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Multi Intuitionistic Fuzzy RW-Open Maps and Multi Intuitionistic Fuzzy RW-Closed Maps in Multi Intuitionistic Fuzzy Topological Spaces

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ABSTRACT

In this paper, we have studied some of the properties of multi intuitionistic fuzzy rw-open mappings and multi intuitionistic fuzzy rw-closed mappings in multi intuitionistic fuzzy topological spaces and prove some results on these.

Introduction

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [23] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. Intuitionistic fuzzy set was introduced and studied by Atanassov.K.T.[2, 3]. The following papers have motivated us to work on this paper. C.L.Chang [8] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like R.H.Warren [22], K.K.Azad [4], G.Balasubramanian and P.Sundaram [5], S.R.Malghan and S.S.Benchalli [14, 15] and many others have contributed to the development of fuzzy topological spaces. On multi fuzzy rw-closed, multi fuzzy rw-open sets in multi fuzzy topological spaces are introduced by Murugan.V, U.Karuppiah & M.Marudai [18, 19]. We have introduced the concept of multi intuitionistic fuzzy rw-open mappings and multi intuitionistic fuzzy rw-closed mappings in a multi intuitionistic fuzzy topological spaces and have established some results.

1. Preliminaries

1.1 Definition[22]

Let X be a non-empty set. A fuzzy subset A of X is a function A: X → [0,1]

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1.2 Definition

A multi fuzzy subset A of a set X is defined as an object of the form $A = \{ \langle x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \rangle / x \in X\}$, where $A_i : X \to [0, 1]$ for all i. It is denoted as $A = \langle A_1, A_2, A_3, \ldots, A_n \rangle$.

1.3 Definition

Let X be a set and $\Im$ be a family of multi fuzzy subsets of X. The family $\Im$ is called a multi fuzzy topology on X if and only if $\Im$ satisfies the following axioms

(i) $\bar{0}$, $\bar{1} \in \Im$, (ii) If $\{ A_i ; i \in I \} \subseteq \Im$, then $\bigcup_{i\in I} A_i \in \Im$, (iii) If $A_1, A_2, A_3, \ldots A_n \in \Im$, then $\bigcap_{i=1}^{i=n} A_i \in \Im$. The pair $(X, \Im)$ is called a multi fuzzy topological space. The members of $\Im$ are called multi fuzzy open sets in X. A multi fuzzy set A in X is said to be multi fuzzy closed set in X if and only if A^c is a multi fuzzy open set in X.

1.4 Definition

Let $(X, \Im)$ be a multi fuzzy topological space. A multi fuzzy set A of X is called multi fuzzy regular w-clsoed(briefly, multi fuzzy rw-closed) if mfcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is multi fuzzy regular semi-open in a multi fuzzy topological space X.

1.5 Definition

A multi fuzzy set A of a multi fuzzy topological space X is called a multi fuzzy regular w-open (briefly, multi fuzzy rw-open) set if its complement A^c is a multi fuzzy rw-closed set in a multi fuzzy topological space X.

1.6 Definition

Let X and Y be multi fuzzy topological spaces. A map $f : X \to Y$ is said to be multi fuzzy rw-continuous if the inverse image of every multi fuzzy open set in Y is multi fuzzy rw-open in X.

1.7 Definition

Let X and Y be multi fuzzy topological spaces. A map $f : X \to Y$ is said to be a multi fuzzy rw-irresolute map if the inverse image of every multi fuzzy rw-open set in Y is a multi fuzzy rw-open set in X.

1.8 Definition[2]

An intuitionistic fuzzy subset(IFS) A of a set X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$, where $\mu_A: X \to [0, 1]$ and $\nu_A: X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.9 Definition

A multi intuitionistic fuzzy subset(MIFS) A of a set X is defined as an object of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$, where $\mu_A(x) = (\mu_{A1}(x), \mu_{A2}(x), \ldots \mu_{An}(x))$, $\mu_{Ai} : X \to [0, 1]$ for all i and $\nu_A(x) = (\nu_{A1}(x), \nu_{A2}(x), \ldots, \nu_{An}(x))$, $\nu_{Ai} : X \to [0, 1]$ for all i, define the degrees of membership and the degrees of non-membership of the element x in X respectively and for every x in X satisfying $0 \leq \mu_{Ai}(x) + \nu_{Ai}(x) \leq 1$ for all i.

