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Nano pre-generalized locally closed sets and Nano pre-generalized locally continuous functions in Nano topological space.

K.Bhuvaneswari¹ and K.Mythili Gnanapriya²

¹Department of Mathematics, Mother Teresa Women's University, Kodaikanal, Tamilnadu, India. ²Research Scholar, Department of Mathematics, Karpagam Academy of Higher Education, Coimbatore, Tamilnadu, India.

ARTICLE INFO Article history: Received: 19 October 2016;	ABSTRACT The purpose of this paper is to introduce a new class of sets called Nano pre-generalized locally closed set in Nano topological space and to discuss some of its properties.
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1. Introduction

Balachandran [1] *et al.*, introduced the concept of generalized locally closed sets in topology. They also investigated the classes of GLC-continuous maps and GLC-irresolute maps. The notion of Nano topology was introduced by Lellis Thivagar [5] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure. He has also defined Nano continuous functions, Nano open mapping, Nano closed mapping and Nano Homeomorphism. Bhuvaneswari [2] et.al introduced and studied some properties of Nano pre-generalized closed sets in Nano topological spaces. In this paper, new class of sets called Nano pre-generalized locally closed sets are introduced and some of its properties are discussed. We also introduce Nano pre-generalized locally continuous function and Nano pre-generalized locally irresolute function and study some of its properties. **2. Preliminaries**

Definition: 2.1 A subset of a topological space (X, τ) is called generalized locally closed (briefly glc) [1] if $A = C \cap D$, where

C is g-open and D is g-closed in (X, τ)

Definition: 2.2 Let $f:(X,\tau) \to (Y,\sigma)$ be a function and f is said to be GLC-continuous [1] if $f^{-1}(V)$ is a glc-set of

 (X,τ) for each open set V of (Y,σ)

Definition : 2.3 [8] Let U be a non-empty finite set of objects called the universe and be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_{p}(X)$.

That is, $L_R(X) = \sum_{x \in U} \{R(x) : R(x) \subseteq X\}$ where R(x) denotes equivalence class determined by x.

(*ii*) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = Y\{R(x) : R(x) \cap X \neq \phi\}$

(*iii*) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Property: 2.4 [8] If (U, R) is an approximation space and $X, Y \subseteq U$, then

⁽ⁱ⁾
$$L_R(X) \subseteq X \subseteq U_R(X)$$

⁽ⁱⁱ⁾ $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$

Tele: E-mail address: drkbmaths@gmail.com

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^(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition:2.5 [5]. Let U be the Universe, R be an equivalence relation on U and $\mathcal{T}_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 8, $\mathcal{T}_R(X)$ satisfies the following axioms: (i) U and $\phi \in \mathcal{T}_R(X)$.

(ii) The union of the elements of any sub-collection of $\mathcal{T}_R(X)$ is in $\mathcal{T}_R(X)$

(iii) The intersection of the elements of any finite sub-collection of $T_{p}(X)$ is

in $\mathcal{T}_{R}(X)$.

That is, $\mathcal{T}_{P}(X)$ is a topology on U called the Nano topology on U with respect to X.

We call $(U, \mathcal{T}_R(X))$ as the Nano topological space. The elements of $\mathcal{T}_R(X)$ are called as Nano-open sets. The elements of the complement of $\mathcal{T}_R(X)$ are called as Nano-closed sets.

Definition: 2.6 [3] A subset A of a Nano topological space $(U, \tau_R(X))$, is called Nano generalized locally closed set (briefly Ng-lc) if $A = G \cap F$ where G is Ng-open in $(U, \tau_R(X))$ and F is Ng- closed in $(U, \tau_R(X))$.

Definition: 2.7 [3] For a subset A of $(U, \tau_R(X))$, $A \in NGLC^*((U, \tau_R(X)))$, if there exist a Ng- open set G and a Nano closed set F of $(U, \tau_R(X))$, respectively, such that $A = G \cap F$

Definition: 2.8 [3] For a subset A of $(U, \tau_R(X))$, $A \in NGLC^{**}(U, \tau_R(X))$, if there exist a Nano open set G and a Ngclosed set F of $(U, \tau_R(X))$, respectively, such that $A = G \cap F$.

3. Nano Pre-Generalized Locally Closed Set in Nano Topological Space

Definition : 3.1 A subset A of a Nano topological space $(U, \tau_R(X))$, is called Nano pre-generalized locally closed set (briefly Npg-lc) if $A = G \cap F$ where G is Npg-open in $(U, \tau_R(X))$ and F is Npg- closed in $(U, \tau_R(X))$.

Definition: 3.2 For a subset A of $(U, \tau_R(X))$, $A \in NPGLC^* - (U, \tau_R(X))$, if there exist an Ng- open set G and a Nano closed set F of $(U, \tau_R(X))$, respectively, such that $A = G \cap F$

Definition : 3.3 For a subset A of $(U, \tau_R(X))$, $A \in NPGLC^{**} - (U, \tau_R(X))$, if there exist a Nano open set G and a Ngclosed set F of $(U, \tau_R(X))$, respectively, such that $A = G \cap F$.

