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The Flow of a Saffman's Dusty Gas with Pressure-Dependent Viscosity through Porous Media

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ABSTRACT

Field equations of Saffman's dusty gas with pressure-dependent viscosity and variable number density through isotropic porous media of variable porosity are developed in this work. The porous microstructure, the Darcy resistance and the Forchheimer micro-inertial effects are accounted for in the intrinsic volume averaging process.

1. Introduction

Modelling dusty gas flow through porous media based on the continuum approach of Saffman's dusty gas model, [1], was first initiated by Barron and Hamdan, [2,3], in the early 1990's. Various other models of flow were subsequently developed and account for popular porous microstructures, flow conditions, and particle behavior, (cf. [4,5] and the references therein).

A missing aspect of the available models are ones dealing with fluid-particle mixtures in which the carrier fluid possesses a pressure-dependent viscosity. This aspect of modelling is initiated in the present work where Saffman's dusty gas equations are intrinsically volume averaged over a porous control volume of variable porosity. Viscosity of the carrier fluid is assumed to be a function of the fluid pressure, and the dust particle number density is a non-uniform function of position.

Effects of the porous microstructure are modelled based on existing geometric microstructure descriptions of porous media, and Darcy resistance and Forchheimer micro-inertia are accounted for in a manner that parallels recent work, [6,7]. The developed model can provide information on the effects of pressure-dependent viscosity of the carrier fluid on the dust particle behavior, and finds applications in liquid-dust separators under high pressures, in lubrication processes involving lubricants with a dispersed phase in addition to other potential industrial and environmental applications. For further details on the applications and available single-phase flow models with pressure-dependent viscosity, one is referred to the many excellent works listed in [8-14] and the references therein.

2. Model Development

The steady flow of an incompressible dusty gas, with a small concentration of dust particles per unit volume, through free-space is described by the following coupled set of field equations developed by Saffman, [1]:

Fluid-phase continuity equation

$$\nabla \bullet \vec{U} = 0 \quad \ldots(1)$$

Fluid-phase momentum equation

$$\rho\nabla \bullet \vec{U}\vec{U} = -\nabla P + \nabla \bullet \vec{T} + KN(\vec{V} - \vec{U}) \quad \ldots(2)$$

Dust-phase continuity equation

$$\nabla \bullet N\vec{V} = 0 \quad \ldots(3)$$

Dust-phase momentum equation

$$\nabla \bullet N\vec{V}\vec{V} = \frac{K}{m}N(\vec{U} - \vec{V}) \quad \ldots(4)$$

where

$$\vec{T} = \mu(\nabla\vec{U} + \nabla\vec{U}^T) \quad \ldots(5)$$

and $\vec{U}$ and $\vec{V}$ are the fluid-phase and dust-phase velocity fields, respectively, P is the fluid pressure, ρ is the fluid density, m is the mass of a dust particle, K is the Stokes' coefficient of resistance, N is the particle distribution (or the particle number density, that is, the number of particles per unit volume), and μ is the fluid viscosity coefficient.

Equations (1) through (4) represent a determinate system of eight scalar equations in the eight functions of position, $\vec{U}, \vec{V}, N$, and P. In order to develop a continuum model to describe the flow of a particle-fluid mixture with non-uniform particle distribution through an isotropic porous material of variable porosity, assuming that the fluid viscosity is a function of pressure, the above equations will be averaged over a Representative Elementary Volume (REV). Letting V be the bulk volume of the REV and V_φ its pore volume, the porosity of the REV (and that of the porous medium) is given by

$$\varphi = \frac{V_\varphi}{V}.$$

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Following [15] and [16], the following notation and rules for volume averaging are listed here for ease of reference.

The volumetric phase average of a quantity F (that is, the volumetric volume average of F over the bulk volume, V) is defined as:

$$<F> = \frac{1}{V}\int_{V_\phi} F dV \qquad \ldots(6)$$

and the intrinsic phase average (that is, the volumetric average of F over the effective pore space, V_ϕ) is defined as:

$$<F>_\varphi = \frac{1}{V_\varphi}\int_{V_\phi} F dV. \qquad \ldots(7)$$

The relationship between the volumetric phase average and the intrinsic phase average is obtained from equations (6), (7) and the definition of porosity, and takes the form:

$$<F> = \varphi <F>_\varphi . \qquad \ldots(8)$$

Averaging theorems are written in the following forms. Let F and H be volumetrically additive scalar quantities, $\vec{F}$ a vector quantity, and c a constant (whose average is itself), then:

(i)… $<cF> = c<F> = c\varphi <F>_\varphi$.

