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A Probability Model for Estimating Under-Five Mortality among Women for Fixed Parity in India

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ABSTRACT

Child mortality refers to the death of infants and children under the age of five or between the age of one month to four years depending on the definition. Child mortality is a core indicator of child health and well-being. In this paper we propose a probability model for child mortality among women with fixed parity under certain assumption and techniques. The suitability of the model is tested through observed data. NFHS-3 data has been used to carry out this study.

Introduction

Child mortality, conventionally, being an important component of public health. Child mortality and infant mortality has been a main concern for the health professionals over the past several decades. Child survival interventions are designed to address the most common causes of child deaths that occur, which include diarrhea, pneumonia, malaria, and neonatal conditions. Of the portion of children under the age of 5 alone, an estimated 9.2 million children die each year mostly from such preventable causes [1]. The factor of child mortality has played a dominant role in determining the growth of population, the size of which fluctuated in the past mainly in response to variations in mortality.

The child survival strategies and interventions are in line with the fourth Millennium Development Goal which focused on reducing child mortality by 2/3 of children under five before the year 2015. Many countries are now devoted to the child survival interventions as a way of reducing the child mortality. In 2013, the world average was 46 percent, down from 90 percent in 1990 [2].The average was 6 in developed countries and 50 in developing countries, including 92 in Sub-Saharan Africa [3]. The highest rate in the world was 167, in Angola [3].

The high rate of infant and child mortality shows a low level development of the health programme and also for the nation's. The representation of mortality data using a probability model has attracted the attention of demographers and statisticians for over a century. Infant and child mortality has been of interest o researchers and demographers because of its apparent relationship with fertility and indirect relationship with the acceptance of modern contraceptive methods [4]. Gompertz [5] given the popular model, still used by demographer and statistician today. Hill [6] have suggested an approach for estimating child mortality for all births which have taken place in last five years before the survey. Singh [7] discussed the number of death of children under the age of five years for fixed parity. Later Arnold [8] used pareto distribution and Krishnan [9] applied finite range model for the same. Keyfitz [10] used a hyperbolic function to study the infant and child mortality. Pandey [11] used bio-demographic hazard model to analyze the effects of the biological and socio-economic conditions on the survival of male and female children under-five.

1. Construction of model

Suppose the number of births to a women be n i.e. women with n parity. Let p be the probability of death of an under-five child to women and let be that random variable X which denotes the total child death to women out of n births. The model is developed under the assumption each child will either die or survive at the age of five.

Let us define for ith (i=1,2,3..n) children born to women be a random variable z_i taking value 1 if the child died before completing 5 years age otherwise 0. It is assumed that the child death (children born to women) is independent of each other with the same probability p, and the total number of child deaths X to women is the sum of independent Bernoulli variables, and hence follows a binomial distribution. Therefore, the distribution of may be given by

$$P[X = x] = \binom{n}{x} p^x (1-p)^{n-x} \qquad (1)$$

where, $0 \le p \le 1$, x = 0, 1, 2……………..n.

2.1.1. Model- II

In this model, we consider that the women consists a high proportion of households having no child death. Due to more observations with zero counts, the frequency of zero cells is inflated and the resulting over dispersion cannot be modeled accurately with the simple binomial model. In such scenario an inflated binomial model may be used. Assume that the proportion of women experience child death be α and (1 –α) be the proportion of women have no child deaths. Therefore the probability density function of zero inflated binomial model is

$$P[X = x] = \begin{cases} (1-\alpha) + \alpha(1-p)^n & \text{for } x=0 \\ \alpha\binom{n}{x} p^x (1-p)^{n-x} & \text{for } x=1,2,3\ldots n \end{cases} \qquad (2)$$

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2.2. Model-II

In the model-I, we have assumed that probability ‘p’ is fixed for all child deaths to a women. But in reality, ‘p’ is affected by a number of factors and therefore assumption of p being constant for all women seems to be questionable. Thus, it seems more logical to consider p as a random variable following some distribution g(p). Beta distribution of first kind with parameters (a, b) is a suitable distribution for risk of child death ‘p’, since ‘p’ the risk varies from 0 to 1 and beta distribution possess the property of flexibility, and capability of accommodating wide range of variability. The probability density function of Beta distribution is:

$$g(p) = \frac{1}{\beta(a,b)} p^{(a-1)} (1-p)^{b-1} \quad ; 0 \le p \le 1,\ a, b>0 \qquad (3)$$

