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# (1,2) * - $\alpha^*$ - CLOSED SETS IN BITOPOLOGICAL SPACES

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**ABSTRACT**

The aim of this paper is to introduce a new class of sets called (1 ,2)*- $\alpha^*$ -closed sets in topological spaces and to study their properties. Further, we define and study (1,2)*- $T_{\alpha^*}$ $-$space , (1,2)*- ${}_gT_{\alpha^*}$ -space and their properties..



## 1.Introduction

The study of bitopological spaces was first initiated by J.C. Kelly [6] in the year 1963. Levine [9] introduced the notions of generalized closed sets and studied their properties. Veronica Vijayan and K.Priya [13] investigated the concept of $\alpha^*$ closed sets in topological space. Lellis Thivagar et al [8] developed the concept of (1,2)*-semi open sets, (1, 2)*-α-open sets, (1, 2)*-generalized closed sets, (1, 2)*-semi generalized closed sets and (1, 2)*-α-generalized closed sets in bitopological spaces. In this paper, we introduce the notion of (1,2)*- $\alpha^*$ -closed sets and investigate their properties. Further, we study (1,2)*- $T_{\alpha^*}$ $-$space , (1,2)*- ${}_gT_{\alpha^*}$ -space and their properties.

## 2. Preliminaries

Throughout this paper, X and Y denote the bitopological spaces $(X,\tau_1,\tau_2)$ and $(Y,\sigma_1,\sigma_2)$respectively, on which no separation axioms are assumed.

Definition: 2.1 [8] A subset S of a bitopological space X is said to be $\tau_{1,2}$-open if S=A∪B where $\tau_1$ ∈ A and $\tau_2$∈ B .A subset S of X is said to be (i) $\tau_{1,2}$- closed if the complement of S is $\tau_{1,2}$-open. (ii) $\tau_{1,2}$-clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed.

Definition: 2.2 [8] Let S be a subset of the bitopological space X. Then the $\tau_{1,2}$- interior of S denoted by $\tau_{1,2}$- int(S) is defined by ∪{G: G⊆S and G is $\tau_{1,2}$- open} and $\tau_{1,2}$- closure of S denoted by $\tau_{1,2}$-cl(S) is defined by∩{F: S⊆F and F is $\tau_{1,2}$- closed}.

Definition: 2.3 A subset A of a bitopological space X is said to be

(i) (1,2)*-pre-open [8] if A$\subseteq \tau_{1,2}-$int($\tau_{1,2}-$cl(A)).

(ii) (1,2)*-semiopen [8] if A$\subseteq \tau_{1,2}-$cl($\tau_{1,2}-$int(A)).

(iii) (1,2)*-regular open [8] if A=$\tau_{1,2}-$int($\tau_{1,2}-$cl(A)).

(iv)(1,2)*-α-open[8]if A$\subseteq \tau_{1,2}-$int($\tau_{1,2}-$cl($\tau_{1,2}-$int(A))).

(v) (1,2)*-generalized closed(briefly (1,2)*-g-closed) set[8] if $\tau_{1,2}-$cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}$ open set in X

vi)(1,2)*-semi-generalized closed(briefly (1,2)*-sg-closed) set[8] if (1,2)*-scl(A)$\subseteq$U whenever A$\subseteq$U and U is (1,2)*-semi-open set in X.

(vii)(1,2)*-generalized semi closed(briefly (1,2)*-gs-closed) set[2] if (1,2)*- scl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}-$open set in X .

(viii)(1,2)*-generalized $\alpha-$closed(briefly(1,2)*-g$\alpha-$closed) set[8] if (1,2)*- $\alpha$ cl(A)$\subseteq$U whenever A$\subseteq$U and U is (1,2)*- $\alpha-$open set in X

(ix)(1,2)*- $\alpha-$generalized closed(briefly (1,2)*- $\alpha$ g-closed) set[8] if (1,2)*- $\alpha$ cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}-$open set in X.

(x)(1,2)*- $\pi$ generalized closed(briefly $(1,2)^*$ -$\pi g-$closed) [10] if $\tau_{1,2}-$cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}-\pi$-open set in X.

(xi)(1,2)*- $\pi$ generalized $\alpha-$closed((briefly $(1,2)^*$ -$\pi g\alpha-$closed ) set[2] if $(1,2)^*$-$\alpha$ cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}-\pi$-open set in X.