1.10 Definition

Let A and B be any two multi intuitionistic fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all x in X.

(ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$, for all x in X.

(iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$.

(iv) $A \cap B = \{ \langle x, \min\{ \mu_A(x), \mu_B(x) \}, \max\{ \nu_A(x), \nu_B(x) \} \rangle / x \in X \}$.

(v) $A \cup B = \{ \langle x, \max \{ \mu_A(x), \mu_B(x) \}, \min\{ \nu_A(x), \nu_B(x) \} \rangle / x \in X \}$.

1.11 Definition

Let X be a set and $\Im$ be a family of multi intuitionistic fuzzy subsets of X. The family $\Im$ is called a multi intuitionistic fuzzy topology on X if and only if $\Im$ satisfies the following axioms (i) $0_X, 1_X \in \Im$, (ii) If $\{ A_i ; i \in I \} \subseteq \Im$, then $\bigcup_{i\in I} A_i \in \Im$, (iii) If $A_1, A_2, A_3, \ldots, A_n \in \Im$, then $\bigcap_{i=1}^{i=n} A_i \in \Im$. The pair $(X, \Im)$ is called a multi intuitionistic fuzzy topological space. The members of $\Im$ are called multi intuitionistic fuzzy open sets in X. A multi intuitionistic fuzzy set A in X is said to be multi intuitionistic fuzzy closed set in X if and only if A^c is a multi intuitionistic fuzzy open set in X.

1.12 Definition

Let $(X, \Im)$ be a multi intuitionistic fuzzy topological space and A be a multi intuitionistic fuzzy set in X. Then $\cap\{ B: B^c \in \Im$ and $B \supseteq A \}$ is called multi intuitionistic fuzzy closure of A and is denoted by mifcl(A).

1.13 Definition

Let $(X, \Im)$ be a multi intuitionistic fuzzy topological space and A be a multi intuitionistic fuzzy set in X. Then $\cup\{ B : B \in \Im$ and $B \subseteq A \}$ is called multi intuitionistic fuzzy interior of A and is denoted by mifint(A).

1.14 Definition

Let $(X, \Im)$ be a multi intuitionistic fuzzy topological space and A be a multi intuitionistic fuzzy set in X. Then A is said to be

(i) multi intuitionistic fuzzy semi-open if and only if there exists a multi intuitionistic fuzzy open set V in X such that $V \subseteq A \subseteq$ mifcl(V),

(ii) multi intuitionistic fuzzy semi-closed if and only if there exists a multi intuitionistic fuzzy closed set V in X such that mifint(V) $\subseteq A \subseteq V$,

(iii) multi intuitionistic fuzzy regular open set of X if mifint(mifcl(A)) = A,

(iv) multi intuitionistic fuzzy regular closed set of X if mifcl(mifint(A)) = A,

(v) multi intuitionistic fuzzy regular semi-open set of X if there exists a multi intuitionistic fuzzy regular open set V in X such that $V \subseteq A \subseteq$ mifcl(V).
We denote the class of multi intuitionistic fuzzy regular semi-open sets in a multi intuitionistic fuzzy topological space X by MIFRSO(X).
(vi) multi intuitionistic fuzzy generalized closed (mifg-closed) if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy open set and A is a multi intuitionistic fuzzy generalized open set if A^c is multi intuitionistic fuzzy generalized closed,
(vii) multi intuitionistic fuzzy rg-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy regular open set in X,
(viii) multi intuitionistic fuzzy rg-open if its complement A^c is a multi intuitionistic fuzzy rg-closed set in X,
(ix) multi intuitionistic fuzzy w-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy semi open set in X,
(x) multi intuitionistic fuzzy w-open if its complement A^c is a multi intuitionistic fuzzy w-closed set in X,
(xi) multi intuitionistic fuzzy gpr-closed if mifpcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy regular open set in X,
(xii) multi intuitionistic fuzzy gpr-open if its complement A^c is a multi intuitionistic fuzzy gpr-closed set in X.

