)\underline{\omega}(s,t,r), & \varphi(x,y,s,t) \geq 0, \\ \varphi(x,y,s,t)\overline{\omega}(x,y,r), & \varphi(x,y,s,t) < 0. \end{cases}$$

$$\Psi_2(x,y,s,t,\underline{\omega}(s,t,r),\overline{\omega}(s,t,r)) = \begin{cases} \varphi(x,y,s,t)\overline{\omega}(s,t,r), & \varphi(x,y,s,t) \geq 0, \\ \varphi(x,y,s,t)\underline{\omega}(x,y,r), & \varphi(x,y,s,t) < 0. \end{cases}$$

for each $0 \le r \le 1$; $a \le x \le b$ and $c \le y \le d$. We solve Eqs. (1) and (2) by using Adomian method. The fuzzy Fredholm-Volterra integral equation of the second kind (2) is as follows:

$$\widetilde{\omega}(x) - \lambda_1 \int_c^y \varphi_1(x,r)\xi\big(r,\widetilde{\omega}(r)\big)\,dr - \lambda_2 \int_a^b \varphi_2(x,r)\mu\big(r,\widetilde{\omega}(r)\big)\,dr = \tilde{f}(x)$$

where $\lambda_1, \lambda_2 \geq 0$, $\tilde{f}$ is a fuzzy function of $x : a \le x \le b$, and φ_1, φ_2 are analytic functions on $[a,b]$. For solving in parametric form of Eq. (2), consider $(\underline{f}(x,t), \overline{f}(x,t))$ and, $\left(\underline{\omega}(x,t), \overline{\omega}(x,t)\right)$, $0 \le t \le 1$; $t \in [a,b]$ are parametric. Then, the parametric of Eq. (2) are as follows:

$$\underline{\omega}(x,t) - \lambda_1 \int_c^y \underline{\varphi_1(x,r)\xi\big(r,\omega(r,t)\big)}\,dr - \lambda_2 \int_a^b \underline{\varphi_2(x,r)\mu\big(r,\omega(r,t)\big)}\,dr = \underline{f}(x,t) \qquad (5)$$

and

$$\overline{\omega}(x,t) - \lambda_1 \int_c^y \overline{\varphi_1(x,r)\xi\big(r,\omega(r,t)\big)}\,dr - \lambda_2 \int_a^b \overline{\varphi_2(x,r)\mu\big(r,\omega(r,t)\big)}\,dr = \overline{f}(x,t) \qquad (6)$$

Let for $a \le t \le b$, we have

$$K_1\big(r,\underline{\omega},\overline{\omega}\big) = \min \varphi_1(r,\theta) : \underline{\omega}(r,t) \le \theta \le \overline{\omega}(r,t) \qquad (7)$$

$$K_2\big(r,\underline{\omega},\overline{\omega}\big) = \min \varphi_2(r,\theta) : \underline{\omega}(r,t) \le \theta \le \overline{\omega}(r,t) \qquad (8)$$

$$E_1\big(r,\underline{\omega},\overline{\omega}\big) = \mathrm{MAX}\, \varphi_1(r,\theta) : \underline{\omega}(r,t) \le \theta \le \overline{\omega}(r,t) \qquad (9)$$

$$E_2\big(r,\underline{\omega},\overline{\omega}\big) = \mathrm{MAX}\, \varphi_2(r,\theta) : \underline{\omega}(r,t) \le \theta \le \overline{\omega}(r,t) \qquad (10)$$

