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Transformation of an Object in Computer Graphics: A Case Study of Mathematical Matrix Theory

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ABSTRACT

Equations of curves or straight lines have reference to certain set of axes. In fact we shall have different equations for the same curve or straight line when referred to different set of co-ordinate axes. This may happen when the origin is shifted to a point keeping the directions of the axes the same or when the axes are rotated through same angle keeping the origin unaltered. Transformation of an object in computer graphics is based on mathematical theory and uses an important concept related to matrix theory. Transformation of an object may be related to 2dimensional or 3 dimensional or more .A transformation which map 3-D objects onto 2-D screen we are going to call it Projection .In general there are two ways to represent the transformation of an object (i) transformation are done by using the rules of co-ordinates axes (ii) transformation are done by representing in the matrix form. In this paper it is tried to study the properties of matrix used to develop transformation of an object related to computer graphics theory. It is tried to study set of zero/non-zero vectors are used to represent a matrix and is helpful to do transformation of an object.

1. Introduction

Transformation of an object is the process by which we can change the shape, position and direction of any object with respect to any co-ordinate system on background by translation, rotation, scaling and reflection. Basically, transformation is described in two categories:

(i)Geometric transformation

(ii)Co-ordinate transformation

When object itself moved relative to stationery co-ordinate system or background, referred as geometric transformation and applied to each point of the object and object is held stationary than this process is termed as co-ordinate transformation.

The transform means to "change". This change can either be of shape, size or position of the object. To perform transformation on any object, object matrix X is multiplied by the transformation matrix T.

[Transformed object matrix]= [original matrix]*[Transformation matrix]

To represent transformation of an object in the computer graphics there is need to understand the basic knowledge of mathematical matrix theory. Let $A=(a_{ij})_{m,n}$, $X^T=(x_1,x_2,x_3,\ldots..x_n)$, $B^T=(b_1,b_2,\ldots\ldots..b_n)$ are matrix.

X^T = transpose of matrix X

B^T = transpose of matrix B

Then the system can be expressed as AX=B. The matrix A is said to be coefficient matrix of the system

A system of m linear equations in n unknowns x_1, $x_2,\ldots..,x_n$ is of the form

$a_{11}x_1+a_{12}x_2+\ldots\ldots\ldots+a_{1n}x_n=b_1$

$a_{21}x_1+a_{22}x_2+\ldots\ldots\ldots+a_{2n}x_n=b_2$

$a_{31}x_1+a_{32}x_2+\ldots\ldots\ldots+a_{3n}x_n=b_3$

…. …… ……….. ………………….

…………………………………

$a_{m1}x_1+a_{m2}x_2+\ldots\ldots\ldots+a_{mn}x_n=b_m$

Where a_{ij}'s and b_i's are elements of a field F, called the fields of scalars, a_{ij}'s are called coefficients of the system. In particular, a_{ij}'s and b_i's are real (or complex) numbers where F is the field R (set of real numbers) or C(set of complex number).

An ordered set $(c_1,c_2,\ldots..,c_n)$where c_i belongs to F, is said to be a solution of the system if each equation of the system is satisfied for$x_1=c_1$, $x_2=c_2$, …….. , $x_n=c_n$. Therefore a solution of the system can be considered as n-tuple vector of $V_n(F)$. In particular, if the field of scalars be R, a solution of the vector in R^n . A system of equations is said to be consistent if it has solution. Matrix representation of linear mapping is helpful to understand the Projection theory in computer graphics.

2. Transformation of an object and its representation with respect to matrix in computer graphics

2.1 Translation of an Object in Graphics: In translation, an object is displayed at a given distance and direction from its original position. A translation moves all points in an object along the same straight-line path to new positions. The path is represented by a vector, called the translation or shift vector.

We can write the components:

$$p'_x = p_x + t_x$$
$$p'_y = p_y + t_y$$

or in matrix form:

$$P' = P + T$$

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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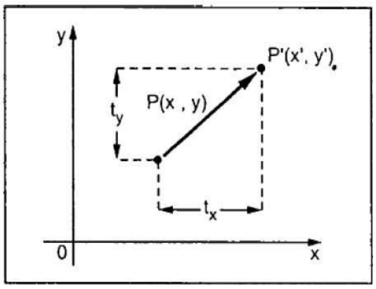
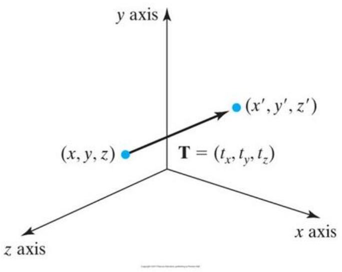


Fig 1 . Image of translation of an object.