1.15 Definition

A multi intuitionistic fuzzy set A of a multi intuitionistic fuzzy topological space (X, $\Im$) is called:
(i) multi intuitionistic fuzzy g-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy open set in X,
(ii) multi intuitionistic fuzzy g-open if its complement A^c is a multi intuitionistic fuzzy g-closed set in X,
(iii) multi intuitionistic fuzzy rg-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy regular open set in X,
(iv) multi intuitionistic fuzzy rg-open if its complement A^c is a multi intuitionistic fuzzy rg-closed set in X,
(v) multi intuitionistic fuzzy w-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy semi-open set in X,
(vi) multi intuitionistic fuzzy w-open if its complement A^c is a multi intuitionistic fuzzy w-closed set in X,
(vii) multi intuitionistic fuzzy gpr-closed if mifpcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a multi intuitionistic fuzzy regular open set in X,
(viii) multi intuitionistic fuzzy gpr-open if its complement A^c is a multi intuitionistic fuzzy gpr-closed set in X.

1.16 Definition

A mapping f : X $\to$ Y from a multi intuitionistic fuzzy topological space X to a multi intuitionistic fuzzy topological space Y is called
(i) multi intuitionistic fuzzy continuous if f^{-1}(A) is multi intuitionistic fuzzy open in X for each multi intuitionistic fuzzy open set A in Y,
(ii) multi intuitionistic fuzzy generalized continuous if f^{-1}(A) is multi intuitionistic fuzzy generalized closed in X for each multi intuitionistic fuzzy closed set A in Y,
(iii) multi intuitionistic fuzzy semi continuous if f^{-1}(A) is multi intuitionistic fuzzy semi-open in X for each multi intuitionistic fuzzy open set A in Y,
(iv) multi intuitionistic fuzzy almost continuous if f^{-1}(A) is multi intuitionistic fuzzy open in X for each multi intuitionistic fuzzy regular open set A in Y,
(v) multi intuitionistic fuzzy irresolute if f^{-1}(A) is multi intuitionistic fuzzy semi-open in X for each multi intuitionistic fuzzy semi-open set A in Y,
(vi) multi intuitionistic fuzzy gc-irresolute if f^{-1}(A) is multi intuitionistic fuzzy generalized closed in X for each multi intuitionistic fuzzy generalized closed set A in Y,
(vii) multi intuitionistic fuzzy completely semi continuous if and only if f^{-1}(A) is a multi intuitionistic fuzzy regular semi-open set of X for every multi intuitionistic fuzzy open set A in Y,
(viii) multi intuitionistic fuzzy w-continuous if and only if f^{-1}(A) is a multi intuitionistic fuzzy w-closed set of X for every multi intuitionistic fuzzy closed set A in Y,
(ix) multi intuitionistic fuzzy rg-continuous if f^{-1}(A) is multi intuitionistic fuzzy rg-closed in X for each multi intuitionistic fuzzy closed set A in Y,
(x) multi intuitionistic fuzzy gpr-continuous if f^{-1}(A) is multi intuitionistic fuzzy gpr-closed in X for each multi intuitionistic fuzzy closed set A in Y,
(xi) multi intuitionistic fuzzy almost-irresolute if f^{-1}(A) is multi intuitionistic fuzzy semi open in X for each multi intuitionistic fuzzy regular semi-open set A in Y.