Theorem : 3.4 Every Nano locally closed set is Nplc set but not conversely.

Proof: Let A be a Nano locally closed set. Then by definition of Nano locally closed set, A is the intersection of Nano open and Nano closed set. Since every Nano open set is Nano pre-open and every Nano closed set is Nano pre-closed, A is Nano pre-locally closed set.

Example: 3.5 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\mathcal{T}_R(X) = \{\Phi, \{a\}, \{a, b, d\}, \{b, d\}, U\}$. The set $\{b\}$ is an Nplc set but not Nlc set.

Theorem : **3.6** Every Nano locally closed set is Npg-lc set but not conversely.

Proof: Let A be a Nano locally closed set. Then by definition of Nano locally closed set , A is the intersection of Nano open and Nano closed set. Since every Nano open set is Npg-open and every Nano closed set is Npg-closed, A is Nano pre-generalized locally closed set.

Example : 3.7 In the example 3.5, Ngp-lc sets are P(U) and hence the converse does not hold.

Remark : 3.8 If a subset S of $(U, \mathcal{T}_R(X))$ is Ng-lc set, then it is necessarily not a Npg-lc set which is shown in the following example.

Example: 3.9 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a\}$. Then $\mathcal{T}_R(X) = \{\Phi, \{a\}, U\}$. Here the set $\{a, b\}$ is Ng-lc set but not Npg-lc set.

Remark : 3.10 Ng-lc^{*} sets and Npg-lc^{*} sets are independent of each other which is shown in the following examples.

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Example: 3.11 Consider $T_P(X)$ from the example 3.5. Consider set $A = \{a, b, c\}$. Then A is Npg-lc* set but it is not Ng-lc* set. That is Npg-lc* sets does not imply Ng-lc* sets.

Example: 3.12 Consider $T_B(X)$ from the example 3.9. Let $B = \{b\}$ which is a Ng-lc* set but B is not Npg-lc* sets. That is Nglc* sets does not imply Npg-lc* sets.

Remark : 3.13 If a subset S of $(U, \mathcal{T}_R(X))$ is Ng-lc^{**} set, then it is necessarily not a Npg-lc^{**} set which is shown in the following example.

Example: 3.14 Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then $\mathcal{T}_R(X) = \{\Phi, \{a\}, U\}$. Let $S = \{a, b\}$ which is a Ng-lc** set whereas it is not Npg-lc** set.

Remark : 3.15 If a subset S of $(U, T_R(X))$ is Nplc set, then it is necessarily not a Npg-lc set which is shown in the following example 3.9 in which Nplc-sets is given by P(U).

Remark : 3.16 If a subset S of $(U, \mathcal{T}_R(X))$ is Nplc* set, then it is necessarily not a Npg-lc* set which is shown in the following example.

Example: 3.17 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a\}$. Then $\mathcal{T}_R(X) = \{\Phi, \{a\}, U\}$. Let $A = \{a, b\}$. Then A is a Nplc* set and it is not Npg-lc* set.

Remark: 3.18 Nplc** sets and Npg-lc** sets are independent of each other which is shown in the following examples.

Example: 3.19 Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then $\tau_R(X) = \{\Phi, \{a\}, U\}$. Let $A = \{a, c\}$ which is a Nplc** set but not Npg-lc** sets. That is Nplc** sets does not imply Npg-lc** sets.

Example: 3.20 Let $U = \{a, b, c\}$. Then $T_R(X) = \{\Phi, \{a, c\}, U\}$ with $U/R = \{\{a, c\}, \{b\}\}$ and $X = \{a, c\}$. Let $A = \{a, b\}$

which is a Npg-lc** set but not Nplc** sets. That is Npg-lc** sets does not imply Nplc** sets.

Example : 3.21 Nplc* sets / Ng-lc*sets

Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\mathcal{T}_R(X) = \{\Phi, \{a\}, \{a, b, d\}, \{b, d\}, U\}$. Let $A = \{a, b\}$. b, c}. Then A is Nplc* set but not Ng-lc* set.

Example: 3.22 Ng-lc**sets Nplc** sets

Consider the example 3.20 in which $A = \{a, b\}$ is Ng-lc** sets. But A is not Nplc** sets.

Remark : 3.23 Every Nlc set is Nplc* set, Nplc** set, Npglc* set and Npglc** set but not conversely.

Remark : 3.24 Every Npg-lc* set or Npg-lc** set is Npg-lc set but not conversely.

Example: 3.25 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a\}$. Then $\mathcal{T}_R(X) = \{\Phi, \{a\}, U\}$. Here the set $\{b\}$ is Npg-lc set but not Npg-lc*set. Similarly, the set {b, c} is Npg-lc set but not Npg-lc** set.

Remark : 3.26 Npg-lc*sets and Npg-lc** sets are independent of each other.

Example: 3.27 Consider $T_R(X)$ of example 3.17. Then the set {b} is Npg-lc** set but not Npg-lc* set. Similarly the set {b, d}

is Npg-lc* set but not Npg-lc** set.

Example : 3.28 Ng-lc^{**} set / → Nplc^{**} set.