(ii)… $<\nabla F> = \varphi\nabla <F>_\varphi + \frac{1}{V}\int_S F^\circ \vec{n} dS$

where S is the surface area of the solid matrix in the REV that is in contact with the fluid, and $\vec{n}$ is the unit normal vector pointing into the solid. The quantity $F^\circ = F - <F>$ is the deviation of the averaged quantity from its true (microscopic) value.

(iii)… $<F \mu H> = <F> \mu <H>$
$= \varphi <F \mu H>_\varphi = \varphi <F>_\varphi \mu \varphi <H>_\varphi$

(iv)…
$<FH> = \varphi <FH>_\varphi = \varphi <F>_\varphi <H>_\varphi + \varphi <F^\circ H^\circ>_\varphi$

(v) $<\nabla \bullet \vec{F}> = \nabla \bullet \varphi <\vec{F}>_\varphi + \frac{1}{V}\int_S \vec{F} \bullet \vec{n} dS.$

(vi)... Due to the no-slip condition, a surface integral is zero if it contains the fluid velocity vector explicitly.

The above averaging rules are applied to equations (1) through (4), term by term, to obtain:

For fluid-phase:

Continuity Equation

$$\nabla \bullet \varphi <\vec{U}>_\varphi + \frac{1}{V}\int_S \vec{U} \bullet \vec{n} dS = 0. \qquad \ldots(9)$$

Momentum Equations

$$\rho\nabla \bullet \varphi <\vec{U}>_\varphi <\vec{U}>_\varphi + \rho\nabla \bullet \varphi <\vec{U}^\circ \vec{U}^\circ>_\varphi$$
$$+ \frac{\rho}{V}\int_S \vec{U}\vec{U} \bullet \vec{n} dS$$
$$= -\varphi\nabla <P>_\varphi - \frac{1}{V}\int_S P^\circ \vec{n} dS + \nabla \bullet \varphi <\vec{T}>_\varphi$$
$$+ \frac{1}{V}\int_S \vec{T} \bullet \vec{n} dS$$
$$+ K\varphi <N>_\varphi [<\vec{V}>_\varphi - <\vec{U}>_\varphi] + K\varphi[<N^\circ \vec{V}^\circ>_\varphi$$
$$- <N^\circ \vec{U}^\circ>_\varphi] \qquad \ldots.(10)$$

For dust-phase

Continuity Equation

$$\nabla \bullet \varphi <N>_\varphi <\vec{V}>_\varphi + \nabla \bullet \varphi <N^\circ \vec{V}^\circ>_\varphi + \frac{1}{V}\int_S N\vec{V} \bullet \vec{n} dS = 0 \qquad \ldots(11)$$

Momentum Equations

$$\nabla \bullet \varphi <N>_\varphi <\vec{V}>_\varphi <\vec{V}>_\varphi + \nabla \bullet \varphi <N^\circ \vec{V}^\circ \vec{V}^\circ>_\varphi + \frac{1}{V}\int_S N\vec{V}\vec{V} \bullet \vec{n} dS$$
$$= \frac{K}{m}\varphi <N>_\varphi (<\vec{U}>_\varphi - <\vec{V}>_\varphi) + \frac{K}{m}\varphi(<N^\circ \vec{U}^\circ>_\varphi - <N^\circ \vec{V}^\circ>_\varphi) \qquad \ldots(12)$$

3. Analysis of Surface Integrals and Deviation Terms

The deviation terms and surface integrals that appear in the averaged equations (9)-(12) contain information on the forces that are exerted by the porous matrix on the fluid and dust phases, and the interactions that take place between the phases involved. Pore space boundaries present additional solid boundary on which the fluid-phase experiences no-slip on its velocity and the dust-phase experiences additional friction that results in dust particle capture mechanisms, settling, and reflection into the flow field.

Tortuosity of the flow path and the converging-diverging pore structure could result in enhancing microscopic inertial effects or may influence dispersion of the dust particles. It is therefore important to accurately analyze the above surface integrals and deviation terms.