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{T} \cdot \mathbf{P}$$

Fig 2 .Image of translation of an object in 3-D dimension.

2.2 Rotation of an object in computer graphics

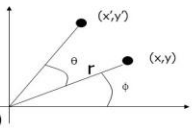
2D Rotation

$x = r\cos(\phi) \quad y = r\sin(\phi)$

$x' = r\cos(\phi+\theta) \quad y = r\sin(\phi+\theta)$

$$\begin{aligned} x' &= r\cos(\phi+\theta) \\ &= r\cos(\phi)\cos(\theta) - r\sin(\phi)\sin(\theta) \\ &= x\cos(\theta) - y\sin(\theta) \end{aligned}$$

$$\begin{aligned} y' &= r\sin(\phi+\theta) \\ &= r\sin(\phi)\cos(\theta) + r\cos(\phi)\sin(\theta) \\ &= y\cos(\theta) + x\sin(\theta) \end{aligned}$$



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In this transformation, object is rotated θ about origin.

We take θ positive for counter clockwise otherwise, negative and co-ordinates of new point from the above equation is given by the following equation

• A rotation repositions all points in an object along a circular path in the plane centered at the pivot point.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

>> Example of 3D- rotation of an object with respect to Y-axis

The equation for Y-axis rotaion

$$\mathbf{x' = x\cos\theta + z\sin\theta}$$
$$\mathbf{y' = y}$$
$$\mathbf{z' = z\cos\theta - x\sin\theta}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Fig3.Image of 3D-rotation about Y-axis.

2.3 Scaling transformation in computer graphics: Scaling is the process of changing the size and proportion of the image. Alternative, it is the process of expanding or compressing the dimension of an object. There are two factors used in scaling transformation i.e. S_x and S_y, where S_x is scale factor for the co-ordinates and S_y is the scale factor for the Y co-ordinates.

Scaling changes the size of an object and involves two scale factors, S_x and S_y for the x- and y- coordinates respectively.

• Scales are about the origin.

• We can write the components:

$p'_x = s_x \bullet p_x$

$p'_y = s_y \bullet p_y$

or in matrix form:

$P' = S \bullet P$

Scale matrix as:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

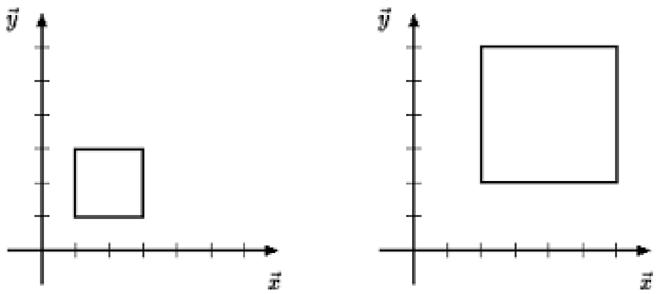


Fig4: An Images of Scaling of an object in 2-Dimension

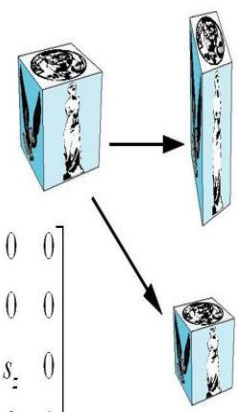
>>Example of 3D scaling of an object:

Expand or contract along each axis (fixed point of origin)

$x' = s_x x$
$y' = s_y x$
$z' = s_z x$

$\mathbf{p' = Sp}$

$$S = S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



2.4Shearing of images : A transformation that distorts the basic shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other is called a shear and this transformation is called shearing transformation.

• Shear is a transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other.

•Two common shearing transformations are those that shift coordinate x values and those that shift y values.

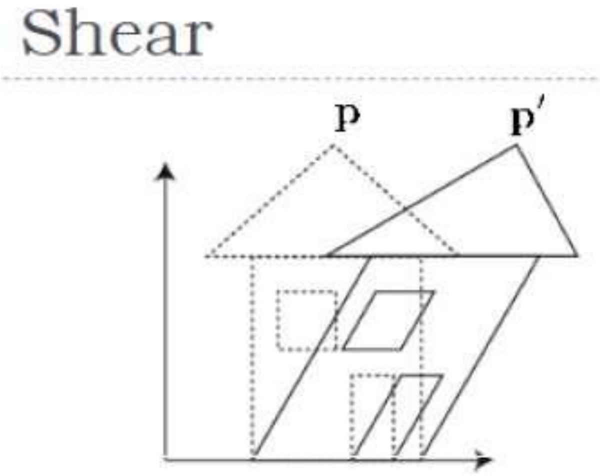


Figure5: Image of shearing of an object

2.5 Reflection of images: Reflection is a transformation which generates the mirror image of an object. The mirror image for two dimensional reflections is generated by rotating the object 180 degree on the x- axis of reflection.

