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Investigations on Nuclear Counting System using Data Acceptance Tests

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ABSTRACT

This investigation uses Multi Channel Analyser (MCA) coupled with Gamma Ray Spectroscope (GRS) to investigate some common counting statistics used for radiation measurements of a ^{137}Cs gamma source. Few statistical tests involving 25 and 100 trials for data acceptance were applied to study the stability of counting system. The statistical analysis evaluated count data on on four primary criteria; the Ratio Test, *Chauvenet's* Criterion, the *Chi*-square test, and a control chart. The control chart also reflected almost accurate statistical data except for a minor error during few points of the 25 trial test. Fano factor was also evaluated for both trials to ascertain the measure of reliability and signal to noise ratio of the equipment. The results demonstrated that the counting system was fairly accurate, with a few exceptions. The Ratio Test, *Chauvenet's* Criterion, and the *Chi*-square test each, was passed successfully. After evaluating the statistical data, Poisson distribution was created to better analyze the data.

Introduction

Any scientific experiment, including radioactivity, is generally subjected to an error in measurement. Primarily two types of error exists for raw data; determinate (or systematic) and indeterminate (or random). The determinate errors includes the correctable factors such as the dead time of detector, impact of background counts or due to improper shielding of detector *etc*. whereas random errors cannot be eliminated as they may be arising from fluctuations in testing and measurement conditions and can be evaluated by statistical methods. In general, the reproducibility of data is a vital aspect but not in case of stochastically random processes of radioactivity[1].Within any given time interval the emitted radiation are subjected to unavoidable statistical fluctuations and different counts are observed in iteration as decay probability of each decaying atom is different. The statistical analysis makes it possible to ascertain the probability of count rate within certain limits of the true or average count rate. The nuclear counting statistics involves the framework to process the raw data and predict about the expected precision of derived quantities. The comparison of observed fluctuation with predicted result from statistical models can tell about existing abnormality in the counting system. A trial or the number of decays in a given interval is independent of all previous measurements, due to randomness of the undergoing processes [2]. For a large data, the dispersion or deviation from the mean count rate adapts in a predictable distribution. The shape of probability distribution function specifies the extent of internal fluctuations in the data set. The width of the curve about its mean value gives the relative measure of existing dispersion or scatter. For finite data the experimental mean value can be regarded true mean value and small deviations from the mean value are much more likely than large deviations. Poisson or Gaussian (Normal) distribution can be utilized to understand the statistical models followed by the observed inherent fluctuations. This framework plays an important role in ascertaining the effectiveness of measurement equipment and procedures, and to know if data belong to the same random distribution [3].

Theoretical Background

The present investigations were undertaken to evaluate the usefulness and accuracy of raw collected data by using data acceptance tests namely, T-ratio test, *Chauvenet's* Criterion, *Chi*-square test, control chart test and Fano factor analysis, each relying on different data information. The T-test, also called the ratio test, verifies the probability of occurrence of any two consecutive values. It is a rapid method to identify background noise affecting data collection in the counting system. Since the ratio test requires only two data points, so it can be implemented on initial observations in an investigation to fix any measurement errors [1]. Using the first two points x_1
and x_2,where

$$T \cong \frac{|x_1 - x_2|}{\sqrt{2}\sqrt{(x_1 + x_2)}} \qquad (1)$$

and if $T > 3.5$, then, there is less than a 1 in 2000 chance that the observed data is statistically accurate and this measurements should be rejected and the counting system must be recalibrated before taking fresh observations.

The *Chauvenet's* Criterion is used to reject statistically "bad data" or "wild points". This test identifies significant *outliers* that skew the data towards one direction so they can be discarded from the set of observations due to large deviations from the calculated mean [4]. For relatively small

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set of counts, *outlier* points can significantly change the mean and standard deviation [1].

If x_i and x_m are i^{th} trial and mean value respectively, then

$$\tau \cong \frac{|x_i - x_m|}{\sqrt{x_m}} \quad (2)$$

The standard table values helps in identifying any significant deviations from the expected values then rejecting them according to *Chauvenet's* Criterion [5].

The next assessment for *goodness of fit* is obtained by the *Chi*-square test to find out whether the observed data is part of the same or any other random distribution [6].