(xiv)(1,2)*- $\pi$ generalized pre-closed(briefly (1,2)*- $\pi$ gp-closed) set[12] if $(1,2)^*$-pcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\tau_{1,2}-\pi$-open set in X.

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(xv)(1,2)*-gsp-closed set [4] if (1,2)*- spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}-$open set in X.

(xvi)(1,2)*-gpr- closed set[7] if $(1,2)^*$-pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $(1,2)^*$-regular open set in X.

(xvii)(1,2)*- $\pi^*$g-closedset[5]ifcl( $\tau_{12}-$int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{12}-\pi$-open set in X.

(xviii) a (1,2)*- weakly generalized closed[5] (briefly (1,2) *-wg- closed ) if $\tau_{1,2}-$cl( $\tau_{1,2}-$int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}-\pi$-open set in X.

The complements of all the above mentioned open sets are called their respective closed sets. The family of all (1,2)*-open ((1,2)*-regular open, (1,2)*-semi open,(1,2)*- α-open) sets of X will be denoted by (1,2)*-O(X)(resp.(1,2)*RO(X), (1,2)*-SO(X),(1,2)*-αO(X)).The (1,2)*-semi-closure(resp. (1,2)*-preclosure,(1,2)*-α-closure) of a subset A of X is denoted by(1,2)*-scl(A) (resp.(1,2)*-pcl(A),(1,2)*-αcl(A)) defined as the intersection of all (1,2)*-semi-closed(resp.(1,2)*-preclosed,(1,2)*α-closed) sets containing A.

## 3. (1,2) * - $\alpha^*$ - closed sets

**Definition : 3.1** A subset A of a bitopological space X is called(1,2) * - $\alpha^*$ - closed set if $\tau_{1,2}-cl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is (1,2) * - $\alpha$ - open in X. The collection of all (1,2)*- $\alpha^*$ - closed sets in X is denoted by (1,2) * - $\alpha^* C(X)$

**Theorem 3.2**

i)Every $\tau_{1,2}-$closed set is (1,2)*- $\alpha^*$-closed set .

ii)Every (1,2)*- $\alpha^*$-closed set is (1,2)*- g-closed set.

iii)Every (1,2)*- $\alpha^*$-closed set is (1,2)*- gs-closed set.

iv)Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $\alpha g$ -closed set.

v) Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $\alpha$ -closed set.

vi) Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $g\alpha$ -closed set.

vii) Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $\pi g$ -closed set.

viii)Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $\pi g\alpha$ -closed set.

x) Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $gp$ -closed set.

xi) Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $\pi gp$ -closed set.

xii) Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $\pi^* g$ -closed set.

xiii)Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $gpr$ -closed set.

xiv)Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $gsp$-closed set.

xv)Every (1,2)*- $\alpha^*$-closed set is (1,2)*- $wg$ -closed set.

**Proof:**

i)Straight forward.

ii)Suppose A is (1,2)*- $\alpha^*$-closed set and U be any $\tau_{1,2}-$open set containing A. Since every $\tau_{1,2}-$open set is (1,2)*- $\alpha$ -open set and then , $\tau_{1,2}-cl(A)\subseteq U$ for every subset A of X. Thus, $\tau_{1,2}-cl(A)\subseteq U$ and hence A is (1,2)*- g-closed set.

iii) Suppose A is (1,2)*- $\alpha^*$-closed set and U be any $\tau_{1,2}-$open set containing A. Since every $\tau_{1,2}-$open set is (1,2)*- $\alpha$ -open set and then , $\tau_{1,2}-cl(A)\subseteq U$ for every subset A of X. Therefore, $(1,2)^*-scl(A)\subseteq\tau_{1,2}-cl(A)\subseteq U$ and hence A is (1,2)*- gs-closed set.

iv)The proof is obvious.

v)The proof is straight forward , since every $\tau_{1,2}-$open set is (1,2)*- $\alpha$ -open set

vi)It is true that , $(1,2)^*-\alpha cl(A)\subseteq\tau_{1,2}-cl(A)\subseteq U$ ,for every subset A of X.

vii) Suppose A is (1,2)*- $\alpha^*$-closed set and U be any $\tau_{1,2}-\pi-$open set containing A. Since every $\tau_{1,2}-\pi-$open set is (1,2)*- $\alpha$ -open set and then , $\tau_{1,2}-cl(A)\subseteq U$ for every subset A of X. Thus, $\tau_{1,2}-cl(A)\subseteq U$ and hence A is (1,2)*- g-closed set.