Thus, the joint distribution of x and p is given by

$P [X = x \cap P = p] = P [X = x/ p] \times g(p) =$

$$\binom{n}{x} p^x (1-p)^{n-x} * \frac{1}{\beta(a,b)} p^{(a-1)} (1-p)^{b-1} \qquad (4)$$

and the marginal distribution of x is given as

$$P[X = x] = \binom{n}{x} \frac{\beta(a+x, b+n-x)}{\beta(a,b)} \quad a, b >0 \qquad (5)$$

The above marginal distribution of x in equation (5) is known as beta-binomial distribution and it is a natural extension of binomial model under the consideration for random nature of ‘p’ in the population. The parameters a and b are its shape parameter. If someone is interested in getting a single value (like p) for comparing the migration of two places, one may take mean i.e.

$$\frac{\hat{a}}{\hat{a}+\hat{b}}$$

an estimate of average number of migrants at the household level. Also with this distribution one can know the distribution of risk of migration which cannot be obtained directly.

2.3Estimation

Model I

The moment estimates of the parameters α and *p* of the proposed model can be obtained as follows:

$$E(X) = \alpha np \qquad (6)$$

$$E(X^2) = \alpha np\,(np + 1 - p - \alpha np) + (\alpha np)^2 \qquad (7)$$

Let μ_1' and μ_2' denotes the first two raw moments about zero for data in hand. Replacing E(X) and E(X^2) by μ_1' and μ_2' in above equations we get two equations with two unknowns α and *p* as given below:

$$\mu_1' = \alpha np \qquad (8)$$

$$\mu_2' = \alpha np(np + 1 - p - \alpha np) + (\alpha np)^2 \qquad (9)$$

With these two equations (8) and (9), estimates of p and α can be obtained easily.

Model-II

The moment estimates of the parameters a and b can be obtained as follows

$$E(X) = \frac{na}{a+b} \qquad (10)$$

$$E(X^2) = \frac{na\,[n\,(1+a)+b}{(a+b)\,(a+b+1)} \qquad (11)$$

As mentioned above replacing *E*(*X*) and *E*(X^2) by μ_1' and μ_2' in above equations we get two equations with two unknowns *a* and *b* as given below:

$$\mu_1' = \frac{na}{a+b} \qquad (12)$$

$$\mu_2' = \frac{na\,[n\,(1+a)+b}{(a+b)\,(a+b+1)} \qquad (13)$$

Substituting the value of b $= \frac{n-\mu 1'}{\mu 1'} a$ from the equation (12) in the above equation and separating the coefficients for *a* we have

$$a\left[\mu_2' - n\,\mu_1' + \left[\frac{n-\mu_1'}{\mu_1'}\right]\left(\mu_2' - \mu_1'\right)\right] = n\,(\mu_1' - \mu_2') \qquad (14)$$

or,

$$a = \frac{\mu_1'^2 - \mu_1'\mu_2'}{n(\mu_2' - \mu_1') - n\mu_1'^2 + \mu_1'^2} \qquad (15)$$

after solving this we can get the estimate of *a* and the using this estimate and equation (12) *b* can be estimated easily.

2. Results and Discussion

In this section we have discussed about the estimate of parameters and fitting of the proposed probabilistic models. Third round national family health survey (NFHS-3) data has been used to carry out the study. NFHS-3 collected information from a nationally representative sample of 124,385 women age 15-49 about their fertility, mortality, family planning and important aspects of reproductive health. NFHS-3 also collects the information of all live births to women and the survival status of all births at the time of survey. To avoid censored cases, we have also taken the females who are not having any birth in the last five years.