Then

$$\underline{\varphi_1(x,r)\zeta(r,\omega(r,t))} = \begin{cases} \varphi_1(x,r)k_1(r,\underline{\omega},\overline{\omega}), & \varphi_1(x,r) \geq 0, \\ \varphi_1(x,r)E_1(r,\underline{\omega},\overline{\omega}), & \varphi_1(x,r) < 0. \end{cases} \qquad (11)$$

$$\underline{\varphi_2(x,r)\mu(r,\omega(r,t))} = \begin{cases} \varphi_2(x,r)k_2(r,\underline{\omega},\overline{\omega}), & \varphi_2(x,r) \geq 0, \\ \varphi_2(x,r)E_2(r,\underline{\omega},\overline{\omega}), & \varphi_2(x,r) < 0. \end{cases} \qquad (12)$$

$$\overline{\varphi_1(x,r)\zeta(r,\omega(r,t))} = \begin{cases} \varphi_1(x,r)E_1(r,\underline{\omega},\overline{\omega}), & \varphi_1(x,r) \geq 0, \\ \varphi_1(x,r)K_1(r,\underline{\omega},\overline{\omega}), & \varphi_1(x,r) < 0. \end{cases} \quad (13)$$

$$\overline{\varphi_2(x,r)\mu(r,\omega(r,t))} = \begin{cases} \varphi_2(x,r)E_2(r,\underline{\omega},\overline{\omega}), & \varphi_2(x,r) \geq 0, \\ \varphi_2(x,r)K_2(r,\underline{\omega},\overline{\omega}), & \varphi_2(x,r) < 0. \end{cases} \quad (14)$$

for each 0 ≤ r ≤ 1 and $a \leq x \leq b$. We can see that Eq. (2) convert to a system of nonlinear Volterra-Fredholm integral equations in crisp case for each 0 ≤ t ≤ 1 and $a \leq r \leq b$. Now, we explain Adomian method as a numerical algorithm for approximating solution of this system of nonlinear integral equations in crisp case. Then, we find approximate solutions for $\tilde{\omega}(x)$, $a \leq x \leq b$.

Adomian Decomposition Method

The ADM has been applied to a wild class of functional equations [1, 10, 12, 19, 20, 22, 23] by scientists and engineers since the beginning of the 1980s. Adomian gives the solution as an infinite series usually converging to a solution consider the following fuzzy Fredholm-Volterra integral equation of the form:

$$\tilde{\omega}(x) - \lambda_1 \int_c^y \varphi_1(x,r)\xi\big(r,\tilde{\omega}(r)\big)\,dr - \lambda_2 \int_a^b \varphi_2(x,r)\mu\big(r,\tilde{\omega}(r)\big)\,dr = \tilde{f}(x)$$

we can write

$$\underline{\omega}(x,t) - \lambda_1 \int_c^y \underline{\varphi_1(x,r)\xi\big(r,\omega(r,t)\big)}\,dr - \lambda_2 \int_a^b \underline{\varphi_2(x,r)\mu\big(r,\omega(r,t)\big)}\,dr = \underline{f}(x,t)$$

and

$$\overline{\omega}(x,t) - \lambda_1 \int_c^y \overline{\varphi_1(x,r)\xi\big(r,\omega(r,t)\big)}\,dr - \lambda_2 \int_a^b \overline{\varphi_2(x,r)\mu\big(r,\omega(r,t)\big)}\,dr = \overline{f}(x,t)$$

The ADM assume an infinite series solution for the unknowns functions $[\underline{\omega},\overline{\omega}]$, given by

$$\underline{\omega}(x) = \sum_{i=0}^{\infty} \underline{\omega}_i(x)\,,\ \overline{\omega}(x) = \sum_{i=0}^{\infty} \overline{\omega}_i(x) \quad (15)$$