viii) It is true that , $(1,2)^*-\alpha cl(A)\subseteq\tau_{1,2}-cl(A)\subseteq U$ ,for every subset A of X.

ix) The proof is obvious.

x)It is true , since $(1,2)^*-pcl(A)\subseteq\tau_{1,2}-cl(A)\subseteq U$ , for evey subset A of X

xi)The proof is straight forward.

xii) Suppose A is (1,2)*- $\alpha^*$-closed set and U be any $\tau_{1,2}-\pi-$open set containing A. Since every $\tau_{1,2}-\pi-$open set is (1,2)*- $\alpha$ -open set and then , $\tau_{1,2}-cl(\tau_{1,2}-\text{int}(A)\subseteq\tau_{1,2}-cl(A)\subseteq U$ for every subset A of X. Thus, $\tau_{1,2}-cl(\tau_{1,2}-\text{int}(A))\subseteq U$ and hence A is (1,2)*- $\pi^* g$ -closed set.

xiii)The proof is obvious.

xiv)The proof is obvious.

xv)The proof is obvious.

**Remark :3.3** The converse of the above results need not be true as seen in the following examples.

**Example: 3.4**

**i)**Let $X=\{a,b,c,d\}$ $\tau_1=\{\phi,X,\{a\}\}$, $\tau_2=\{\phi,X,\{c.d\}\}$ Then the set $\{b,c\}$ is (1,2)*- $\alpha^*-$ closed set but not $\tau_{1,2}-$closed.

ii)Let $X=\{a,b,c,d\}$
$\tau_1=\{\phi,X,\{a\},\{d\},\{a.d\},\{c,d\},\{a,c,d\}\}$,
$\tau_2=\{\phi,X,\{a.c\}\}$
Then the set $\{a,b,d\}$ is (1,2)*-g- closed set but not (1,2)*- $\alpha^*-$ closed.

iii) In example 3.4 (ii), $\{a\} and \{a,c\}$ are (1,2)*-gs- closed set but not (1,2)*- $\alpha^*-$ closed.

iv) ) In example 3.4 (ii), $\{c\}$ is (1,2)*- $\alpha g$ - closed set but not (1,2)*- $\alpha^*-$ closed.

v)Let $X=\{a,b,c,d\}$ $\tau_1=\{\phi,X,\{a\},\{b,c,d\}\}$,
$\tau_2=\{\phi,X,\{c\}\{a,b,d\}\}$

Then the set $\{b\}$ is (1,2)*- $\alpha-$ closed set but not (1,2)*- $\alpha^*-$ closed.

vi) In example 3.4 (ii), $\{c\}$ is (1,2)*- $g\alpha$ - closed set but not (1,2)*- $\alpha^*-$ closed.

vii)In example 3.4 (ii), $\{a,b,d\}$ is (1,2)*- $\pi g$ - closed set but not (1,2)*- $\alpha^*-$ closed.

viii) In example 3.4 (ii), $\{a\}$and $\{c\}$ are (1,2)*- $\pi gs$ - closed set but not (1,2)*- $\alpha^*-$ closed.

ix) In example 3.4 (ii), $\{c\}$ is (1,2)*- $\pi g\alpha$ - closed set but not (1,2)*- $\alpha^*-$ closed.

x) In example 3.4 (ii), $\{c\}$ is (1,2)*- $gp$ - closed set but not (1,2)*- $\alpha^*-$ closed.

xi)In example 3.4 (ii), $\{c\}$ is (1,2)*- $\pi gp$ - closed set but not (1,2)*- $\alpha^*-$ closed.

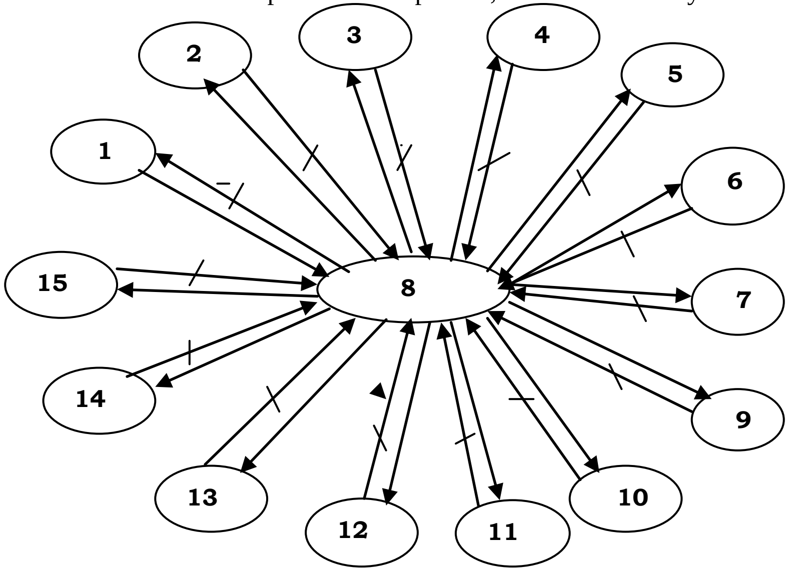
xii) In example 3.4 (ii), $\{a,b,d\}$ is (1,2)*- $\pi^* g$ - closed set but not (1,2)*- $\alpha^*-$ closed.

