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Mixed Convection Couette Flow of a Nanofluid through a Vertical Channel

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ABSTRACT

The study of fully developed mixed convection Couette flow of a nano fluid between two vertical parallel plates, with asymmetric thermal and nanoparticle concentration conditions at the walls, filled by a nanofluid has been studied. The nanofluid model used in this paper takes into account the lower plate moving velocity, Brownian diffusion and the thermophoresis effects and the analysis is based on analytical solutions. Analytical expressions for the fully developed velocity, temperature and nanoparticle concentration profiles as well as for the Nusselt and Sherwood numbers at the left wall of the channel are obtained and analysed.

Introduction

Nanotechnology [1] has been broadly used in several industrial applications. It aims at manipulating the structure of the matter at the molecular level with the goal for innovation in virtually every industry and public endeavour including biological sciences, physical sciences, electronics cooling, transportation, the environment and national security. Low thermal conductivity of convection heat transfer fluids such as water, oil and ethylene glycol mixture is a primary limitation in enhancing the performance and the compactness of many engineering electronic devices. To overcome this drawback, there is a strong motivation to develop advanced heat transfer fluids with substantially higher conductivities to enhance thermal characteristics. Small particles (nano - particles) stay suspended much longer than larger particles. If particles settle rapidly (micro particles), more particles need to be added to replace the settled particles, resulting in extra cost and degradation in the heat transfer enhancement. As such an innovative way in improving thermal conductivities of a fluid is to suspend metallic nano-particles within it. The resulting mixture referred to as a nanofluid possesses a substantially larger thermal conductivity compared to that of traditional fluids. Nanofluids demonstrate anomalously high thermal conductivity, significant change in properties such as viscosity and specific heat in comparison to the base fluid, features which have attracted many researchers to perform in engineering applications. The popularity of nanofluids can be gauged from the researchers done by scientists for its frequent applications and can be found in the literature [2-6]. Mixed convection flows or combined free and forced convection flows occur in many technological and industrial applications in nature, e.g.; solar receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown and so on. The vertical channel is a frequently encountered configuration in thermal engineering equipments, for example, collector of solar energy, cooling devices of electronic etc. Due to its wide applications, numerous investigations have been done toward the understandings of fully developed mixed convection flow in a vertical channel filled with nanofluids. The case of fully developed mixed convection in a vertical channel filled by a nanofluid was solved by Grosan and Pop [7]. Das et al. [8] solved the problem of mixed convective magneto hydrodynamic flow in a vertical channel filled with nanofluids. Barletta et al. [9] have described a dual mixed convective flow in a vertical channel. Hang and Pop [10] have examined the fully developed mixed convection flow in a vertical channel filled with nanofluids. The mixed convection flow of a nanofluid in a vertical channel has been presented by Xu et al. [11]. Fakour et al. [12] have described the mixed convection flow of a nanofluid in a vertical channel.

Couette flow is important flow phenomenon with respect to engineering applications involving shear-driven flow such as aerodynamic heating and polymer technology [13]. In particular, the study of MHD Couette flow is useful for acquiring a better understanding of electrostatic precipitation and MHD power generators [14]. Singh [15] investigated the effect of natural convection on unsteady Couette flow. Jha [16] investigated natural convection in unsteady MHD Couette flow.

The aim of the present paper is to study the fully developed laminar mixed convection couette flow in a vertical channel filled with a nanofluid using the model proposed by Grosan [7]. Both walls of the channel are kept at constant temperatures and concentrations. The effects of the moving plate velocity, mixed convection, buoyancy ratio, Brownian motion and thermophoresis parameters are discussed about temperature and velocity distributions in the channel.

We consider a nanofluid that steadily flows between two vertical and parallel plane walls. The distance between the walls is L. We select a coordinate system in which the x-axis is aligned parallel to the gravitational acceleration vector g, but with the opposite direction (Fig. 1).

