



U. S. Raiput and Neetu kanaujia/Elixir Appl. Math. 99 (2016) 43202-43207 Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics



Elixir Appl. Math. 99 (2016) 43202-43207

MHD Flow through Porous Media past an Impulsively Started Vertical Plate with Variable Temperature in the Presence of Hall Current

U. S. Rajput and Neetu kanaujia*

Department of Mathematics and Astronomy, University of Lucknow, Lucknow – U.P, India.

ARTICLE INFO							
Article history:							
Received: 5 September 2016;							
Received in revised form:							
4 October 2016;							
Accepted: 14 October 2016;							

Keywords MHD, Hall Current, Porous Media, Variable Temperature.

ABSTRACT

In the present paper, MHD flow through porous media past an impulsively started vertical plate with variable temperature in the presence of Hall current is studied. The fluid considered is electrically conducting, absorbing-emitting radiation in a nonscattering medium. The Laplace transform technique has been used to find the solutions for the velocity profile and Skin friction. The velocity profile and skin friction have been studied for different parameters like Prandtl number, Hall parameter, magnetic field parameter, Thermal Grashof number, and time. The effect of parameters are shown graphically and the value of the Skin-friction for different parameters has been tabulated.

© 2016 Elixir All rights reserved.

Introduction

In recent years, MHD flow through porous media became very important in the field of research because it is applicable in many branches of science and technology. Manyonge et al [4] have studied Steady MHD Poiseuille Flow between two infinite parallel porous plates in an inclined magnetic field. Ram et al [7] studied Hall effects on heat and mass transfer flow through porous medium. Ahmed et al [6] have studied unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. Sulochana [1] have analyzed Hall effects on unsteady MHD three dimensional flow through a porous medium in a rotating parallel plate channel with effect of inclined magnetic field. Rajput and Sahu [2] have studied effects of chemical reactions on free convection MHD past an exponentially accelerated infinite vertical plate through a porous medium with variable temperature and mass diffusion. Rajput and Kumar [5] have studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. We are extending their [5] work to study the effect of Hall current in the flow model.

The effect of Hall current on the velocity is observed with the help of graphs, and the skin friction has been tabulated.

Mathematical Analysis

An unsteady viscous incompressible electrically conducting fluid past an impulsively started vertical plate is considered here. The plate is electrically non-conducting. A uniform magnetic field B is assumed to be applied on the flow. Initially, at time $t \le 0$ the temperature of the fluid and the plates are same as T_{∞} , and the concentration of the fluid is C_{∞} . At time t > 0, temperature of the plate is raised to T_w and the concentration of the plate is C_w . Using the relation $\nabla \cdot B = 0$, for the magnetic field $\overline{B} =$

 (B_x, B_y, B_z) , we obtain B_y (say B_0) = constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field.

Due to Hall effect, there will be two components of the momentum equation, which are as under. The usual assumptions have been taken in to consideration. The fluid model is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_{\infty}) - \frac{\sigma B_0^2}{\rho(1 + m^2)} (u + mw) - \frac{\upsilon}{K} u, \qquad (1)$$

$$\frac{\partial W}{\partial t} = \upsilon \frac{\partial W}{\partial y^2} - \frac{\partial B_0}{\rho(1+m^2)} (w - mu) - \frac{\partial}{K} w,$$
(2)

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P} \frac{\partial^2 T}{\partial y^2}.$$
(3)

The following boundary conditions have been assumed:



(4)

Tele: E-mail address: rajputneetulu@gmail.com

© 2016 Elixir All rights reserved

43203

U. S. Rajput and Neetu kanaujia/Elixir Appl. Math. 99 (2016) 43202-43207

Here *u* is the velocity of the fluid in *x*- direction, *w* - the velocity of the fluid in *z*- direction, *m* is the Hall parameter, $g - acceleration due to gravity, <math>\beta$ - volumetric coefficient of thermal expansion, *t*- time, T_{∞} - the temperature of the fluid near the plate, T_w - temperature of the plate, *T* - the temperature of the fluid far away from the plate, k - the thermal conductivity, υ - the kinematic viscosity, ρ - the fluid density, σ - electrical conductivity, μ - the magnetic permeability, *K* - the permeability of the medium, and C_P is the specific heat at constant pressure. Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (3) in dimensionless from, we introduce the following non - dimensional quantities:

$$\overline{u} = \frac{u}{u_0}, \overline{w} = \frac{w}{u_0}, \overline{y} = \frac{yu_0}{v}, \operatorname{Pr} = \frac{\mu C_p}{k}, M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \overline{K} = \frac{K u_0}{v^2}, \overline{t} = \frac{t u_0^2}{v}, G_r = \frac{g \beta v (T_w - T_w)}{u_0^3}, \theta = \frac{(T - T_w)}{(T_w - T_w)}.$$
(5)

Here the symbols used are:

 \overline{u} - dimensionless velocity, \overline{w} - dimensionless velocity, Pr -Prandtl number, M- magnet field parameter, \overline{y} - dimensionless coordinate axis normal to the plate, θ - dimensionless temperature, and Gr - thermal Grashof number.