xiii) In example 3.4 (ii), $\{c\}$and $\{a,d\}$ are (1,2)*- $gpr$ - closed set but not (1,2)*- $\alpha^*-$ closed.

xiv) In example 3.4 (ii), $\{a\}$is (1,2)*- $gsp$ - closed set but not (1,2)*- $\alpha^*-$ closed.

xv )In example 3.4 (ii), $\{c\}$and $\{a,b,d\}$ are (1,2)*- $wg$ - closed set but not (1,2)*- $\alpha^*-$ closed.

**Remark: 3. 5** The following diagram shows the relationships of (1, 2)*- $\alpha^*-$closed sets with other known existing sets. A $\rightarrow$ B represents A implies B, but not conversely.




1. $\tau_{12}$ -closed set
2. (1, 2)*-g-closed set
3. (1, 2)*-gs-closed set.
4. (1, 2)*- αg-closed
5. (1, 2)*- α-closed set
6. (1, 2)*- gα-closed set
7. (1, 2)*- πg-closed set
8. (1, 2)*- $\alpha^*$-closed set
9. (1, 2)*- πgα-closed
10. (1, 2)*- gp-closed set
11. (1, 2)*- πgp-closed set
12. (1, 2)*- $\pi^*$g-closed set.
13. (1, 2)*- gpr-closed set.
14. (1, 2)*- gsp-closed set.
15. (1, 2)*- wg-closed set.

**Proposition 3.6**:Finite union of (1,2)* -$\alpha^*-$closed sets are (1,2)* -$\alpha^*-$ closed set .

**Proof:** Let $A\cup B\subseteq U$ where $U$ is $\alpha-$open. Since A and B are (1,2)*- $\alpha^*-$closed sets, $\tau_{1,2}-cl(A)\subseteq U$ and $\tau_{1,2}-cl(B)\subseteq U$ .Now $\tau_{1,2}-cl(A\cup B)\subseteq U$ . Hence $A\cup B$ is (1,2)* -$\alpha^*-$ closed set.

**Theorem 3.7** :If a subset A of X is both (1,2)* -$\alpha-$ open and (1,2)* -$\alpha^*-$ closed, then it is $\tau_{1,2}-closed$ .

**Proof**:Let A be a subset of X which is both (1,2)* -$\alpha-$ open and (1,2)* -$\alpha^*-$ closed.Then $\tau_{1,2}-cl(A)\subseteq U$ whenever $A\subseteq U$ and U is (1,2)* -$\alpha-$ open in X. Also $A\subseteq\tau_{1,2}-cl(A)$ .This implies $\tau_{1,2}-cl(A)=A$.Hence A is $\tau_{1,2}-closed$ .

**Proposition 3.8:**For any $x\in X$ ,its complement $X-\{x\}$ is (1,2)* -$\alpha^*-$ closed or (1,2)* -$\alpha-$ open.

**Proof :**Suppose $X-\{x\}$ is not (1,2)* -$\alpha-$ open.Then $X$ is the only (1,2)* -$\alpha-$ open set containing $X-\{x\}$ .That is, $X-\{x\}\subseteq X$ and $X$ is (1,2)* -$\alpha-$ open.Then, $\tau_{12}-cl(X-\{x\})\subseteq X$ whenever $X-\{x\}\subseteq X$ and $X$ is (1,2)* -$\alpha-$ open.This implies $X-\{x\}$ is (1,2)* -$\alpha^*-$ closed.

**Proposition 3.9**:If A is (1,2)* -$\alpha^*-$ closed and $A\subseteq B\subseteq\tau_{1,2}-cl(A)$ ,then B is also (1,2)* -$\alpha^*-$closed.