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Mathematical Analysis

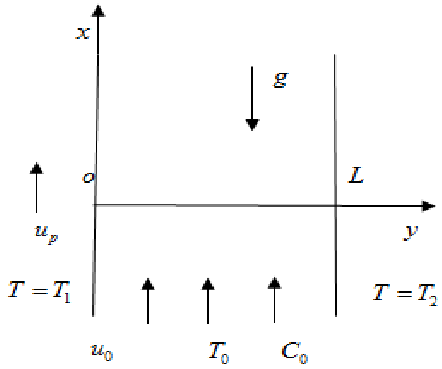

Fig 1. physical model and coordinate system

The y-axis is orthogonal to the channel walls, and the origin of the axes such that the positions of the channel walls are $y=0$ and $y=L$, respectively.

We consider a nanofluid that steadily flows between two vertical and parallel plane walls. The distance between the walls is L. We select a coordinate system in which the x-axis is aligned parallel to the gravitational acceleration vector g, but with the opposite direction (Fig. 1). The y-axis is orthogonal to the channel walls, and the origin of the axes such that the positions of the channel walls are $y=0$ and $y=L$, respectively. It is assumed that the temperature and the nanoparticles concentration at the wall at $y=0$ are T_1 and C_1, while the temperature and the nanoparticles concentration at the wall at $y=L$ are T_2 and C_2, respectively. It is also assumed that the values of velocity at the channel entrance are u_0 and lower plate velocity u_p.

The governing equations are:

$$-\frac{dp}{dx}+\mu\frac{d^2u}{dy^2}+[(1-c_0)\rho_{f0}\beta(T-T_0)-(\rho_p-\rho_{f0})(C-C_0)]g=0 \tag{1}$$

$$k\frac{d^2T}{dy^2}+(\rho c)_p\left[D_B\frac{dC}{dy}\frac{dT}{dy}+\left(\frac{D_T}{T_0}\right)\left(\frac{dT}{dy}\right)^2\right]=0 \tag{2}$$

$$D_B\frac{d^2C}{dy^2}+\left(\frac{D_T}{T_0}\right)\frac{d^2T}{dy^2}=0 \tag{3}$$

and are subjected to the boundary conditions

$$u=u_p,\quad T=T_1,\quad C=C_1 \quad \text{at} \quad y=0 \tag{4}$$

$$u=0,\quad T=T_2,\quad C=C_2 \quad \text{at} \quad y=L \tag{4}$$

In order to determine the pressure gradient from Eq. (1), the mass flux conservation Q is required

$$\int_0^1 u\,dy=Q \tag{5}$$

we introduce now the following dimensionless variables:

$$Y=\frac{y}{L},\ U(Y)=\frac{u(y)}{u_0},\ P(Y)=\frac{p(y)}{(\rho u_0^2)},$$

$$U_p=\frac{u_p}{u_0} \tag{6}$$

$$\theta(Y)=(T-T_0)/(T_2-T_0),$$

$$\phi(Y)=(C-C_0)/(C_2-C_0)$$

We assume that, $T_0=(T_1+T_2)/2$ and $C_0=(C_1+C_2)/2$. Substituting these variables into Eqs. (1)-(3). We get the following ordinary differential equations:

$$\frac{d^2U}{dY^2}+\frac{Gr}{\mathrm{Re}}\theta-Nr\phi+\alpha=0 \tag{7}$$

$$\frac{d^2\theta}{dY^2}+Nb\frac{d\theta}{dY}\frac{d\phi}{dY}+Nt\left(\frac{d\theta}{dY}\right)^2=0 \tag{8}$$

$$\frac{d^2\phi}{dY^2}+\frac{Nt}{Nb}\frac{d^2\theta}{dY^2}=0 \tag{9}$$

and the boundary conditions (4) become

$$U(0)=U_p,\quad \theta(0)=-1,\quad \phi(0)=-1 \tag{10}$$

$$U(1)=0,\quad \theta(1)=1,\quad \phi(1)=1 \tag{10}$$

along with the mass flux conservation relation (5), which becomes

$$\int_0^1 U\,dY=1 \tag{11}$$

Where we have taken $Q=U_0L$.