1.17 Definition

Let (X, $\Im$) be a multi intuitionistic fuzzy topological space. A multi intuitionistic fuzzy set A of X is called multi intuitionistic fuzzy regular w-closed (briefly, mifrw-closed) if mifcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is multi intuitionistic fuzzy regular semi-open in a multi intuitionistic fuzzy topological space X.

1.18 NOTE

We denote the family of all multi intuitionistic fuzzy regular w-closed sets in a multi intuitionistic fuzzy topological space X by MIFRWC(X).

1.19 Definition

A multi intuitionistic fuzzy set A of a multi intuitionistic fuzzy topological space X is called a multi intuitionistic fuzzy regular w-open (briefly, mifrw-open) set if its complement A^c is a multi intuitionistic fuzzy rw-closed set in a multi intuitionistic fuzzy topological space X.

1.20 NOTE

We denote the family of all multi intuitionistic fuzzy rw-open sets in a multi intuitionistic fuzzy topological space X by MIFRWO(X).

1.21 Definition

Let X and Y be multi intuitionistic fuzzy topological spaces. A map $f : X \longrightarrow Y$ is said to be multi intuitionistic fuzzy rw-continuous if the inverse image of every multi intuitionistic fuzzy open set in Y is multi intuitionistic fuzzy rw-open in X.

1.22 Definition

Let X and Y be multi intuitionistic fuzzy topological spaces. A map $f : X \longrightarrow Y$ is said to be a multi intuitionistic fuzzy rw-irresolute map if the inverse image of every multi intuitionistic fuzzy rw-open set in Y is a multi intuitionistic fuzzy rw-open set in X.

1.23 Definition

Let $(X, \Im)$ be a multi intuitionistic fuzzy topological space and A be a multi intuitionistic fuzzy set of X. Then multi intuitionistic fuzzy rw-interior and multi intuitionistic fuzzy rw-closure of A are defined as follows.

mifrwcl(A) = $\cap$ { K: K is a multi intuitionistic fuzzy rw-closed set in X and $A \subseteq K$ }.

mifrwint(A) = $\cup$ { G: G is a multi intuitionistic fuzzy rw-open set in X and $G \subseteq A$ }.

1.24 Remark

It is clear that $A \subseteq$ mifrwcl(A) $\subseteq$ mifcl(A) for any multi intuitionistic fuzzy set A.

1.25 Definition

A mapping $f : X \longrightarrow Y$ from a multi intuitionistic fuzzy topological space X to a multi intuitionistic fuzzy topological space Y is called a

(i) multi intuitionistic fuzzy open mapping if f(A) is multi intuitionistic fuzzy open in Y for every multi intuitionistic fuzzy open set A in X,

(ii) multi intuitionistic fuzzy semi-open mapping if f(A) is multi intuitionistic fuzzy semi-open in Y for every multi intuitionistic fuzzy open set A in X.

1.26 Definition

A mapping $f : X \longrightarrow Y$ from a multi intuitionistic fuzzy topological space X to a multi intuitionistic fuzzy topological space Y is called a

(i) multi intuitionistic fuzzy closed mapping if f(A) is multi intuitionistic fuzzy closed in Y for every multi intuitionistic fuzzy closed set A in X,

(ii) multi intuitionistic fuzzy semi-closed mapping if f(A) is multi intuitionistic fuzzy semi-closed in Y for every multi intuitionistic fuzzy closed set A in X.

1.27 Definition

A bijective mapping $f : X \longrightarrow Y$ from a multi intuitionistic fuzzy topological space X to another multi intuitionistic fuzzy topological space Y is called multi intuitionistic fuzzy homeomorphism if f and f^{-1} are multi intuitionistic fuzzy continuous.

1.28 Definition

Let $(X, \Im_1)$ and $(Y, \Im_2)$ be two multi intuitionistic fuzzy topological spaces. A map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is called multi intuitionistic fuzzy rw-open if the image of every multi intuitionistic fuzzy open set in X is multi intuitionistic fuzzy rw-open in Y.