**Proof** :Let $B\subseteq U$ ,where U is (1,2)* -$\alpha-$open in X. Then $A\subseteq B$ implies $A\subseteq U$.Since A is (1,2)* -$\alpha^*-$ closed implies $\tau_{1,2}-cl(A)\subseteq U$ .Given $B\subseteq\tau_{1,2}-cl(A)$ implies $\tau_{1,2}-cl(B)\subseteq\tau_{1,2}-cl(A)\subseteq U\Rightarrow\tau_{1,2}-cl(B)\subseteq U$ .Therefore B is (1,2)* -$\alpha^*-$closed.

**Proposition 3.10:** A is (1,2)* -$\alpha^*-$ closed set if and only if $\tau_{1,2}-cl(A)-A$ does not contain any (1,2)* -$\alpha-$closed set.

**Proof** :**Necessity** : Let F be an (1,2)* -$\alpha-$closed.set such that $F\subseteq\tau_{1,2}-cl(A)-A$.Since A is (1,2)* -$\alpha^*-$ closed, $\tau_{1,2}-cl(A)\subseteq U$ whenever $A\subseteq U$ and U is (1,2)* -$\alpha-$ open in X. Then $A\subseteq X-F$. Since A is (1,2)* -$\alpha^*-$ closed, $\tau_{1,2}-cl(A)\subseteq X-F$ whenever $A\subseteq X-F$ and $X-F$ is (1,2)* -$\alpha-$ open in X. So $F\subseteq X-\tau_{1,2}-cl(A)$. But $F\subseteq\tau_{1,2}-cl(A)-A$. Therefore $F\subseteq[(X-\tau_{1,2}-cl(A))\cap(\tau_{1,2}-cl(A)-A)]=\phi$.Thus $F=\phi$.

**Sufficiency**: Let A be a subset of X such that $\tau_{1,2}-cl(A)-A$ does not contain any non-empty (1,2)* - $\alpha-$closed set. Let U be (1,2)* - $\alpha-$open set such that $A\subseteq U$. If $\tau_{1,2}-cl(A)\not\subset U$, then $\tau_{1,2}-cl(A)\cap U^c\neq\phi$ and $\tau_{1,2}-cl(A)\cap U^c$ is (1,2)* - $\alpha-$closed set. But $\phi=\tau_{1,2}-cl(A)\cap U^c\subseteq\tau_{1,2}-cl(A)-A$. That is, $\tau_{1,2}-cl(A)-A$ contains a non-empty (1,2)* - $\alpha-$closed set which is a contradiction. Therefore $\tau_{1,2}-cl(A)\subseteq U$ and hence A is (1,2)* - $\alpha-$closed set.

**Proposition 3.11** :If A is (1,2)* - $\alpha^*-$ closed set ,then $(1,2)^*-\alpha cl(x)\cap A\neq\phi$ holds, for each $x\in\tau_{1,2}-cl(A)$

Proof:Suppose $(1,2)^*-\alpha cl(x)\cap A=\phi$ holds, for each $x\in\tau_{1,2}-cl(A)$.We $A\subseteq X-(1,2)^*-\alpha cl(\{x\})\cap A\neq\phi$.Since A is (1,2)* - $\alpha^*-$ closed set , $\tau_{1,2}-cl(A)\subset X-[(1,2)^*-\alpha cl(x)]$ implies $x\notin\tau_{1,2}-cl(A)$,which is a contradiction. Hence $(1,2)^*-\alpha cl(x)\cap A\neq\phi$ holds, for each $x\in\tau_{1,2}-cl(A)$.

## 4. (1,2)*-$\alpha^*$-Open Sets

**Definition 4.1** A set $A\subseteq X$ is called (1,2)*- $\alpha^*$-open set iff its complement is (1,2)*- $\alpha^*$-closed set and the collection of all (1,2)*- $\alpha^*$-open sets jnX is denoted by (1,2)*- $\alpha^*$ O(X).

**Remark 4.2i**) $\tau_{1,2}-cl(X-A)=X-\tau_{1,2}-\text{int}(A)$

ii) For any $A\subseteq X,(\tau_{1,2}-\text{int}(\tau_{1,2}-cl(A)-A))=\phi$

**Theorem: 4.3** A subset $A\subseteq X$ is (1,2)*- $\alpha^*$-open set iff $F\subseteq\tau_{1,2}-\text{int}(A)$ wherever F is (1,2)*- $\alpha$ -closed set such that $F\subseteq A$.