In the above equations, $\alpha=-dp/dx$ is the pressure parameter, Gr is the Grashof number, Re is the Reynolds number and Gr/Re is the mixed convection parameter. Further, Nr is the buoyancy-ratio parameter, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter, U_P is the lower plate moving velocity, T_0 is the reference temperature, C_0 is the reference nano-particles volume fraction concentration, ρ_p is the nano-particle mass density, ρ_f is the fluid density, $(\rho c)_f$ is the heat capacity of the fluid, and $(\rho c)_p$ is the effective heat capacity of the nano-particle material. These parameters are given by

$$\alpha=-dp/dx,\quad Gr=\frac{(1-C_0)g\beta(T_2-T_0)L^3}{\nu^2},$$

$$\mathrm{Re}=\frac{u_0L}{\nu},\quad Nr=\frac{g(\rho_p-\rho_{f0})(C_2-C_0)L^2}{\mu u_0}$$

$$Nb=\frac{D_B(C_2-C_0)}{k/(\rho c)_p},\quad Nt=\frac{D_T(T_2-T_0)}{T_0k/(\rho c)_p} \tag{12}$$

We notice that when Nr, Nb and Nt are all zero, Eqs. (7)-(9) involve just two dependent variables, namely U and θ.

Solution of the Problem

The equations (7)-(9) along with the boundary conditions (10) and the mass flux conservation equation (11) have been solved analytically. The general solution has the form

$$\theta(Y) = A_3 + A_4 e^{-A_2 Y}$$

$$\phi(Y) = -\frac{Nt}{Nb}\theta(Y) + A_1 Y - \frac{A_1}{2}$$

$$U(Y) = \left(U_0 + \frac{A_5}{A_2^2}\right) + \left(\frac{A_7}{2} + \frac{A_5 e^{-A_2 Y}}{A_2^2} + \frac{A_6}{2} + \frac{\alpha}{2}\right)Y - \quad (13)$$

$$\left(U_0 + \frac{A_5}{A_2^2}\right)Y - \left(\frac{A_7+\alpha}{2}\right)Y^2 - \frac{A_6}{6}Y^3 - \frac{A_5 e^{-A_2 Y}}{A_2^2}$$

$$\alpha = 12 + 12\frac{A_5}{A_2^3}\left(1-e^{-A_2}\right) - 6\frac{A_5}{A_2^2}\left(1+e^{-A_2}\right) - \frac{A_6}{12} - A_7 - 6U_0$$

where $A_i, i = 1-7$ are constants, which are given by

$$A_1 = 2\left(1+\frac{Nt}{Nb}\right), \quad A_2 = NbA_1, \quad A_3 = \frac{e^{A_2}+1}{e^{A_2}-1}$$

$$A_4 = \frac{-2e^{A_2}}{e^{A_2}-1}, \; A_5 = A_4\left(\frac{Gr}{\mathrm{Re}} + Nr\frac{Nt}{Nb}\right), \; A_6 = -NrA_1, \quad (14)$$

$$A_7 = A_3\left(\frac{Gr}{\mathrm{Re}} + Nr\frac{Nt}{Nb}\right) + Nr\frac{A_1}{2}$$

The physical quantities of interest are the Nusselt (Nu) and the Sherwood (Sh) numbers defined as

Graphs

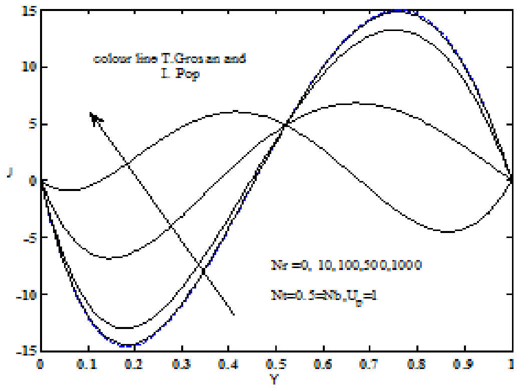

Fig 2. Variation of dimensionless velocity for different values of Nr.