The *Chi*-square value is given by

$$\chi^2 = \frac{\sum_{i=1}^{N} (x_i - x_m)^2}{x_m} \quad (3)$$

The χ^2 values can be calculated for the entire data set and compared to the values in standard table using the value of statistical degrees of freedom as f = N-1 to find the probability function $P(\chi^2)$ where N is the number of measurement s the experimental mean has been obtained from the same data [7]. In case of large samples, for a perfectly fit to the Poisson distribution the χ^2 value is 0.50. The malfunction of setup is indicated by very large fluctuations in the data set. When $0.01 < P(\chi^2) < 0.90$, the data is considered acceptable. When $0.05 < P(\chi^2) < 0.10$ data is considered marginal and finally, when $0.90 < P(\chi^2) < 0.95$, then data should be rejected [8].

The control chart evaluations are to ascertain that dispersion in the data points. If points are too scattered then the experimental mean is not a true or faithful or effective representation of the entire data set. The acceptability of data can be ensured with its help. The smaller value of the standard deviation (SD = σ) implies the greater the reproducibility of measurement. In the control test, each data point is classified according to its location away from the mean line in terms of the number of standard deviations σ. If one data point exceeds the limit of ±3σ, then the measurement must be repeated as it has crossed the control limit (CL). If 2 any out of 3 consecutive data points are outside the limit of ±2σ then the measurement must be repeated and is referred as Warning Limit (WL). If any 4 data points exceed consecutively the limit of ±σ, then add another measurement in the counts. If this point also crosses the SD limit of ±σ, then the apparatus needs fixation before further use and should be recalibrated. Furthermore, if any 6 data points are on the same side of the mean line consecutively, and then add a 7^{th} point, if it lies on the same side of the mean line, then *Mean Line Rule* suggests that this data should be rejected and the equipment must be recalibrated.

The last test to measure of coefficient of variance is Fano factor or noise which gives the signal to noise ratio. It is used to account for reliability with which random variable could be estimated in the analysis [9]. These four data acceptance tests were applied to counts recorded from a gamma rays emitting ^{137}Cs source in 25 and 100 trials.

Materials and method

In the present investigations a radioactive source (^{137}Cs) was used which undergoes radioactive decay. The emitted gamma ray follows fixed mean rate but in a random manner as decay events are statistically independent [10]. It was assumed that background gamma rays from other sources also occur but at a fixed mean rate. Whenever an atom of ^{137}Cs decays (event), a burst of 661.7keV gamma ray are emitted which excites some of the NaI (Tl) molecules in the detector. In this process, they lose some energy and then scintillation is produced from the excited crystal. The intensity of photon so emitted, is dependent on the energy of the exciting radiation. These photons strike the photocathode located at the end of the photomultiplier (PMT) tube and eject photo-electrons by undergoing the photoelectric effect. The high voltage is applied from the power supply to maintain a large potential difference between the two ends of the PMT. Several electrodes, called as dynodes, are arranged along the length of the tube with increasing potential. As these electrons travel down the tube, they gain energy and get accelerated, then on striking another dynode releases furthermore electrons. These electrons are multiplied as they travel through a series of dynode layers. This causes an avalanche or cascade of ejected electrons, and finally an output electrical pulse is obtained at end of the photomultiplier tube. The output is a resulting slightly amplifed pulse, which is then fed into a preamplifer, where the signal is inverted and then fed to the amplifer. Typical gains of such dynode chains ranges from several thousand to one million and in the present setup coarse gain of 1K was used. This electrical pulse passes through a preamplifer and then in an amplifer for further amplification of the signal. This signal is then passed through a discriminator, which rejected all pulses below a certain threshold voltage. Finally, the resulting signal is fed to the counter which records the number of pulses received in tunable time intervals. The signal gain, the distance between source and detectors *etc*, are so adjusted that nearly mean count rates of 8.36 and 351.8 sec $^{-1}$ (Hz) are obtained. The numbers of counts were recorded for fixed time interval of 10 second for each trial of 25 and 100 data points respectively. The numbers of counts over a period of 100 seconds for each of these mean count rates were also recorded to ascertain the range of mean count rate. The schematics of system setup are illustrated in Fig.1. In this study, Multi-Channel Analyzer (8K MCA, Type MC 1000U, Make: Nucleonix, Hyderabad) was used which took several hundred channels counts simultaneously while retaining a low dead time, creating a more reliable and accurate spectrum. The PHAST MCA software was used for this study as default spectroscopy software, along with the other compatible equipments were provided by Nucleonix Systems, Hyderabad [11]. This unit in conjunction with PHAST allowed the simultaneous measurement of all peaks including photo peak, and calculated counts in the specific regions of interest.