**Proof**:Let F is (1,2)*- $\alpha$ -closed set such that $F\subseteq A$.Since X-A is (1,2)*- $\alpha$ -closed and $X-A\subseteq X-F$ ,we have $F\subseteq\tau_{1,2}-\text{int}(A)$.Conversely, $F\subseteq\tau_{1,2}-\text{int}(A)$ where F is (1,2)*- $\alpha$ -closed set and $F\subseteq A$.Since $F\subseteq A$ and $X-F$ is (1,2)*- $\alpha$ -open , $\tau_{1,2}-cl(X-A)=X-\tau_{1,2}-\text{int}(A)\subseteq X-F$ .Therefore A is (1,2)*- $\alpha^*$ -open set.

**Theorem :4.4** If $\tau_{1,2}-\text{int}(A)\subseteq B\subseteq A$ and $A$ is (1,2)*- $\alpha^*$-open set ,then $B$ is (1,2)*- $\alpha^*$ -open set.

**Proof** :Since $\tau_{1,2}-\text{int}(A)\subseteq B\subseteq A\Rightarrow X-A\subseteq X-B\subseteq X-\tau_{1,2}-\text{int}(A)$. That is $X-A\subseteq X-B\subseteq\tau_{1,2}-cl(X-A)$.Since $X-A$ is (1,2)*- $\alpha^*$-closed set, by Theorem 4.3 $X-B$ is (1,2)*- $\alpha^*$-closed set. This implies , $B$ is (1,2)*- $\alpha^*$ -open set.

**Theorem :4.5** If $A\subseteq X$ is (1,2)*- $\alpha^*$-closed set ,then $(\tau_{1,2}-cl(A))-A$ is (1,2)*- $\alpha^*$-open set.

**Proof:**Let A be (1,2)*- $\alpha^*$-closed and F be (1,2)*- $\alpha$ -closed set, such that $F\subseteq(\tau_{1,2}-cl(A))-A$. Then by proposition 3.10, $F=\phi$. $F\subseteq(\tau_{1,2}-\text{int}(\tau_{1,2}-cl(A)-A))=\phi$ This implies, $(\tau_{1,2}-cl(A))-A$ is (1,2)*- $\alpha^*$ -open set.

**Theorem :4.6** If $A$ and $B$ are (1,2)*- $\alpha^*$-open sets ,then $A\cap B$ is (1,2)*- $\alpha^*$ -open set.

**Proof**:Let $X-(A\cap B)=(X-A)\cup(X-B)\subseteq F$ ,where $F$ is (1,2)*- $\alpha$ -open . Since $X-A\subseteq F, X-B\subseteq F$ and $A$ and $B$ are (1,2)*- $\alpha^*$-open sets, we have $\tau_{1,2}-cl(X-A)\subseteq F,\tau_{1,2}-cl(X-B)\subseteq F$ .So, $\tau_{1,2}-cl[X-(A\cap B)]\subseteq F$.Therefore , $A\cap B$is (1,2)*- $\alpha^*$ -open set.

**Theorem:4.7** For any $A\subseteq X,\tau_{1,2}-\text{int}(A)\subseteq(1,2)^*-\alpha^*-\text{int}(A)\subseteq A$.

**Proof:** The proof follows immediately, since every $\tau_{1,2}$ - open set is (1,2)*- $\alpha^*$ -open set.

**Definition :4.8** For a subset A of X , we define the (1,2)*- $\alpha^*$-closure of A as (1,2)*- $\alpha^*$-cl(A) = $\cap\{F:A\subseteq F, F\text{ is }(1,2)^*-\alpha^*-\text{closed in }X\}$.

**Lemma :4.9** Let A be a subset of X and $x\in X$ .Then $x\in$(1,2)*- $\alpha^*$-cl(A) if and only if $V\cap A\neq\phi$,for every (1,2)*- $\alpha^*$ -open set V containing x.

**Proof**:**Necessity**: Suppose that there exists a (1,2)*- $\alpha^*$ -open set V containing x such that $V\cap A=\phi$.Since $A\subseteq X-V$, (1,2)*- $\alpha^*$-cl(A) $\subseteq X-V$, and this implies $x\notin$(1,2)*- $\alpha^*$ -cl(A), a contradiction.

**Sufficiency:** Suppose that, $x\notin$(1,2)*- $\alpha^*$ -cl(A).Then, there exists (1,2)*- $\alpha^*$ -closed set F containing A such that $x\notin$F. Then $x\in X-F$ and $X-F$ is (1,2)*- $\alpha^*$ -open. Also, $(X-F)\cap A=\phi$ ,a contradiction.