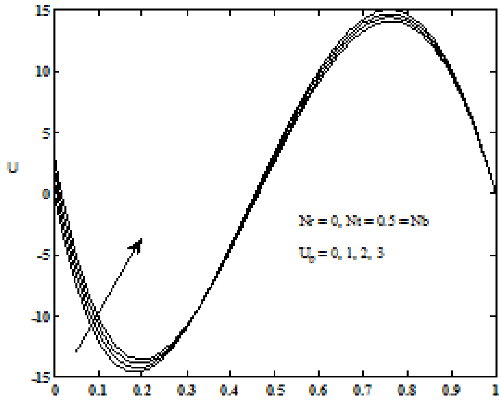

Fig 3. Variation of dimensionless velocity for different values of U_p

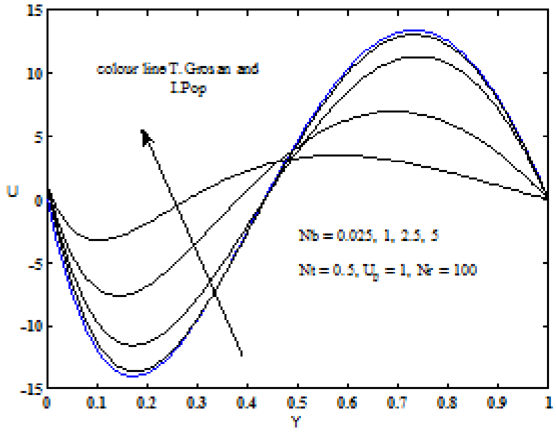

Fig 4. Variation of dimensionless velocity for different values of Nb.

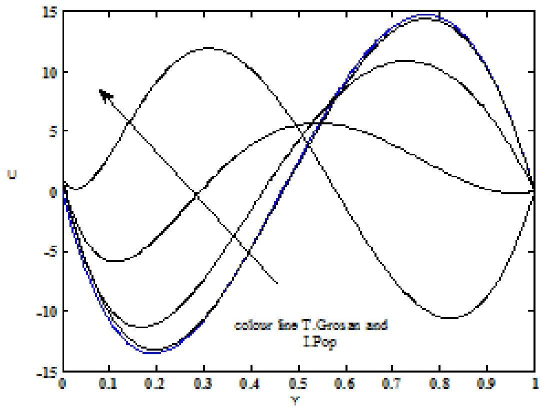

Fig 5. Variation of dimensionless velocity for different values of Nt = 0, 1, 2.5, 5. For fixed values of Nb = 0.5, Nr = 100, U_p = 1.

Table 1. Comparison between $U_p = 0$ and $U_p = 1$

$\frac{Gr}{\mathrm{Re}}$	Nr	Nt	Nb	α		$\theta'(0)$		$\phi'(0)$	
				U_p = 0	U_p = 1	U_p = 0	U_p = 1	U_p = 0	U_p = 1
				I.Pop	**Present**	**I.Pop**	**Present**	**I.Pop**	**Present**
0	0	0	0.2	12	6	2.4266	2.4266	2	2
	0	0.2	0.2	12	6	2.9055	2.9055	1.0945	1.0945
	5	0.2	0.2	11.2090	5.2090	2.9055	2.9055	1.0945	1.0945
1000	0	0	0.2	-67.7723	-73.7723	2.42659	2.42659	2	2
	0	0.2	0.2	-146.1992	-152.1992	2.90554	2.90554	1.0945	1.0945
	5	0.2	0.2	-146.9902	-152.9902	2.90554	2.90554	1.0945	1.0945