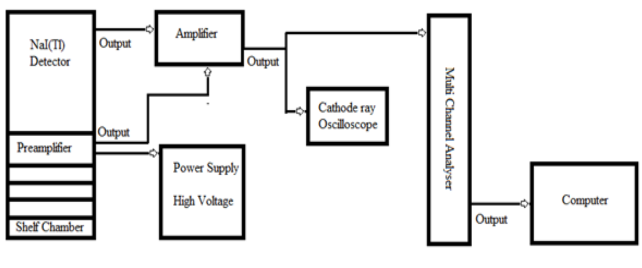


Fig.1 Schematic of Experimental setup

It can also be used to accurately identify unknown radio nuclides by measuring counts from reference samples and matching the photo peaks of the unknown nuclide with the known radioactive sources It consisted of a NaI(Tl) detector connected to both the Amplifier and the High Voltage Power Supply. The Multi-Channel Analyzer (MCA) and the Oscilloscope were both connected to the Amplifier, and the MCA connected straight to the computer.

Data and Results

Table 1. Statistical analysis of 25 trial data sample with mean, Ratio test, standard deviation and *Chi*-squared values

Count Data	Theoretical Std deviation	Ratio Test	*Chauvenet's* Criterion
366.9	19.15	0.24	0.80
357.8	18.92	0.02	0.32
358.5	18.93	0.39	0.36
344.0	18.55	0.26	0.42
353.6	18.80	0.55	0.09
333.1	18.25	0.03	1.00
334.1	18.28	0.12	0.94
338.5	18.40	0.21	0.71
330.8	18.19	1.05	1.12
370.3	19.24	0.29	0.99
359.1	18.95	0.02	0.39
359.8	18.97	0.04	0.43
358.4	18.93	0.43	0.35
342.4	18.5	0.25	0.50
351.6	18.75	0.06	0.01
349.2	18.69	0.07	0.14
346.7	18.62	0.51	0.27
327.8	18.11	0.56	1.28
348.5	18.67	0.56	0.18
369.6	19.22	0.03	0.95
368.6	19.20	0.21	0.89
360.4	18.98	0.17	0.46
354.0	18.81	0.08	0.12
357.1	18.90	0.06	0.28
354.7	18.83	--	0.15

Sample Mean: 351.82, Standard Deviation: 18.76, *Chi*-Square:10.17

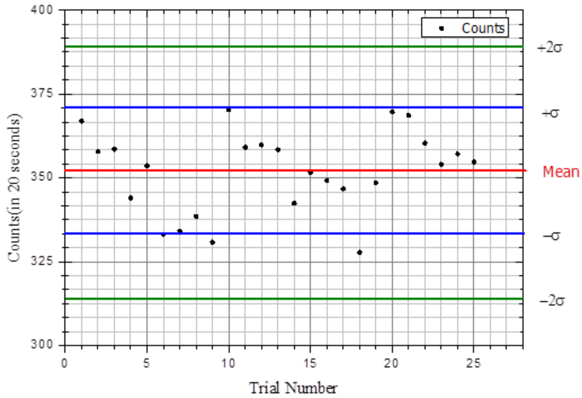
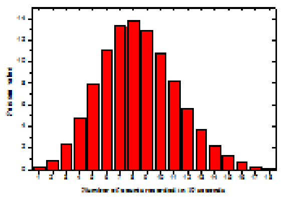


Fig 2. Control chart of data of 25 trial count data

Bin	Actual frequency	Poisson Value
0	0	0.20
1	0	0.82
2	2	2.28
3	3	4.76
4	2	7.96
5	8	11.1
6	13	13.3
7	10	13.8
9	13	10.8
10	18	8.17
11	9	5.69
12	5	3.66
13	1	2.19
14	1	1.22
15	2	0.64
16	0	0.31
17	0	0.15
18	1	0.20