In this study two models have been proposed for fixed number of child deaths to the women so that after obtaining the estimate of parameters for different parity, we obtained the estimated frequencies for both the models. Tables 1 to 2 show the expected frequencies along with the observed frequencies for women with parity 4 to 5 in India. The p-value and values of χ2 shown in the tables clearly indicate that both the models described well the distribution of child deaths to the women for fixed parity. However, the expected number of child deaths for Inflated Binomial and Beta-Binomial model is very close to the observed number of child deaths. From table 1 we can observe that the expected frequencies of beta binomial distribution are closer to the observed frequencies than the expected frequencies obtained by inflated binomial distribution. Similar result has been obtained in table 2, beta binomial fit well compared to inflated binomial. So, one may prefer binomial model over inflated binomial model in various real-world problems. Figures 1 and 2 portrayed the plot of observed and expected frequency distributions of inflated binomial distribution and beta binomial distribution for women with parity 4 and 5 respectively.

Table 1. Expected and observed frequency distribution of child deaths to the women with parity 4 in India, 2005-06.

		Expected number of women	
Number of child deaths	Observed	Inflated Binomial	Beta-Binomial
0	1303	1306.26	1307.09
1	496	485.64	490.77
2	112	123.54	111.62
3	19	13.97	19.09
4	0	0.59	1.43
Total	1930	1930	1930
Parameters		p = 0.1450	a = 0.4450
		α = 0.6941	b = 3.9766
Chi-square		3.7125	1.5005
p value		0.4463	0.8266
Mean	0.4026		
Variance	0.5778		

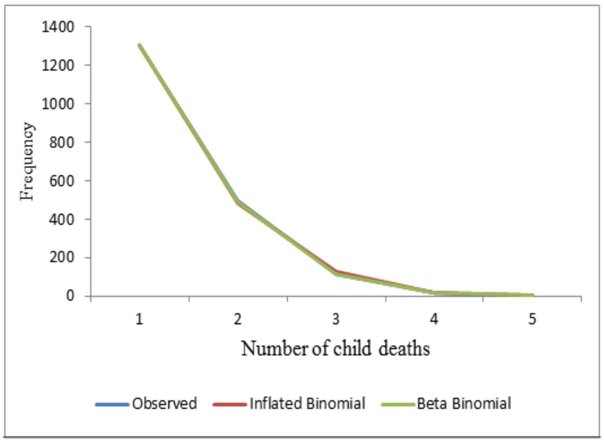

Figure 1. Expected and observed frequency distribution of child deaths to the women with parity 4 in India, 2005-06.

Table 2. Expected and observed frequency distribution of child deaths to the women with parity 5 in India, 2005-06.

		Expected number of women	
Number of child deaths	**Observed**	**Inflated Binomial**	**Beta Binomial**
0	1037	1057.83	1038.03
1	940	890.69	937.62
2	312	338.72	309.05
3	57	64.40	58.24
4	8	6.12	9.45
5	4	0.23	5.56
Total	2358	2358	2358
Parameters		p = 0.1597	a = 1.5388
		α = 0.9487	b = 8.61374
Chi-square		7.3972	0.7194
p value		0.1927	0.9819
Mean	0.7578		
Variance	1.2421		

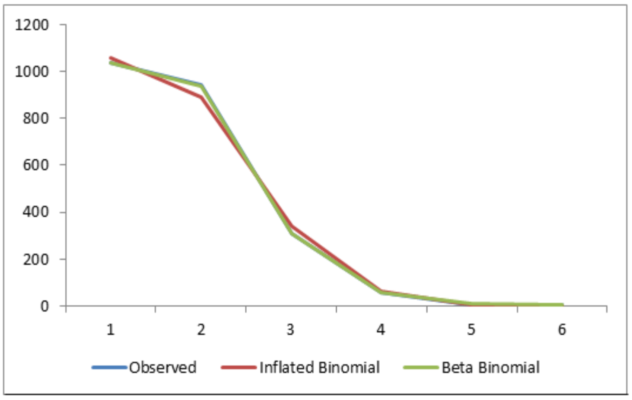

Figure 2. Expected and observed frequency distribution of child deaths to the women with parity 5 in India, 2005-06.

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