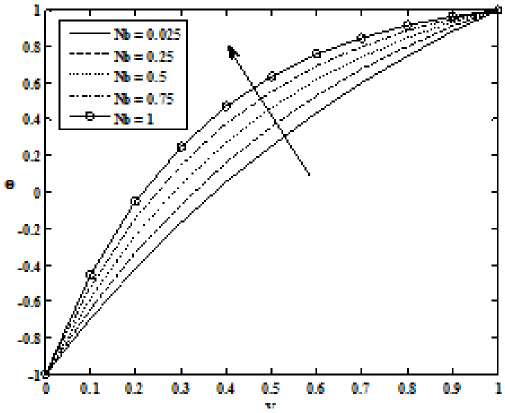

Fig 6. Variation of dimensionless temperature for fixed values of Nt=0.5.

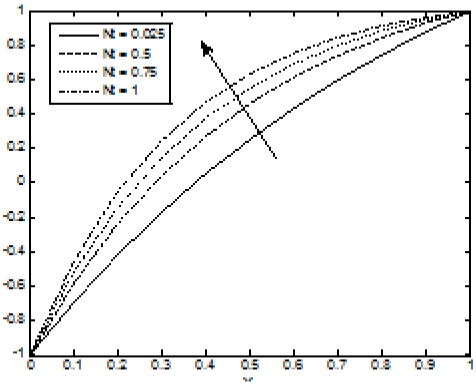

Fig 7. Variation of dimensionless temperature for fixed values of Nb=0.5.

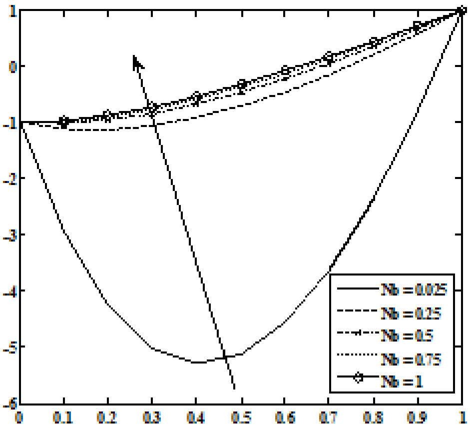

Fig 8. Variation of dimensionless concentration for fixed values of Nt = 0.5.

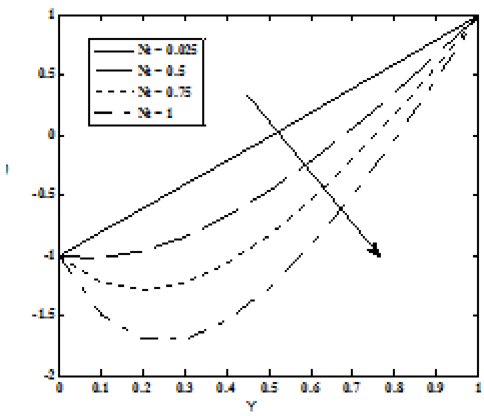

Fig 9. Variation of dimensionless concentration for fixed values of Nb=0.5.

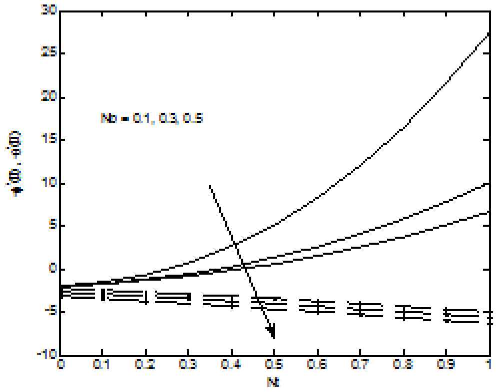

Fig 10. Variation of reduced Nusselt number $-\theta'(0)$ (dashed line) and reduced Sherwood number $-\phi'(0)$ (full line) with respect to Nb.