The nonlinear operators $\xi\big(r,\underline{\omega}(r,t)\big), \xi\big(r,\overline{\omega}(r,t)\big)$ and $\mu\big(r,\underline{\omega}(r,t)\big), \mu\big(r,\overline{\omega}(r,t)\big)$ into an infinite series of polynomials given by:

$$\xi\big(r,\underline{\omega}(r,t)\big) = \sum_{n=0}^{\infty} \underline{A}_n\,, \qquad \xi\big(r,\overline{\omega}(r,t)\big) = \sum_{n=0}^{\infty} \overline{A}_n$$

$$\mu\big(r,\underline{\omega}(r,t)\big) = \sum_{n=0}^{\infty} \underline{B}_n\,, \qquad \mu\big(r,\overline{\omega}(r,t)\big) = \sum_{n=0}^{\infty} \overline{B}_n \quad (16)$$

where the $\tilde{A}_n = [\underline{A}_n, \overline{A}_n]$, $\tilde{B}_n = [\underline{B}_n, \overline{B}_n]$, n ≥ 0, are the so-called Adomian polynomial defined by:

$$\underline{A}_n = \frac{1}{n}\Big[\frac{d^n}{d\phi^n}\Big(\zeta \sum_{i=0}^{n} \phi^i \underline{\omega}_i\Big)\Big]_{\phi=0},$$

$$\overline{A}_n = \frac{1}{n}\Big[\frac{d^n}{d\phi^n}\Big(\zeta \sum_{i=0}^{n} \phi^i \overline{\omega}_i\Big)\Big]_{\phi=0} \quad (17)$$

$$\underline{B}_n = \frac{1}{n}\Big[\frac{d^n}{d\phi^n}\Big(\mu \sum_{i=0}^{n} \phi^i \underline{\omega}_i\Big)\Big]_{\phi=0}, \quad (18)$$

$$\overline{B}_n = \frac{1}{n}\Big[\frac{d^n}{d\phi^n}\Big(\mu \sum_{i=0}^{n} \phi^i \overline{\omega}_i\Big)\Big]_{\phi=0} \quad (19)$$

(20)

substituting Eqs. (15) and (16) into Eqs. (5) and (6), n ≥ 0 we get:

$$\underline{\omega}_0 = \underline{f}(x,t)$$

$$\underline{\omega}_1 = \lambda_1 \int_a^x \underline{\varphi_1(x,r)A_0}\,dr + \lambda_2 \int_a^b \underline{\varphi_2(x,r)B_0}\,dr$$

.
.
.

$$\underline{\omega}_{n+1} = \lambda_1 \int_a^x \underline{\varphi_1(x,r)A_n}\,dr + \lambda_2 \int_a^b \underline{\varphi_2(x,r)B_n}\,dr \quad (21)$$

and

$$\overline{\omega}_0 = \overline{f}(x,t)$$

$$\overline{\omega}_1 = \lambda_1 \int_a^x \overline{\varphi_1(x,r)A_0}\,dr + \lambda_2 \int_a^x \overline{\varphi_2(x,r)B_0}\,dr$$

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.
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$$\overline{\omega}_{n+1} = \lambda_1 \int_a^x \overline{\varphi_1(x,r)A_n}\,dr + \lambda_2 \int_a^x \overline{\varphi_2(x,r)B_n}\,dr \quad (22)$$

we approximate $\tilde{\omega}(x,t) = [\underline{\omega}(x,t), \overline{\omega}(x,t)]$, by:

$$\underline{\eta}_n = \sum_{i=0}^{n-1} \underline{\omega}_i(x,t)\,,\ \overline{\eta}_n = \sum_{i=0}^{n-1} \overline{\omega}_i(x,t) \qquad (23)$$

where

$$\lim_{n\to\infty} \underline{\eta}_n = \underline{\omega}(x,t), \lim_{n\to\infty} \overline{\eta}_n = \overline{\omega}(x,t).$$