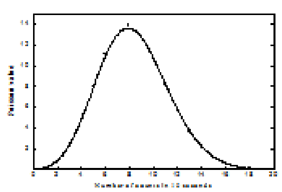


Fig 3. Poisson distribution curve of data collected in the 100 trial test

Table 2. Standard deviation values for each trial of 100

Count	SD	Count	SD	Count	SD	Count	SD	Count	SD
6	2.45	9	3.00	5	2.24	6	2.45	10	3.00
10	3.16	7	2.65	10	3.16	8	2.83	8	3.16
5	2.24	7	2.65	6	2.45	11	3.32	9	2.83
8	2.83	10	3.16	8	2.83	6	2.45	7	3.00
2	1.41	15	3.87	5	2.24	8	2.83	3	2.65
8	2.83	10	3.16	12	3.46	10	3.16	13	1.73
8	2.83	5	2.24	10	3.16	9	3.00	9	3.61
6	2.45	11	3.32	8	2.83	9	3.00	9	3.00
14	3.74	12	3.46	7	2.65	9	3.00	3	3.00
5	2.24	11	3.32	10	3.16	12	3.46	11	1.73
10	3.16	3	1.73	10	3.16	10	3.16	10	3.32
11	3.32	11	3.32	11	3.32	10	3.16	7	3.16
6	2.45	10	3.16	7	2.65	11	3.32	6	2.65
9	3.00	9	3.00	12	3.46	11	3.32	9	2.45
6	2.45	6	2.45	5	2.24	6	2.45	10	3.00
7	2.65	10	3.16	8	2.83	8	2.83	6	3.16
8	2.83	6	2.45	10	3.16	9	3.00	6	2.45
4	2.00	10	3.16	12	3.46	4	2.00	5	2.45
8	2.83	7	2.65	5	2.24	15	3.87	2	2.24
7	2.65	18	4.24	9	3.00	9	2.45	7	1.41

Sample mean=8.36, Standard deviation=2.85, *Chi*-square χ^2=96.30

Analysis and Discussion

Initially, only first five counts of the 25 trials were analyzed to ascertain the working of experimental setup. Table 1 gives standard deviation, Ratio test and *Chauvenet's* Criterion, assuming data of 25 trials adopting Poisson distribution. All first five standard deviation values of this data set were within 1 count of the true standard deviation, but not significantly farther from the experimental standard deviation. The difference existing between these values lies due to their different origin. The experimental standard deviation is derived from the difference between raw data and the experimental mean. For this reason, outliers dragged the standard deviation somewhat higher. True standard deviation, however, relies more exclusively on the experimental mean. Because none of the first five points are significant outliers, they all fall closer to the true standard deviation than to the experimental standard deviation.

The experimental standard deviation was computed as 12.21 and the true standard of deviation was calculated to be 18.76 counts and it was observed that both values failed to match exactly. The observed difference in two values is due to the more spread of raw data than would be deemed for that experimental mean. The Ratio Test was applied to the first five data points in the sample to test for statistically credible behavior. Also, because each value is less than the generally accepted value of 3.5, the data can be considered statistically credible. In fact, the ratio tests were not even close to the accepted value of 3.5, demonstrating the accuracy of the first five points evaluated.

Chauvenet's Criterion was applied to the first five trial results initially then to remaining trials to identify data points as not meeting the criteria. The results of the first five trials matched closely, verifying the code used for *Chauvenet's* Criterion. According to the Standard Table, the acceptable value for *Chauvenet's* Criterion for 25 trials is 2.33, because all 25 of the values for *Chauvenet's* Criterion are less than this value so this data can be deemed acceptable. Table1 shows the performed *Chi*-square test for the 25 trial sample data. For 25 trials, the obtained value of *Chi*-square for data is 10.27. As χ^2 falls in the probability range of 0.995- 0.975, so the data can be considered to be acceptable. Based on the *Chi*-square test performed, the data set is considered to be from the same random distribution and the data can be deemed acceptable [12].