3.1 Analysis of the Deviation Terms

Deviations from microscopic values are present in the fluid-phase momentum equations and in the dust-phase continuity and momentum equations. Products of deviations have been identified with hydrodynamic dispersion of the average phase velocities in the porous medium [15, 16]. Hydrodynamic dispersion through porous media is the sum of mechanical dispersion due to tortuosity of the flow path in the porous microstructure, and molecular diffusion of vorticity, (cf. [4, 5] and the references therein). The inertial deviation terms, $\nabla \bullet \varphi <\vec{U}^\circ \vec{U}^\circ>_\varphi$ of equation (10) and in $\nabla \bullet \varphi <N^\circ \vec{V}^\circ \vec{V}^\circ>_\varphi$ of equation (12), involve averages of products of deviations of average phase velocities and are representative of mechanical dispersion due to the porous microstructure. We can write:

$$<\vec{U}^\circ \vec{U}^\circ>_\varphi = <\vec{U}^\circ>_\varphi <\vec{U}^\circ>_\varphi + <\vec{U}^{\circ\circ} \vec{U}^{\circ\circ}>_\varphi \qquad (13)$$

$$<N^\circ \vec{V}^\circ \vec{V}^\circ>_\varphi = <N^\circ>_\varphi <\vec{V}^\circ>_\varphi <\vec{V}^\circ>_\varphi + <N^{\circ\circ} \vec{V}^{\circ\circ} \vec{V}^{\circ\circ}>_\varphi . \qquad (14)$$

The leading terms on the right-hand-sides of (13) and (14) involve products of averages of deviations that are arguably small in porous media where velocity and porosity gradients are not high, hence can be neglected. However, they may be of significance in media with high porosity gradients, hence a need arises to model them using dynamic diffusivity, [4, 5].

The term $<N^\circ \vec{U}^\circ>_\varphi - <N^\circ \vec{V}^\circ>_\varphi$ represents dispersion of the dust particles due to fluctuations in the average relative velocity vector. This term can be written in the form:

$$<N^\circ \vec{U}^\circ>_\varphi - <N^\circ \vec{V}^\circ>_\varphi$$

$$=<N^{\circ}>_{\varphi}(<\vec{U}^{\circ}>_{\varphi}-<\vec{V}^{\circ}>_{\varphi})+(<N^{\infty}\vec{U}^{\infty}>_{\varphi}-<N^{\infty}\vec{V}^{\infty}>_{\varphi}). \quad \ldots(15)$$

in which the leading term on the right-hand-side involves the difference between average deviations of the dust and fluid velocities. If $<\vec{U}^{\circ}>_{\varphi}$ and $<\vec{V}^{\circ}>_{\varphi}$ are of similar magnitudes, their difference is small and can be neglected.

In cases where significant dispersion due to fluctuations in the average relative velocity vector exist, hydrodynamic dispersion may be modelled as a diffusion process involving a product of a diffusion coefficient vector, $\vec{\delta}$, and a number density driving differential, $<N>_{\varphi}-N_d$, where N_d is an average reference particle distribution, [4, 5]. We can thus write

$$<N^{\circ}>_{\varphi}[<\vec{U}^{\circ}>_{\varphi}-<\vec{V}^{\circ}>_{\varphi}]=\vec{\delta}[<N>_{\varphi}-N_d]. \quad \ldots(16)$$

The term $\nabla\bullet\varphi<\vec{T}>_{\varphi}$ appearing in the averaged fluid-phase momentum equation can be written as $\nabla\bullet\varphi<\mu\vec{I}>_{\varphi}$, where $\vec{I}=\nabla\vec{U}+\nabla\vec{U}^{T}$, and can be expanded into the form

$$\nabla\bullet\varphi<\vec{T}>_{\varphi}=\nabla\bullet\varphi<\mu\vec{I}>_{\varphi}=\nabla\bullet[\varphi<\mu>_{\varphi}<\vec{I}>_{\varphi}+\varphi<\mu^{\circ}\vec{I}^{\circ}>_{\varphi}]$$
$$=\nabla\bullet\varphi<\mu>_{\varphi}<\vec{I}>_{\varphi}+\nabla\bullet[\varphi<\mu^{\circ}>_{\varphi}<\vec{I}^{\circ}>_{\varphi}+\varphi<\mu^{\infty}\vec{I}^{\infty}>_{\varphi}] \quad \ldots(17)$$

The last term on the right-hand-side of (17) involves products of deviations from the average velocity gradients and deviations from the average viscosity, which are small in the absence of large velocity and viscosity gradients. Hence, they are ignored here and (17) takes the form

$$\nabla\bullet\varphi<\vec{T}>_{\varphi}=\nabla\bullet\varphi<\mu>_{\varphi}<\vec{I}>_{\varphi}. \quad \ldots(18)$$

3.2 Analysis of the Surface Integrals

The surface integrals appearing in continuity equations (9) and (11) can be evaluated by invoking Gauss' Divergence Theorem, and writing the surface integrals as:

$$\int_S \vec{U}\bullet\vec{n}dS=\int_{V_\varphi}\nabla\bullet\vec{U}dV \quad \ldots(19)$$

$$\int_S N\vec{V}\bullet\vec{n}dS=\int_S\nabla\bullet N\vec{V}dV. \quad \ldots(20)$$

Making use of continuity equations (1) and (3), the integrals (19) and (20) vanish.