Now, we explain the Adomian method of the following fuzzy Volterra-Fredholm integral equation of the form:

$$\tilde{\omega}(x,y) = \tilde{f}(x,y) + \int_c^y \int_a^b \varphi(x,y,s,t)\tilde{\omega}(s,t)dsdt$$

and $\varphi(x,y,s,t) = [\varphi_{ij}(x,y,s,t)]$, $i = 1,\dots,n$; j = 1,...,n consider the i^{th} equation of (1) as:

$$\omega_i(x,y) = f_i(x,y) + \int_c^y \int_a^b \sum_{j=1}^{n} \varphi_{ij}(x,y,s,t)\omega_j(s,t)dsdt \qquad (24)$$

if

$$T_i(\omega_1,\omega_2,\dots,\omega_n)(x,y) = \int_c^y \int_a^b \sum_{j=1}^{n} \varphi_{ij}(x,y,s,t)\omega_j(s,t)dsdt$$

then, we have:

$$\omega_i(x,y) = f_i(x,y) + T_i(\omega_1,\omega_2,\dots,\omega_n)(x,y) \qquad (25)$$

to use the Adomian method, let

$$\omega_i(x,y) = \sum_{j=1}^{n} \omega_{ij}(x,y)$$

and

$$T_i(x,y) = \sum_{j=1}^{n} A_{ij}$$

where A_{ij} , $i = 0,\dots,n$ are polynomials depending on $\omega_{10},\dots,\omega_{1j},\dots,\omega_{n0},\dots,\omega_{nj}$ that called Adomian polynomials and then has been approximated the solution by $\rho_{ik}(x,y) = \sum_{j=0}^{k-1} \omega_{ij}(x,y)$, and by substituting in (24), we have:

$$\omega_i(x,y) = \sum_{j=0}^{\infty} \omega_{ij}(x,y) = f_i(x,y) + \sum_{j=0}^{\infty} A_{ij}(\omega_{10},\dots,\omega_{1j},\dots,\omega_{n0},\dots,\omega_{nj})$$

to use Adomian method, let

$$\omega_{i\beta}(x,y) = \sum_{j=0}^{\infty} \omega_{ij}(x,y)\beta^j \qquad (26)$$

$$T_{i\beta}(x,y) = \sum_{j=0}^{\infty} A_{ij}\beta^j \qquad (27)$$

where β is a parameter introduced for convenience.

By substituting (26) and (27) in (25), we have:

$$\sum_{m=0}^{\infty} \omega_{im}(x,y)\beta^m = f_i(x,y) + \int_c^y \int_a^b \sum_{j=1}^{n} \varphi_{ij}(x,y,s,t)\omega_j(s,t)\Big(\sum_{m=0}^{\infty} \omega_{im}(x,y)\beta^m\Big)dsdt \qquad (28)$$

equating powers of β on both sides of Eq. (28) gives:

$$\omega_{io}(x,y) = f_i(x,y)\omega_{i,k+1} = \int_c^y \int_a^b \sum_{j=1}^{n} \varphi_{ij}(x,y,s,t)\omega_{jk}(s,t)dsdt$$

we practice, of course, the sum of the infinite series has to be truncated at some order k. The quantity, can thus be reasonable approximation of the exact solution, provided k is sufficiently large, as k → ∞, the series converge smoothly toward the exact solution for 0 ≤ t ≤ 1 [24].

Numerical Examples

In this section, we used the ADM which is discussed of the previous section for solve two examples.

Example 1. Consider the fuzzy Volterra-Fredholm integral equation as:

$$\tilde{\omega}(x,y) - \int_0^y \int_{-1}^1 ze^r\tilde{\omega}(z,r)dzdr = (x + \sin y + \frac{2}{3} - \frac{2}{3}e^y)(\frac{t^2+t}{9}, \frac{4-t^3-t}{9}) \qquad (29)$$

By Eqs. (24), (28) and (29) for n = 3, 0 ≤ t ≤ 1 we obtain:

$$\tilde{\omega}(x,y) = (\underline{\omega}(x,y;t), \overline{\omega}(x,y;t)) = \frac{1}{9}(x + y - \frac{y^3}{3})(t^2+t, 4-t^3-t)$$

The exact and approximate solution of Adomian method in this example at x = 0.5 and y = 0.5 for n = 3 are shown in figure 1.