Thus the statistical analysis of the 25 trial test demonstrates that the data sets can be relied on. Fig.2 shows a control chart created by using 25 data points set. The data points were evaluated by using the 4 different criteria to determine whether the counting system is under control or not. The control chart displayed the mean line (red line) as well as the standard deviation levels (blue line), warning levels (green line), and critical levels for the data set distribution. All 25 count trials are shown in relation to these levels of deviation. For this data, counts were recorded using ^{137}Cs for 10 second intervals. Based on the control limit (3σ), the warning limit (2σ), and the standard deviation (σ) rules, the data is acceptable and under control. As per the mean line rule, consecutive six measurements should not be above or below the line. From the chart, it is evident that the last 6 measurements are all above the mean line. However, two outliners of these values are very near to the mean line. Thus the data technically clears this test and it is reasonable to continue with further analysis. No data points had to be thrown out and all 4 acceptance tests were deemed reasonably valid. The only exception during the 25 trial test was the mean line test used with the control chart. Although this test was not officially passed, but two data points were very close to the mean line and the above average trend did not continue with the rest of the data. For these two reasons, the control chart was accepted as reasonably justified [13].

Table 2 gives the data of counts obtained from 100 trials test. The data analyzed for the 100 trial test using ^{137}Cs source kept at the lowest shelf to have lower count rate proved to be an accurate Poisson distribution with an experimental mean of 8.36 counts and a standard deviation of 2.852 counts per 10 second interval. The relatively low standard deviation reflects a less varied data set. Using the equation the Poisson Distribution and experimental mean, Poisson probabilities P(x) for the possible number of counts (x) per trial in a sample of 100 trials were computed by starting with the lowest number of counts recorded, *e.g.* x = 0 counts in ten seconds, then continued the process for next data trial. The theoretical standard deviation (σ) for data sample was computed. The theoretical standard deviation for this sample was 2.891. The experimental standard deviation calculated, 2.852 counts, was approximately 1.37% higher than the theoretical standard deviation. This discrepancy shows that the sample data was somewhat slightly varied than expected for the corresponding sample mean. Table 2 describes the development of histogram for 100 trial giving the bin value, actual frequency and the calculated Poisson value. Fig.3 describes the histogram of the number of trials *i.e.*, frequencies of counts recorded during the 100 trial test. Fig.3 shows the estimated Poisson distribution using the experimental data with a maximum between 13 and 14 counts. It should be noted that the maximum of the Poisson distribution only corresponds to the most common counts recorded and is not directly related to the experimental mean. It also depicts a curve of adapted Poisson distribution of data collected in 100 trial test. The background data was taken for 1000 seconds by removing the radioactive source from the vicinity of the detector. Then the standard deviation was calculated for all background counts calculated in 200 second intervals to record any abrupt change in data. The *Chi*-square test of 100 trials test yielded a χ^2 value 96.30, which falls in the acceptable probability range of 0.975 - 0.20 as given in the *Chi*-square reference table. The data set can be considered acceptable and from the same random distribution.

The measure of dispersion of probability distribution is given by Fano Noise F ($=\sigma^2/x_m$), it is the ratio of variance to mean value of random data, which is also referred as coefficient of variance. It gives the measure of reliability with which random variable could be estimated from the time window that on average contains several random events. Since in Poisson process, variance of counts is equal to mean count, thus its value becomes unity. In 25 trial data, Fano noise F= $[(18.76)^2]$ /351.82 = 1.0003 ~1.0. In 100 trial data, Fano noise F= $[(2.852)^2]$ /8.36 = 0.9729 ~1.0. [14].

Conclusions

The count rate for the counting system using ^{137}Cs of 25 and 100 trial data points was investigated using the statistical analysis. In both cases, the effectiveness of the counting system and the accuracy of the data collected were established. The acceptance tests were used to evaluate the data. Using all the tests together, a complete analysis of the system was performed. It was concluded that one can proceed with the experiment without any change in the setup. The apparatus is highly efficient and well organized. All tests were passed, so little follow-up is necessary. The only concern is the later part of the control chart test. It was worthwhile to investigate signal to noise ratio of the equipment used for the data. As these tests verified relatively small data sets, so the present system setup can be expanded to encompass a much larger set of data for detailed radiation analysis.

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