Reflection is a transformation that produces a mirror image of an object. It is obtained by rotating the object by 180 deg about the reflection axis

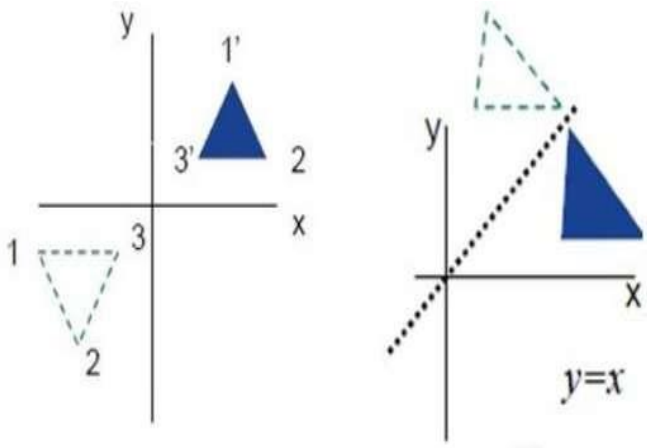


Figure 6. Image of reflection of an object.

2.6 Use of matrix theory in translation, rotation, scaling, reflection of images in computer graphics and matrix representation of a linear transformation

2.6.1 Matrix : Let $F^{m\times n}$ denote the set of m×n matrices with entries in F. Then $F^{m\times n}$ is vector space over F. Vector addition is just matrix addition and scalar multiplication is defined in the obvious way (by multiplying each entry by the same scalar). The zero vectors are just the zero matrixes. The dimension of $F^{m\times n}$ is mn.

2.6.2 Vector Space : A space consisting of vectors, together with the associative and commutative operation of addition of vectors, and the associative and distributive operation of multiplication of vectors by scalars.

2.6.3 Basis: A basis of a vector space V is defined as a subset $v_1, \ldots, v_n$ of vectors in V that are linearly independent and vector space span V. Consequently, if $(v_1, v_2, \ldots, v_n)$ is a list of vectors in, then these vectors form a basis if and only if every can be uniquely written as $v = a_1 v_1 + a_2 v_2 + \ldots + a_n v_n$,

Where $a_1, a_2 \ldots\ldots, a_n$ are elements of the base field.

2.6.4 Vector Space Span : The span of subspace generated by vectors $\mathbf{v}_1$ and $\mathbf{v}_2 \in \mathbb{V}$ is

$$\text{Span}(\mathbf{v}_1, \mathbf{v}_2) \equiv \{r\,\mathbf{v}_1 + s\,\mathbf{v}_2 : r, s \in \mathbb{R}\}.$$

2.6.5 Co-ordinates :If the vectors of the basis set be enumerated in some well-defined way, then we call it an ordered basis. Let V be a finite dimensional vector space defined over the field F and let B={ a_1, $a_2, a_3, \ldots\ldots, a_n$} be an orered basis of V.

Let x belongs to V then there exists a unique n-tuple(c_1, $c_2, \ldots\ldots, c_n$) of scalars such that

$x = c_1a_1 + c_2a_2 + c_3a_3 + \ldots\ldots + c_na_n$.

this n-tuple(c_1, $c_2, \ldots\ldots, c_n$) is called n-tuple of coordinates relative to the ordered basis B.