The dimensionless form of Equations (1), (2), and (3) are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\theta - \frac{M(\bar{u} + m\bar{w})}{(1+m^2)} - \frac{1}{K}u,$$
(6)

$$\frac{\partial \bar{w}}{\partial \bar{v}} = \frac{\partial^2 \bar{w}}{\partial \bar{y}} - M(\bar{w} - m\bar{u}) = 1$$

$$\frac{\partial w}{\partial \bar{t}} = \frac{\partial w}{\partial \bar{y}^2} - \frac{M(w-mw)}{(1+m^2)} - \frac{1}{K}w,$$
(7)

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial \bar{y}^2},\tag{8}$$

with the corresponding boundary conditions: $\overline{t} \le 0, \overline{u} \le 0, \theta = 0, \ W = 0, \ for \ all \ values \ of \ \overline{y},$

$$\vec{t} > 0, \vec{u} = 1, \vec{w} = 0, \theta = t, at \ \vec{y} = 0,$$

$$\vec{u} \to 0, \theta \to 0, \vec{w} \to 0 \ as \ \vec{y} \to \infty.$$

$$(9)$$

Dropping the bars and combining the Equations (7) and (8), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + Gr\theta - \left(\frac{M}{1+m^2}(1-mi) + \frac{1}{K}\right)q,\tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2},\tag{11}$$

where q = u + iw,

with corresponding boundary conditions

$$t \le 0: q = 0, \theta = 0, \text{ for all values of } y, t < 0: q = 1, w = 0, \theta = t, at y = 0, q \to 0, \theta \to 0, as y \to \infty.$$

$$(12)$$

The solution of above equations obtained by Laplace - transform method, which is an under

$$q = \frac{1}{2} e^{-\sqrt{a} \cdot y} (1 + Erf[\frac{2\sqrt{at} - y}{2\sqrt{t}}] + e^{2\sqrt{a} \cdot y} Erfc[\frac{2\sqrt{at} + y}{2\sqrt{t}}]) + \frac{1}{4a^{2}} y \ Gr\{A(1-a) + \sqrt{a} \ e^{-\sqrt{a} \cdot y}(1 - e^{2\sqrt{a} \cdot y} + Erf[\frac{2\sqrt{at} - y}{2\sqrt{t}}]) + e^{2\sqrt{a} \cdot y} Erf[\frac{2\sqrt{at} + y}{2\sqrt{t}}]) + B(1 - Pr) - APr\} + \frac{1}{2a^{2}\sqrt{\pi}} y \ Gr\{a(2e^{\frac{-y^{2} Pr}{4t}} \sqrt{t} + \sqrt{\pi} \cdot y)(-1 + Erf[\frac{y\sqrt{Pr}}{2\sqrt{t}}]) \sqrt{Pr}) + (\frac{C}{ye^{\frac{-u}{-1+Pr}} \sqrt{\sqrt{pr}}} - \frac{C\sqrt{Pr}}{(y\sqrt{Pr})e^{\frac{-u}{-1+Pr}} \sqrt{\sqrt{pr}}}) + (-1 + a + Pr)D\} \sqrt{Pr}$$

$$\theta = t \left[(1 + 2\eta^{2} Pr)erfc(\eta\sqrt{Pr}) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}} e^{-\eta^{2} Pr} \right]$$
(14)

The expressions for the constants involved in the above equations are given in the appendix.

3. Skin Friction

The dimensionless skin friction at the plate y = 0 is computed by

$$\left(\frac{dq}{dy}\right)_{y=0} = \tau_x + i\tau_z$$

Separating real and imaginary parts in

$$\left(\frac{dq}{dy}\right)_{y=0},$$

the dimensionless skin- friction components:

$$\tau_x = \left(\frac{du}{dy}\right)_{y=0}$$

and

$$\tau_z = \left(\frac{dw}{dy}\right)_{y=0}$$

Results and Discussion

The numerical values of velocity and and skin friction are computed for different parameters like, thermal Grashof number Gr, magnetic field parameter M, Hall parameter m, Prandtl number Pr and time t. The values of main parameters considered are

Gr = 10, 15, 20; M = 1, 3, 5; m = 0.4, 0.8, 1; Pr = 5, 7, 13, t = 0.4, 0.5, 0.6; K = 0.5, 1.0, 1.5.

From figures-1, 2, 5 and 6 it is observed that primary velocity (u) increases when m, Gr, K and t are increased. However, it decreases when M and Pr are increased (figure - 3, 4). And from figures - 7, 8, 9, 10 and 12 it is observed that the secondary velocity (w) increases when m, Gr, M, Pr, and t are increased. However, it decreases when K is increased (figure - 11). The values of the skin frictions τ_x and τ_z are tabulated in table -1. τ_x decreases when thermal Grashof number, magnetic field parameter and time are increased. But, it increases when Prandtl number, Permeability of the medium and Hall parameter are increased. The skin friction τ_z increases with increase in thermal Grashof number, time, Hall parameter, Permeability of the medium and magnetic field parameter. However τ_z decreases when Prandtl number is increased.

