Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 100 (2016) 43827-43829

ISSN: 2229-712X

# Maximin Zero Suffix Method for Solving Assignment Problems is Symmetric

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**ARTICLE INFO**

**Article history:**

Received: 23 September 2016;

Received in revised form: 17 November 2016;

Accepted: 28 November 2016;

**Keywords**

Min Max Zero suffix method, Symmetric.

**ABSTRACT**

Assignment problem is an important subject discussed in real physical world. We endeavor in this paper introduce by using Min Max Zero Suffix Method (MMZS) is Symmetric.This method offers significant advantages over similar methods, in the process, first we define the assignment matrix, apply the transpose for original assignment problem then by using determinant representation we obtain a reduced matrix which has at least one zero in each row and columns. Then by using Min Max Zero Suffix Method (MMZS), we obtain an optimal solution for assignment problem by assigning zeros to each row and each column. The Min Max Zero Suffix Method (MMZS) is based on creating some zeros in the assignment matrix and then try to find a complete assignment to their Zeros.The proposed method is a systematic procedure, easy to apply and can be utilized for all types of assignment problem with maximize or minimize objective functions.



## Introduction

An important topic, put forward immediately after the transportation problem, is the assignment problem. This is particularly important in the theory of decision making. The assignment problem is one of the earliest applications of linear integer programming problem. Different methods have been presented for assignment problem and various articles have been published on the subject. See [1], [2] and [3] for the history of these methods.

A considerable number of methods has been so far presented for assignment problem in which the Hungarian method [4], [5], [6], is more convenient method among them. This iterative method is based on add or subtract a constant to every element of a row or column of the cost matrix, in a minimization model and create some zeros in the given cost matrix and then try to find a complete assignment in terms of zeros. By a complete assignment for a cost matrix n × n, we mean an assignment plan containing exactly n assigned independent zeros, one in each row and one in each column. The main concept of assignment problem is to find the optimum allocation of a number of resources to an equal number of demand points. An assignment plan is optimal if optimizes the total cost or electiveness of doing all the jobs.

This paper attempts to propose a method for solving assignment problem which is different from the preceding methods.

## 2. Mathematical Formulation of Assignment Problem

Associated to each assignment problem there is an n×n cost matrix as follows:

The mathematical model for the classic assignment problem may be expressed as

$$Min\ z = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} C_{ij}X_{ij}$$

$$\begin{array}{cc|c|c|c|c|c|}
 & & \multicolumn{5}{c}{\text{Agent}} \\
 & & 1 & \ldots & j & \ldots & n \\
\hline
 & 1 & c_{11} & \ldots & c_{1j} & \ldots & c_{1n} \\
 & . & & \ldots & & \ldots & \\
 & . & & & & & \\
\hline
Job & i & c_{i1} & \ldots & c_{ij} & \ldots & c_{in} \\
 & . & & \ldots & & \ldots & \\
 & . & & & & & \\
\hline
 & n & c_{n1} & \ldots & c_{nj} & \ldots & c_{nn} \\
\hline
\end{array}$$

**Subject to the constrains**

$$\sum_{i=1}^{\infty} x_{ij} = 1, j = 1,2, \ldots \ldots . n.$$

$$\sum_{j=1}^{\infty} x_{ij} = 1, i = 1,2, \ldots \ldots . n.$$

$$x_{ij} = 0\ or\ 1$$

where $x_{ij} = 1$ if ith task is assigned to jth agent, and $x_{ij} = 0$ otherwise; and $c_{ij}$ = the cost of assigning ith task to jth agent. The first set of constraints ensures that one task is assigned to only one agent and the second set of constraints ensures that one agent is assigned to one task.

## 3. Maximin Zero Suffix Method

In this section a new method named, "A maximin zero suffix method" [9] for solving the assignment problem has been presented. We would call it "A maximin zero suffix method" because of making assignment to zero cell having maximum zero suffix value. To obtain the minimum assignment cost we search the least cost cell for possible assignment.

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In the assignment table, after subtracting the lowest cost of each row from all the costs of corresponding row and then subtracting the lowest cost of each column from all the costs of corresponding column we get at least one zero in each row and in each column. The lowest cost cells in each row and in each column are detected by these zeros. On minimum cost distribution, zero cells (Lowest cost cells) should be assigned as far as possible satisfying the condition that a task is assigned to one agent and each agent being assigned to at most one task ( a one - to - one assignment) [10].