Consider $\tau_R(X)$ of example 3.20. Then the set {a, c} is Ng-lc** set but not Nplc** set.

Remark : 3.29 From the above discussions, we have the following implications.



4. NPGLC-continuous and NPGLC-irresolute functions.

In this section we define NPGLC-continuous, NPGLC^{*}-continuous and NPGLC^{**}-continuous which are weaker than Nano locally continuous function.

Definition: 4.1 A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called Nano pre-generalized locally continuous function [shortly, NPGLC- continuous], (resp. NPGLC^{*-} continuous , resp. NPGLC^{**}-continuous) if $f^{-1}(B) \in NPGLC - (U, \tau_R(X))$ (resp.

 $f^{-1}(B) \in NPGLC^* - (U, \mathcal{T}_R(X)), f^{-1}(B) \in NPGLC^{**} - (U, \mathcal{T}_R(X)) \text{ for each } B \in \mathcal{T}_{R'}(Y).$

Example: 4.2 Let $U = V = \{a, b, c, d\}$ and $\tau_R(X) = \{\Phi, \{a\}, \{a, b, d\}, \{b, d\}, U\}$ and $\tau_{R'}(Y) = \{\Phi, \{a\}, V\}$. Then $NPGLC - (U, \tau_R(X)) = P(U)$ and the identity map $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is NPGLC-continuous.

Definition: 4.3 A function $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called Nano pre-generalized locally irresolute function [shortly, NPGLC- irresolute], (resp. NPGLC^{*}- irresolute , resp. NPGLC^{**}-irresolute) if $f^{-1}(B) \in NPGLC - (U, \mathcal{T}_R(X))$ (resp.

$$f^{-1}(B) \in NPGLC^* - (U, \mathcal{T}_R(X)), f^{-1}(B) \in NPGLC^{**} - (U, \mathcal{T}_R(X))$$
 for each $B \in NPGLC - (V, \mathcal{T}_{R'}(Y))$ (resp. B
 $\in NPGLC^* = (V, \mathcal{T}_R(X)), \text{ resp. } B \in NPGLC^{**} = (V, \mathcal{T}_R(X))$).

$$\sim NPGLC^* - (V, \mathcal{T}_{R'}(Y))^{\text{resp. b}} \sim NPGLC^{**} - (V, \mathcal{T}_{R'}(Y))^{\text{resp. b}}$$

Example: 4.4 Let $U = V = \{a, b, c, d\}$ and $\tau_R(X) = \{\Phi, \{a, c\}, U\}$ and $\tau_{R'}(Y) = \{\Phi, \{a\}, V\}$. Then the identity map $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is NPGLC-irresolute.

Theorem: 4.5 Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a function.

- (i) If f is NLC-continuous, then it is NPGLC^{*}-continuous. (ii) If f is NPGLC^{*}-continuous, then it is NPGLC-continuous.
- (iii) If f is NPGLC-irresolute (resp. NPGLC^{*}-irresolute, resp. NPGLC^{**}-irresolute), then it is NPGLC-continuous (resp. NPGLC^{*}-continuous, resp. NPGLC^{**}-continuous)

Proof: (i) Suppose that f is NLC-continuous. Let B be Nano open set of $(V, \tau_{R'}(Y))$. Then $f^{-1}(B)$ is Nano locally closed

in $(U, \tau_R(X))$ by definition. Then by Remark 9.2.26, $f^{-1}(B) \in NPGLC^* - (U, \tau_R(X))$ for each $B \in \tau_{R'}(Y)$. Therefore

f is NPGLC^{*}-continuous.

(ii) Since every NGLC* set is NGLC set and every NGLC** set is NGLC set , the proof follows.

(iii) Follows from the fact that every Nano open set is Npg-lc set, Npg-lc* set and Npg-lc** set.

Remark : 4.6 The converse of the above theorem is not true as seen from the following examples.

Example : 4.7 Let $U = V = \{a, b, c, d\}$ and $\mathcal{T}_R(X) = \{\Phi, \{a, c\}, V\}$ and $\mathcal{T}_{R'}(Y) = \{\Phi, \{a\}, \{a, b, d\}, \{b, d\}, U\}$. Define $f:(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as an identity mapping. Then f is NPGLC-continuous, NPGLC^{*}-continuous and NPGLC^{**}not NLC-continuous continuous but because $NLC(U, \tau_R(X)) = \{\Phi, \{a, c\}, \}$ {b, U. d}, $NPGLC(U, \tau_{R}(X)) = NPGLC^{*}(U, \tau_{R}(X)) = P(U) \text{ and } NPGLC^{**}(U, \tau_{R}(X)) = P(U)^{-\{\{a, b, c\}, \{a, c, d\}\}}.$

Example : 4.8 Let $U = V = \{a, b, c, d\}$ and $\tau_R(X) = \{\Phi, \{a\}, U\}$ and $\tau_{R'}(Y) = \{\Phi, \{a\}, \{a, b, d\}, \{b, d\}, V\}$. Define

 $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as an identity mapping. Then $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is NPGLC-continuous but not NPGLC^{**}-continuous.

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