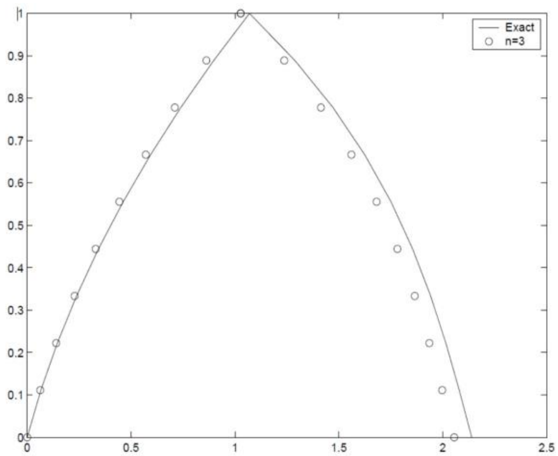

Figure 1: Compares the exact and approximate solutions.

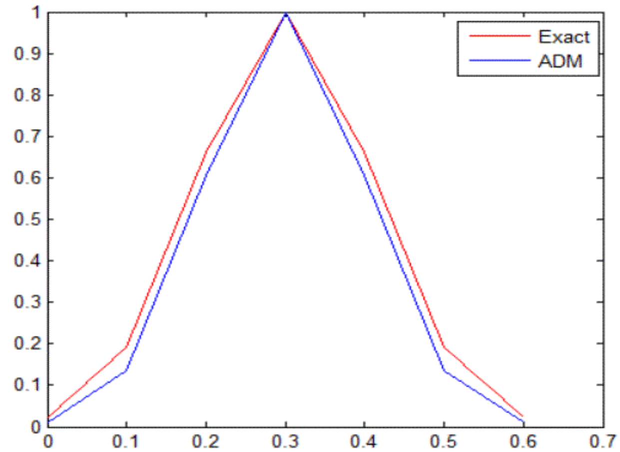

Figure 2: Compares the exact and approximate solutions.

Example 2. In this example, the method in this paper will be applied by the Mathematica 5.0 software package. Consider the fuzzy Fredholm-Volterra integral equation as follows:

$$\tilde{\omega}(x) - \int_0^x \sin(x)\sin(\frac{r}{2})\tilde{\omega}^3(r)dr - \int_0^{0.6} \sin(r)\sin(\frac{x}{2})(1+\tilde{\omega}^2(r))dr = \tilde{f}(x) \quad (30)$$

where

$$\underline{f}(x,t) = \sin(\frac{x}{2})(\frac{13}{15}(t^2+t) + \frac{2}{15}(4-t^3-t)) \; (31)$$

$$\overline{f}(x,t) = \sin(\frac{x}{2})(\frac{2}{15}(t^2+t) + \frac{13}{15}(4-t^3-t)) \; (32)$$

by equations. $(15-23)$ and $(30-32)$ with

$\varepsilon = 10^{-2}, n = 11, (t = 0.3, 0 \le x \le 0.6)$.

x	EXACT	APPROXIMATE (ADM)
0.1	0.2204663982	0.2203548375
0.2	0.3063488741	0.3062332542
0.3	0.4037996457	0.4035946723
0.4	0.5234862761	0.5233741235
0.6	0.6524855123	0.6523678927

Table 1: The exact and approximate solution of example 2.

Conclusion

We solved fuzzy Volterra-Fredholm integral equations by using ADM was converted to a system of Volterra-Fredholm integral equations. In this paper, the ADM has been successfully employed to obtain the approximate solution of the fuzzy Volterra-Fredholm integral equation. For this purpose, in examples we used ADM to find the approximate solution of this system and hence obtain an approximation for fuzzy solution of the fuzzy Volterra-Fredholm integral equation, are used to solve the linear and nonlinear system. The iterative method conjugate gradient method and Homotopy analysis method are proposed to solving the fuzzy Volterra-Fredholm integral equation.

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