If the transformed assignment table does not contain more than one zero in any row/column then assignment corresponding to all the zero cells is the optimal assignment. Unfortunately, each row and each column of the transformed assignment table does not always contains exactly one zero. After the occurrence of more than one zero cells in a row/column the entire zero cells of that row/column cannot be assigned. The problem is to which one zero cell of that row/column should be assigned and which is crossed out for getting over all minimum assignment cost. Our aim is to assign exactly n zero cells one by one in which a manner that each row/column must contained one assigned cell and over all assignment cost becomes Optimal one. The question is to which zero cells should start to assign firstly for getting an optimal assignment schedule..

To overcome this critical situation we focus on two zero cells ( p,q) and (r,s) of the transformed table. Let (p, q) cell is assigned and (r, s) cell is not assigned. It means that rth task is not assigned to sth agent. But rth task must be assigned to any one of the agent. To assign rth task for minimum cost, we select a cell in the rth row whose cost is the next lowest cost in its row. Let $c'_{rv}$ is the next minimum in the rth row. Then rth task must be assigned to vth agent. So excess cost in the absence of not assigning (r, s) cell is $c'_{rv}$ . Also in the absence of not assigning (r, s) cell, sth agent is not assigned rth task. To assign a task to sth agent for minimum cost, we select a cell in sth column whose cost is the next smallest in its column. Let $c'_{us}$ is the next minimum in the sth column. Then sth agent must be assigned uth task. So excess cost, in the absence of not assigning (r, s) cell is $c'_{us}$ . Finally, we conclude that, in the absence of not assigning (r, s) cell, the assignment cost may be increased at most by max $\{c'_{us},c'_{rv}\}$

= max {lowest cost in its row, lowest cost in its column excluding itself}

= Suffix value of zero of (r, s) cell

Similarly, in the case of not assigning (p, q), cell we may have to face excess cost which is equal to the suffix value of zero of (p, q) cell. Note that $c'_{ij}$ is the value of $c_{ij}$ in the transformed table. We conclude that to get minimum assignment cost it is better to assign that zero cell whose suffix value is the greatest one.

The methodology of the paper is to block the path of exceeding cost from optimal one in the process of assigning the zero cells. The process of assigning the cell is to obtain the suffix value of all zeros of the transformed table and assign the cell having the greatest suffix value [11]. Then, we have to delete the row and column of assigned cell to get reduced table. Again, in the reduced table, we have to subtract row minima and column minima to ensure at least one zero in each row and column and the suffix value of all zeros to be obtained. Again, we have to assign the cell having the greatest suffix value and cross out its rows and column. This process continues till all cells are exhausted and finally we get the optimal assignment.

**4. Algorithm of the Maximin Zero Suffix Method**

The algorithm of the zero suffix method for finding an optimal solution of assignment problem consists of the following steps:

Step 1. Subtract the least element of each row from all the elements of the corresponding row of cost matrix.

Step 2. Subtract the least element of each column from all the elements of the corresponding column obtained in step 1, to get the transformed cost matrix.

Step 3. Find the suffix value S of each zero in the transformed cost matrix as follows:S= Maximum of the least element of its row and the least element of its column excluding itself.

Step 4. Search the greatest suffix value of a zero and allocate the corresponding cell.Delete its row/column to get the reduced table. Go to step 1.

Step 5. Repeat steps 1 to 4 until all the cells are exhausted.

**Notes**

I. To convert the maximization assignment problem into minimization assignment problem (a) multiply each element of the cost matrix by -1 or (b) replace each $c_{ij}$ by c max $-c_{ij}$ , where c maxis the maximum of the $c_{ij}$ 's and then apply the algorithm.

II. If tie occur in the greatest suffix value of zero, then search the second suffix value of that zero in which tie occur. Second suffix value of a zero is the maximum of the second lowest element in its row and the second lowest element in its column excluding itself.

III. For the case of non uniqueness of the second suffix value of zero, third suffix value of zero can be obtained as per the change in rule of the second suffix value.

IV. If tie occur for any of the greatest suffix value of zero and if there is no possibility of getting the next suffix value of zero due to the non availability of the next smallest element(s) in its row/column, allocation is made to the tied cell in which the original cost of the cell is minimum.