**Definition :4.10** Let A and B be subset of X. Then A and B are said to be (1,2)* -separated sets if $A\cap\tau_{1,2}-cl(B)=B\cap\tau_{1,2}-cl(A)=\phi$.

**Definition :4.11** A space X is said to be a (1,2)*- $T_{\alpha^*}$ -space if every (1,2)*- $\alpha^*$ -closed set is $\tau_{1,2}-$closed set.

**Definition:4.12** Let X be a space and $A\subseteq X$ ,(1,2)*- $\alpha^*-\text{int}(A)$ is the union of all (1,2)*- $\alpha^*-$ open sets contained in A.

**Theorem:4.13** If $A$ and $B$ are (1,2)*- separated sets in a (1,2)*- $T_{\alpha^*}$ -space ,then $A\cup B$ is (1,2)*- $\alpha^*$ -open.

**Proof :** Since A and B are (1,2)* -separated sets , $A\cap\tau_{1,2}-cl(B)=B\cap\tau_{1,2}-cl(A)=\phi$.Let F be a (1,2)*- $\alpha$ -closed set and $F\subseteq A\cup B$.$F\cap\tau_{1,2}-\text{cl}(A)\subseteq(A\cup B)\cap\tau_{1,2}-cl(A)=(A\cap\tau_{1,2}-cl(B))\cup(B\cap\tau_{1,2}-cl(A))=A\cup\phi=A$. Sin ce X is a (1,2)*- $T_{\alpha^*}$ -space ,every (1,2)*- $\alpha^*$ -closed set is $\tau_{1,2}$ $-$closed ,F is $\tau_{1,2}$ $-$closed and F$\cap$ $\tau_{1,2}$ $-$cl$(A)$ is $\tau_{1,2}$ $-$closed.But every $\tau_{1,2}$ $-$closed set is (1,2)*- $\alpha^*$ -closed set .So it is (1,2)*- $\alpha^*$ -closed.Then A is (1,2)*- $\alpha^*$ -open. This implies, F$\cap$ $\tau_{1,2}$ $-$cl$(A) \subseteq (1,2)*-$ int (A).Similarly, F$\cap$ $\tau_{1,2}$ $-$cl(B) $\subseteq$ (1,2)*-int (B).Now ,

F$\subseteq$(F$\cap$A)$\cup$(F$\cap$B)$\subseteq$

$(F \cap \tau_{1,2}\text{-cl(A)}) \cup (F \cap \tau_{1,2}\text{-cl(B)}) \subseteq$

(1,2)*-int (A) $\cup$(1,2)*-int (B) $\subseteq$ (1,2)*-int (A$\cup$B) .That is, F$\subseteq$ (1,2)*-int (A$\cup$B) . Hence, A$\cup$B is (1,2)*- $\alpha^*$ -open.

**Theorem:4.12** Let A be a subset of a space X. Then $X-\left[(1,2)^* - \alpha^* - \text{int}(A)\right]$ =(1,2)*- $\alpha^*$ -cl(X- A).

**Proof:**Let $x \in X-\left[(1,2)^* - \alpha^* - \text{int}(A)\right]$ .Then $x \notin (1,2)^* - \alpha^* - \text{int}(A)$ .That is, every (1,2)*- $\alpha^*$ -open set B containing x is such that B$\not\subset$A. This implies, every (1,2)*- $\alpha^*$ -open set B containing x intersects X-A. This means, $x \in (1,2)^* - \alpha^* - cl(X-A)$.Conversely, let $x \in (1,2)^* - \alpha^* - cl(X-A)$. Then, every (1,2)*- $\alpha^*$ -open set B containing x intersects X-A.

That is, every (1,2)*- $\alpha^*$ -open set B containing x is such that B$\not\subset$A. This implies $x \notin (1,2)^* - \alpha^* - \text{int}(A)$ .That is, $x \in X-\left[(1,2)^* - \alpha^* - \text{int}(A)\right]$ .

Thus, (1,2)*- $\alpha^*$ -cl(X- A) $\subseteq X-\left[(1,2)^* - \alpha^* - \text{int}(A)\right]$ .

Hence $X-\left[(1,2)^* - \alpha^* - \text{int}(A)\right]$ =(1,2)*- $\alpha^*$ -cl(X- A).