The surface integrals $\int_S \vec{U}\vec{U}\bullet\vec{n}dS$ and $\int_S N(\vec{V}\vec{V}\bullet\vec{n})dS$ that appear in the fluid- and dust-phase momentum equations are representative of shear forces. In the absence of a dust-phase viscosity, particle shear is zero and the surface integral $\int_S N(\vec{V}\vec{V}\bullet\vec{n})dS$ vanishes. In addition, the vanishing of normal component of fluid velocity on solid boundary translates into a no-slip condition on the solid matrix, hence averaging rule (vii). Accordingly, the surface integral $\int_S \vec{U}\vec{U}\bullet\vec{n}dS$ vanishes.

The surface integrals, $-\frac{1}{V}\int_S P^{\circ}\vec{n}dS$ and $\frac{1}{V}\int_S\vec{T}\bullet\vec{n}dS$, that appear in the fluid-phase momentum equations can be combined into the surface filter $\frac{1}{V}\int_S(\vec{T}\bullet\vec{n}-\vec{n}P^{\circ})dS$. This form of surface integral has been abundantly analyzed and quantified in the literature on both single-phase and dusty gas flows in porous media, [15-24]. Since the solid porous matrix affects the dusty gas through the portion of the surface area of the solid that is in contact with it, the above surface integral contains the information necessary to identify and quantify the forces exerted by the porous matrix on the flowing fluid, and give rise to the Darcy resistance and the Forchheimer inertial effects. Darcy resistance, hence the above surface integral, in the case of dusty gas flow through a porous structure may be expressed in terms of the relative velocity as:

$$-\frac{\mu}{\eta}(<U>_{\varphi}-<V>_{\varphi}),$$

where η is the hydrodynamic permeability. While this seems natural in light of the fact that seepage flow in this case is expressible in terms of the relative velocity, the basis is missing for expressing the surface integral in this manner. Hence, the surface integral can be expressed in the form

$$\frac{1}{V}\int_S(\vec{T}\bullet\vec{n}-\vec{n}P^{\circ})dS=-\frac{\mu}{\eta}(<U>_{\varphi} \quad \ldots(21)$$

The analysis provided by in [15-24] points to the decomposition of the above surface integral into two parts: one is a shear force integral (which accounts for the viscous drag effects that predominate in the Darcy regime, that is, for small Reynolds number flow), and the other is an inertial force integral (which accounts for inertial drag effects that predominate in the Forchheimer regime, that is, for high Reynolds number flow).

$$=-\frac{1}{V}\int_S[-p^{\circ}\vec{n}+\mu\nabla\vec{U}\bullet\vec{n}]dS$$
$$-<\mu>_{\varphi}f\varphi<\vec{U}>_{\varphi}=-<\mu>_{\varphi}(f_1+f_2)\varphi<\vec{U}>_{\varphi} \quad \ldots(22)$$

where f_1 be the *velocity-independent* viscous shear geometric factor that depends on the geometry of the porous medium and gives rise to the Darcy resistance, and f_2 the *velocity-dependent* inertial geometric factor that gives rise to the Forchheimer inertial term. Furthermore, in terms of the factor f_1, hydrodynamic permeability, η, is given by (*cf.* [17]):

$$\eta=\frac{\varphi}{f_1}. \quad \ldots(23)$$

Expressions for f_1 and f_2 require a mathematical description of the porous matrix and its microstructure, (cf. [17,18] where detailed analysis of porous microstructures for various types of media is provided). It is customary, however, to express the Darcy resistance and the Forchheimer term as

$$-\frac{<\mu>_{\varphi}}{\eta}\varphi<\vec{U}>_{\varphi}$$

and

$$-\frac{\rho C_d}{\sqrt{\eta}}\varphi<\vec{U}>_{\varphi}\left|\varphi<\vec{U}>_{\varphi}\right|,$$

respectively, where C_d is the Forchheimer drag coefficient.