abic	1. Skin friedon for uniterent i arankter						
т	Gr	Μ	К	Pr	t	$ au_x$	$ au_z$
1.0	10	1	0.2	7	0.5	-4.546	0.161
1.0	10	3	0.2	7	0.5	-4.873	0.467
1.0	10	5	0.2	7	0.5	-5.190	0.755
0.4	10	2	0.2	7	0.5	-4.934	0.214
0.8	10	2	0.2	7	0.5	-4.779	0.307
1.0	10	2	0.2	7	0.5	-4.710	0.316
1.0	15	2	0.2	7	0.5	-5.830	0.375
1.0	20	2	0.2	7	0.5	-6.959	0.433
1.0	10	2	0.5	7	0.5	-3.698	0.368
1.0	10	2	1.0	7	0.5	-3.318	0.398
1.0	10	2	1.5	7	0.5	-3.185	0.410
1.0	10	2	0.2	7	0.4	-3.979	0.261
1.0	10	2	0.2	7	0.5	-4.710	0.316
1.0	10	2	0.2	7	0.6	-5.602	0.394
1.0	10	2	0.2	5	0.5	-5.839	0.446
1.0	10	2	0.2	7	0.5	-4.710	0.316
1.0	10	2	0.2	13	0.5	3712	0.230

Table 1 Skin friction for different Parameters



Figure 1. velocity u for different values of m











Conclusion

Some conclusions of the study are as under:

1. Primary velocity increases with the increase in thermal Grashof number, Hall parameter, Permeability of the medium and time. 2. Primary velocity decreases with the increase in magnetic field parameter and Prandtl number.

3. Secondary velocity increases with increase in thermal Grashof number, time, Hall parameter, Prandtl number and magnetic field parameter.

4. Secondary velocity decreases with increases in permeability of the medium.

5. τ_x decreases with increase in thermal Grashof number, magnetic field parameter and time, and it increases when Prandtl number, Hall parameter permeability of the medium are increased.

6. τ_z increases with increase in thermal Grashof number, time, Hall current, permeability of the medium and magnetic field parameter, and it decreases when Prandtl number is increased.

Appendix

$$\begin{split} A &= \frac{2e^{-\sqrt{a} y} (1 + e^{2\sqrt{a} y} + Erf[\frac{2\sqrt{at - y}}{2\sqrt{t}}] - e^{2\sqrt{a} y} Erf[\frac{2\sqrt{at + y}}{2\sqrt{t}}])}{y}, \\ B &= \frac{1}{y} 2e^{-1 + \Pr} - y\sqrt{\frac{a \Pr}{-1 + \Pr}} \left[-1 - e^{2y\sqrt{\frac{a \Pr}{-1 + \Pr}}} + Erf[\frac{y - 2t\sqrt{\frac{a \Pr}{-1 + \Pr}}}{2\sqrt{t}}] + e^{2y\sqrt{\frac{a \Pr}{-1 + \Pr}}} Erf[\frac{y + 2t\sqrt{\frac{a \Pr}{-1 + \Pr}}}{2\sqrt{t}}] \right], \\ C &= \sqrt{\pi} \left(-1 - e^{2y\sqrt{\frac{a}{-1 + \Pr}}} - Erf[\frac{2t\sqrt{\frac{a}{-1 + \Pr}} - y\sqrt{\Pr}}{2t} + e^{2y\sqrt{\frac{a}{-1 + \Pr}}} Erf[\frac{2t\sqrt{\frac{a}{-1 + \Pr}} + y\sqrt{\Pr}}{2t}] \right), \\ D &= \frac{2\sqrt{\pi}(-1 + Erf[\frac{y\sqrt{\Pr}}{2\sqrt{t}}])}{y\sqrt{\Pr}}, \\ a &= \frac{M}{1 + m^2} (1 - im) + \frac{1}{K}, \ \eta = \frac{y}{2\sqrt{t}} \end{split}$$

References

[1] P Sulochana, "Hall effects on unsteady MHD three dimensional flow through a porous medium in a rotating parallel plate channel with effect of inclined magnetic field", (2014), American journal of computational mathematics, 4, 396-405.

[2] U S Rajput and P K Sahu, "Effects of chemical reactions on free convection MHD past an exponentially accelerated infinite vertical plate through a porous medium with variable temperature and mass diffusion", (2012), Elixir Appl. Math, Vol. 43, 6560-6527-5962

[3] W A Manyonge, D W Kiema, C C W Iyaya, "Steady MHD Poiseuille Flow between two infinite parallel porous plates in an inclined magnetic field", (2012), Inter. Journal of Pure and Applied Maths, 76(5): 661-668.

[4] U S Rajput and S kumar, "MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion", (2011), mathematical sciences, vol.5, no 3, 149-157.

[5] N Ahmed, H kalita and D P Barua, "Unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source",(2010), international journal of engineering, science and technology,Vol. 2, No. 6, pp. 59-74.

[6] P C Ram, C B Singh, U Singh, "Hall effects on heat and mass transfer flow through porous medium", (1984), Astrophysics space science, 100, 45-51.

43207