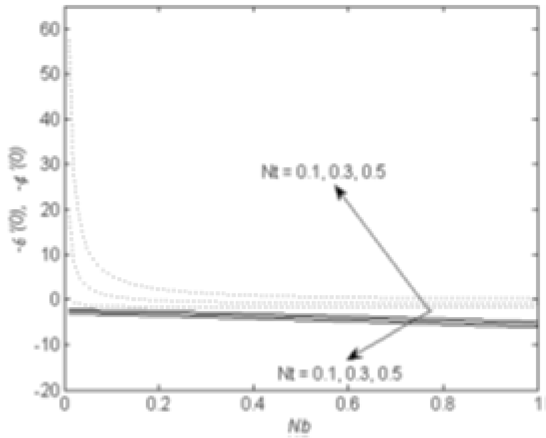

Fig.11. Variation of reduced Nusselt number $-\theta'(0)$ (full line) and reduced Sherwood number $-\phi'(0)$ (dotted line) with respect to Nt.

$$Nu = -\frac{L}{T_2 - T_0}\left(\frac{\partial T}{\partial y}\right)_{y=0},$$

$$Sh = -\frac{L}{C_2 - C_0}\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{15}$$

Substituting Eq. (6) into Eq. (13), we get

$$Nu = -\theta'(0), \quad Sh = -\phi'(0) \tag{16}$$

using Eqs. (13) and (14), the expressions for the Nusselt and Sherwood numbers defined by Eq.(16) become

$$Nu = -\theta'(0) = A_2 A_4,$$

$$Sh = -\phi'(0) = -\left(A_1 + A_2 A_4 \frac{Nt}{Nb}\right) \tag{17}$$

Results and Discussion

In order to check the analytical solution (13), we have the Eqs. (7)-(11) using analytical method for some values of the governing parameters. Representative for a fixed value of the mixed convection parameter Gr/Re =1000. It can be seen from Figs 2-5 increasing values of the buoyancy-ratio parameter Nr, the lower plate moving velocity U_p, the Brownian motion parameter Nb, and the thermophoresis parameter Nt, the flow decelerate near the hot wall and accelerate near the cold wall. However, we notice the presence of the reversed flow near both (hot and cold) wall for large values of Nr (see fig.2) and Nt (see fig. 5).

Fig.2. shows the effects of buoyancy-ratio parameter Nr on velocity profile. We observed that the velocity increases for increasing values of the buoyancy-ratio parameter Nr. The effects U_p on velocity distribution are presented in Fig.3. From this figure we noticed that the velocity increases as lower plate velocity increases. Fig.4 depicts the effects of Brownian motion parameter Nb on velocity. From this figure we observed that the velocity increases as the values of Nb increasing in case of cooling of the plate. The effect of thermophoresis parameter Nt on velocity is shown in fig.5. It shows that the velocity increases as Nt increases. Figs. 6 and 7 represent the effects of Brownian motion parameter Nb and thermophoresis parameter Nt on temperature. An increase in the values of Nb or Nt the temperature increases.

Figs.8 and 9 represent the profiles of concentration distribution which increases with the increase of Brownian motion parameter Nb and decreases with the increase of thermophoresis parameter Nt. Finally Figs.10 and 11 represent the variation of reduced Nusselt number $-\theta'(0)$ (dashed line) and reduced Sherwood number $-\phi'(0)$ (full line) with respect to Nb and Nt. These figures show that the Nusselt number decreases with the increasing of Nb and Nt, while the Sherwood number decreases with the increasing of Nb and increases with the increasing of Nt, which is in agreement with the temperature and the nanoparticle volume fraction profiles shown in figs 5,6,7 and 8. We notice that the reduced Sherwood number is more sensitive with the variation of the parameters Nb and Nt than the reduced Nusselt number.The comparison is shown in Table 1 where a very good agreement is seen. Therefore, we are confident that the obtained results for the present problem are correct. The temperature is enhancing with increasing Nb or Nt.

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