Let V be some abstract n-dimensional vector space over field F and let W be an abstract m-dimensional vector space over field F. Let B = {$\alpha_1, \alpha_2, \ldots, \alpha_n$} be a basis for V and B' = {$\beta_1, \beta_2, \ldots, \beta_n$} be a basis for W. Let T: V →W be a linear transformation from V into W. Let $\alpha_1', \alpha_2', \ldots, \alpha_n'$ be the images in W under transformation T of the basis vectors $\alpha_1, \alpha_2, \ldots, \alpha_n$ as referred to the W basis B' ={$\beta_1, \beta_2, \ldots, \beta_n$}:

$$(1)\qquad \begin{aligned} \alpha_1' &= T(\alpha_1) = a_{11}\beta_1 + a_{12}\beta_2 + \cdots + a_{1m}\beta_m \\ \alpha_2' &= T(\alpha_2) = a_{21}\beta_1 + a_{22}\beta_2 + \cdots + a_{2m}\beta_m \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \alpha_n' &= T(\alpha_n) = a_{n1}\beta_1 + a_{n2}\beta_2 + \cdots + a_{nm}\beta_m \end{aligned}$$

or, in matrix form,

$$(2)\qquad \begin{bmatrix} \alpha_1' \\ \alpha_2' \\ \vdots \\ \alpha_n' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}$$

Transformation of an object follows the matrix rules as stated above.

2.7 Projection of images and uses of linear mapping theory in matrix representation form

Projection of image is a linear mapping. Generally, projection of image of an object may be a mapping from 3-dimension to 2- dimension. Or it may be a mapping from n-dimension to m- dimension. In the matrix theory, linear mapping works as follows which is shown with following example:

>>Example :A linear mapping T : R^3--→ R^2 is defined by $T(x_1,x_2,x_3)=(3x_1-2x_2+x_3, x_1-3x_2-2x_3)$, (x_1,x_2,x_3) belongs to R^3 . Find the matrix of T relative to the ordered bases

(i) (1,0,0), (0,1,0),(0,0,1) of R^3 and (1,0),(0,1) of R^2 ;

(ii) (0,1,0),(1,0,0) ,(0,0,1) of R^3 and (0,1),(1,0) of R^2 ;

(iii) (0,1,1) , (1,0,1),(1,1,0) of R^3 and (1,0),(0,1) of R^2

(i)T(1,0,0)=(3,1)=3(1,0)+1(1,0)

T(0,1,0)=(-2,-3)=-2(1,0)-3(0,1)

T(0,0,1)=(1,-2)=1(1,0)-2(0,1)

Therefore the matrix of T=$\begin{pmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}$

(ii)T (0,1,0) =(-2,-3)=-3(0,1)-2(1,0)
T(1,0,0) =(3,1)=1(0,1)+3(1,0)
T(0,0,1)=(1,-2)=-2(0,1)+1(1,0)

Therefore matrix of T= $\begin{pmatrix} -3 & 1 & -2 \\ -2 & 3 & 1 \end{pmatrix}$

(iii) T(0,1,1)=(-1,-5) =-1(1,0)-5(0,1)
T(1,0,1)=(4,-1)=4(1,0)-1(0,1)
T(1,1,0)=(1,-2)=1(1,0)-2(0,1)

Therefore matrix of T = $\begin{pmatrix} -1 & 4 & 1 \\ -5 & -1 & -2 \end{pmatrix}$

3.Properties and importance of matrix used in transformation / projection of object/ image.

The matrix used in transformation or projection of images may be as follows:

(i) Matrix multiplication rules should be follows. Rules of Matrix Multiplication

Let $A=[a_{ij}]$, i=1,2,3,…,m ; j=1,2,3,….,n

And $B=[b_{jk}]$, j=1,2,……,n; k=1,2,…….,p

Be two m x n and n x p matrices respectively such that numbers of rows of B is the same as the number of column of A. Then the product of these two matrices is defined as an m x p matrix.

$C=AB=[C_{ik}]$, i=1,2,…..,m ;k=1,2,…..,p

Where $C_{ik}=a_{i1}b_{1k}+a_{i2}b_{2k}+\ldots\ldots+a_{in}b_{nk}$

(ii) Dimension and co-ordinates representation with respect to object may be same or different.

(iii) It is helpful to understand important of matrix multiplication in understanding transformation of object.

4. Advantages

In general we solve the linear equation with different variables but we cannot understand what its practical importance is. Representation of transformation of an object or projection of (images) in computer graphics by using mathematical matrix form is helpful to understand about it. Also, linear mapping on a matrix is helpful to understand the projection of images.

5. Conclusion and future Scope

"Transformations are the operations applied to geometrical description of an object to change its position, orientation, or size is called geometric transformations". The matrix representation relative to transformation of an object is helpful to understand that there exists non-zero vector in the matrix. Also this thing is helpful to understand the linear dependency / independency of vector. The aim of this paper is regarding utility of the matrix in the transformation of an object. Also the linear mapping on Matrix is helpful to study the projection of images.

In future, there is scope to study regarding transformation and projection of different dimensional objects by using matrix theory.

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