Similarly, we have $X-\left[(1,2)^* - \alpha^* - cl(A)\right]$ =(1,2)*- $\alpha^*$ -int (X-A).

**5.(1,2)* -$T_{\alpha^*}$ - Spaces** :

**Definition :5.1** A space X is called (1,2)*- $T_{\alpha^*}$ - space if every (1,2)* -$\alpha^*$ -closed set is $\tau_{12}$ closed set.

**Definition :5.2** A space X is called (1,2)*- ${}_gT_{\alpha^*}$ - space if every(1,2)* -g-closed set is (1,2)* $\alpha^*$ -closed set.

**Theorem: 5.3** For a bitopological space (X, $\tau_1, \tau_2$) the following conditions are equivalent.

(i) X is (1,2)*- $T_{\alpha^*}$ -space

(ii) Every singleton of X is either (1,2)* -$\alpha$ - closed or $\tau_{1,2}$ -Open

**Proof:(i) $\rightarrow$ (ii):** Let x$\in$X and assume that {x} is not (1,2)* - $\alpha$ - closed. Then by Proposition 3.8, X – {x} is not (1,2)* - $\alpha$ -open and is trivially (1,2)*- $\alpha^*$-closed set . Since X is (1,2)*- $T_{\alpha^*}$ -space , every (1,2)*- $\alpha^*$ -closed set is $\tau_{1,2}$ -closed. This implies, X-{x} is $\tau_{1,2}$ -closed and hence {x} is $\tau_{1,2}$ -open .

**(ii) $\rightarrow$ (i):** Assume every singleton of X is either (1,2)* -$\alpha$ -closed or $\tau_{1,2}$ -open. Let A$\subset$X be (1,2)* -$\alpha$ -closed or $\tau_{1,2}$ -open. Obviously A$\subset$ $\tau_{1,2}$ -cl(A) and x $\in$ $\tau_{1,2}$ -cl(A).

To prove that A= $\tau_{1,2}$ -cl(A).

**Case 1**:Suppose the set $\{x\}$ is (1,2)* -$\alpha$ -closed. If x$\notin$A, then

A$\subset$X- $\{x\}$ .Since A is (1,2)* -$\alpha$* closed and X- $\{x\}$is (1,2)* -$\alpha$ - open, $\tau_{1,2}$ -cl(A)$\subset$ X- $\{x\}$ and x$\notin$ $\tau_{1,2}$ -cl(A) which is a contradiction.

Therefore x $\in$ $\tau_{1,2}$ cl(A).

**Case 2:**The set $\{x\}$ is $\tau_{1,2}$ -open. Since x $\in$ $\tau_{1,2}$ -cl(A), then $\{x\} \cap$ A$\neq \phi$ implies x$\in$ A.In both the cases ,x$\in$ A .So , $\tau_{1,2}$ -cl(A)= A or equivalently A is $\tau_{1,2}$ -closed.This shows that X is X is (1,2)*- $T_{\alpha^*}$ -space

**Theorem 5.4**

(i)$(1,2)^*$- $\alpha^*$O(X) $\subset$ $(1,2)^*$- GO(X)

(ii)A space is (1,2)*- ${}_gT_{\alpha^*}$ -space iff $(1,2)^*$- $\alpha^*$O(X) = $(1,2)^*$-GO(X).

Proof:(i)Let A be (1,2)*- $\alpha^*$ open. Then X–A is (1,2)*- $\alpha^*$ closed and so (1,2)*- g- closed. Hence A is (1,2)*-g-open. Hence (1,2)*- $\alpha^*$ O(X) $\subset$ (1,2)*-GO(X)

(ii)**Necessity:** Let X be (1,2)*- $T_{\alpha^*}$ -space. Let A $\in$(1,2)*-GO(X). Then X–A is (1,2)*-g-closed. Since the space X is (1,2)*- ${}_gT_{\alpha^*}$ -space , X–A is (1,2)*- $\alpha^*$ closed.

The above implies A is (1,2)*- $\alpha^*$ open in X. Hence (1,2)*-$\alpha^*$O(X)=(1,2)*-GO(X)

**Sufficiency**: Let (1,2)*- $\alpha^*$O(X)=(1,2)*-GO(X).

Let A be (1,2)*-g-closed. Then X–A is (1,2)*-g-open and X–A $\in$(1,2)*- $\alpha^*$ O(X) .Hence A is (1,2)*- $\alpha^*$ closed and hence A is (1,2)*- ${}_gT_{\alpha^*}$ -space.

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