3.3 Final Forms of Averaged Equations

In light of the discussion in Sub-sections 3.1 and 3.2, the following values of deviation terms and surface integrals are substituted in equations (9)-(12):

$$\frac{1}{V}\int_S \vec{U}\bullet\vec{n}dS=\frac{1}{V}\int_S N\vec{V}\bullet\vec{n}dS=\frac{1}{V}\int_S N\vec{V}\vec{V}\bullet\vec{n}dS=\frac{\rho}{V}\int_S \vec{U}\vec{U}\bullet\vec{n}dS=0 \quad \dots(24)$$

$$\nabla\bullet\varphi<N^{\circ}\vec{V}^{\circ}>_{\varphi}=\nabla\bullet\varphi<N^{\circ}\vec{V}^{\circ}\vec{V}^{\circ}>_{\varphi}=\rho\nabla\bullet\varphi<\vec{U}^{\circ}\vec{U}^{\circ}>_{\varphi}$$

$$=\frac{K}{m}\varphi(<N^{\circ}\vec{U}^{\circ}>_{\varphi}-<N^{\circ}\vec{V}^{\circ}>_{\varphi})=0 \quad \dots(25)$$

$$\nabla\bullet\varphi<\vec{T}>_{\varphi}=\nabla\bullet\varphi<\mu>_{\varphi}<\vec{I}>_{\varphi} \quad \dots(26)$$

$$-\frac{1}{V}\int_S P^{\circ}\vec{n}dS+\frac{1}{V}\int_S \vec{T}\bullet\vec{n}dS=-\mu(f_1+f_2)\varphi<\vec{U}>_{\varphi} \quad \dots(27)$$

Letting $\vec{q}_f=\varphi<\vec{U}>_{\varphi}$, $\vec{q}_d=\varphi<\vec{V}>_{\varphi}$, $p=<P>_{\varphi}$, $\mu^*=\varphi<\mu>_{\varphi}$, $n=<N>_{\varphi}$, equations (9)-(12) take the following final form:

For fluid-phase

Continuity Equation

$$\nabla\bullet\vec{q}_f=0. \quad \dots(28)$$

Momentum Equations

Term by term averaging of the fluid-phase momentum equations is accomplished by applying averaging rules (i)-(vi) to obtain

$$\rho\nabla\bullet\frac{\vec{q}_f\vec{q}_f}{\varphi}=-\varphi\nabla p+\nabla\bullet\mu^*\left\{\nabla\left(\frac{\vec{q}_f}{\varphi}\right)+\left(\nabla\left(\frac{\vec{q}_f}{\varphi}\right)\right)^T\right\}-\frac{\mu^*}{\eta}\frac{\vec{q}_f}{\varphi}$$

$$-\frac{\rho C_d}{\sqrt{\eta}}\vec{q}_f\left|\vec{q}_f\right|+Kn[\vec{q}_d-\vec{q}_f] \quad \dots(29)$$

For dust-phase

Continuity Equation

$$\nabla\bullet n\vec{q}_d=0 \quad \dots(30)$$

Momentum Equations

$$\nabla\bullet n\vec{q}_d\vec{q}_d/\varphi=\frac{K}{m}n(\vec{q}_f-\vec{q}_d). \quad \dots(31)$$

It should be noted that in the dependence of viscosity on pressure, $\mu^*=\mu^*(p)$, the pressure function, α, that is used in [10-14] can be defined as

$$\alpha(p)=\frac{f_1}{\varphi}\mu^*. \quad \dots(32)$$

If (32) is used in (29), in the absence of the dust-phase, equation (29) reduces to a same model reported in [10-14].

For constant porosity media, equations (29) and (31) take the following forms, respectively:

$$\rho(\vec{q}_f\bullet\nabla)\vec{q}_f=-\nabla p^*+\nabla\bullet\mu^*\left\{\nabla\vec{q}_f+\left(\nabla\vec{q}_f\right)^T\right\}-\frac{\mu^*}{\eta}\vec{q}_f$$

$$-\frac{\rho C^*_d}{\sqrt{\eta}}\vec{q}_f\left|\vec{q}_f\right|+K^*n[\vec{q}_d-\vec{q}_f] \quad \dots(33)$$

$$m(\vec{q}_d\bullet\nabla)(n\vec{q}_d)=K^*n(\vec{q}_f-\vec{q}_d). \quad \dots(34)$$

where

$K^*=\varphi K$, $p^*=\varphi p$, $C_d{}^*=\varphi C_d$.

4. Conclusion

In this work, we used the method of intrinsic volume averaging to initiate the modelling of dusty gas flow through variable porosity media when the fluid-phase viscosity is a function of pressure.

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