**Example**: Consider the returns (in dollars) maximizing assignment problem with 5 jobs and 5 machines

Jobs

Machines

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 12 | 8 | 7 | 15 | 4 |
| 2 | 7 | 9 | 1 | 14 | 10 |
| 3 | 9 | 6 | 12 | 6 | 7 |
| 4 | 7 | 6 | 14 | 6 | 10 |
| 5 | 9 | 6 | 12 | 10 | 6 |

Row entry – Row min

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 8 | 4 | 3 | 11 | 0 |
| 2 | 6 | 8 | 0 | 13 | 9 |
| 3 | 3 | 0 | 6 | 0 | 1 |
| 4 | 1 | 0 | 8 | 0 | 4 |
| 5 | 3 | 0 | 6 | 4 | 0 |

Column entry – Column min

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 7 | 4 | 3 | 11 | $0_3$ |
| 2 | 5 | 8 | $0_5$ | 13 | 9 |
| 3 | 2 | $0_0$ | 6 | $0_0$ | 1 |
| 4 | 0 | $0_0$ | 8 | $0_0$ | 4 |
| 5 | 2 | $0_0$ | 6 | 4 | $0_0$ |

| | 1 | 2 | 4 | 5 |
|---|---|---|---|---|
| 1 | 7 | 4 | 11 | $0_4$ |
| 3 | 2 | $0_0$ | $0_0$ | 1 |
| 4 | $0_2$ | $0_0$ | $0_0$ | 4 |
| 5 | 2 | $0_0$ | 4 | $0_0$ |

| | 1 | 2 | 4 |
|---|---|---|---|
| 3 | 2 | $0_0$ | $0_0$ |
| 4 | $0_2$ | $0_0$ | $0_0$ |
| 5 | 2 | $0_2$ | 4 |

| | 2 | 4 |
|---|---|---|
| 3 | $0_0$ | $0_4$ |
| 5 | $0_0$ | 4 |

Assignment is (2,3),(1,5),(4,1),(3,4),(5,2)

The Cost is = 1 + 4 + 7 + 6 + 6 = 24

The transpose of the given Assignment problem is solving by using Zero suffix method given below

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 12 | 7 | 9 | 7 | 9 |
| 2 | 8 | 9 | 6 | 6 | 6 |
| 3 | 7 | 1 | 12 | 14 | 12 |
| 4 | 15 | 14 | 6 | 6 | 10 |
| 5 | 4 | 10 | 7 | 10 | 6 |

**Row entry – Row min**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 5 | $0_0$ | 2 | $0_0$ | 2 |
| 2 | 2 | 3 | $0_0$ | $0_0$ | $0_2$ |
| 3 | 6 | $0_6$ | 11 | 13 | 11 |
| 4 | 9 | 8 | $0_0$ | $0_0$ | 4 |
| 5 | $0_2$ | 6 | 3 | 6 | 2 |

**Column entry – Column min**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 5 | $0_0$ | 2 | $0_0$ | 2 |
| 2 | 2 | 3 | $0_0$ | $0_0$ | $0_2$ |
| 3 | 6 | $0_6$ | 11 | 13 | 11 |
| 4 | 9 | 8 | $0_0$ | $0_0$ | 4 |
| 5 | $0_2$ | 6 | 3 | 6 | 2 |

| | 1 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 5 | 2 | $0_2$ | 2 |
| 2 | 2 | $0_0$ | $0_0$ | $0_0$ |
| 4 | 9 | $0_0$ | $0_0$ | 4 |
| 5 | $0_2$ | 3 | 6 | 2 |

| | 1 | 3 | 5 |
|---|---|---|---|
| 2 | 2 | $0_0$ | $0_2$ |
| 4 | 9 | $0_4$ | 4 |
| 5 | $0_2$ | 3 | 2 |

| | 1 | 5 |
|---|---|---|
| 2 | 2 | $0_2$ |
| 5 | $0_2$ | 2 |

The Assignment is

= 1 +7 +6 + 6 + 4

=24

## Conclusion

The above Example concludes that every assignment problem using Zero suffix method satisfies the Symmetric rule. That is the result of original matrix is equal to the result of transpose of that matrix. Hence it is symmetric.

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