1.29 Definition

Let $(X, \Im_1)$ and $(Y, \Im_2)$ be two multi intuitionistic fuzzy topological spaces. A map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is called multi intuitionistic fuzzy rw-closed if the image of every multi intuitionistic fuzzy closed set in X is a multi intuitionistic fuzzy rw-closed set in Y.

1.30 Definition

Let X and Y be multi intuitionistic fuzzy topological spaces. A bijection map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is called multi intuitionistic fuzzy rw-homeomorphsim if f and f^{-1} are multi intuitionistic fuzzy rw-continuous.

1.31 NOTE

The family of all multi intuitionistic fuzzy rw-homeomorphisms from $(X, \Im)$ onto itself is denoted by MIFRW-H$(X, \Im)$.

1.32 Definition

A bijection map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is called a multi intuitionistic fuzzy rwc-homemorphism if f and f^{-1} are multi intuitionistic fuzzy rw-irresolute. We say that spaces $(X, \Im_1)$ and $(Y, \Im_2)$ are multi intuitionistic fuzzy rwc-homeomorphism if there exsists a multi intuitionistic fuzzy rwc-homeomorphism from $(X, \Im_1)$ onto $(Y, \Im_2)$.

1.33 NOTE

The family of all multi intuitionistic fuzzy rwc-homeomorphisms from $(X, \Im)$ onto itself is denoted by MIFRWC-H$(X, \Im)$.

2. Some Properties

2.1 Theorem

Every multi intuitionistic fuzzy open map is a multi intuitionistic fuzzy rw-open map.

Proof

Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a multi intuitionistic fuzzy open map and A be a multi intuitionistic fuzzy open set in a multi intuitionistic fuzzy topological space X. Then f(A) is a multi intuitionistic fuzzy open set in a multi intuitionistic fuzzy topological space Y. Since every multi intuitionistic fuzzy open set is multi intuitionistic fuzzy rw-open, f(A) is a multi intuitionistic fuzzy rw-open set in a multi intuitionistic fuzzy topological space Y. Hence f is a multi intuitionistic fuzzy rw-open map.

2.2 Remark

The converse of the above theorem 2.1 need not be true in general.

Proof

Consider the example, let X = Y = { 1, 2, 3 } and the multi intuitionistic fuzzy sets A, B, C are defined as A = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) > }, C ={ < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) > }. Consider $\Im_1$ = { 0_X, 1_X, A, B, C } and $\Im_2$ = { 0_Y, 1_Y, A }. Then (X, $\Im_1$) and (Y, $\Im_2$) are multi intuitionistic fuzzy topological spaces. Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) be defined as f(1) = f(2) = 1 and f(3) = 3. Then this function is multi intuitionistic fuzzy rw-open but it is not multi intuitionistic fuzzy open, since the image of the multi intuitionistic fuzzy open set C in X is the multi intuitionistic fuzzy set C in Y which is not multi intuitionistic fuzzy open.

2.3 Theorem

Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) and g : (Y, $\Im_2$) $\longrightarrow$ (Z, $\Im_3$) be two multi intuitionistic fuzzy rw-open maps. Show that g•f need not be multi intuitionistic fuzzy rw-open.

Proof

Consider the following example. let X = Y = Z = { 1, 2, 3 } and the multi intuitionistic fuzzy sets A, B, C, D are defined as A = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) > }, C ={ < 1, (1, 1, 1), (0, 0, 0) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) > }, D = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (1, 1, 1), (0, 0, 0) > }. Consider $\Im_1$ = { 0_X, 1_X, A, D } and $\Im_2$ = { 0_Y, 1_Y, A } and $\Im_3$ = { 0_Z, 1_Z, A, B, C }. Then (X, $\Im_1$), (Y, $\Im_2$) and (Z, $\Im_3$) are multi intuitionistic fuzzy topological spaces. Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) and g : (Y, $\Im_2$) $\longrightarrow$ (Z, $\Im_3$) be the identity maps. Then f and g are multi intuitionistic fuzzy rw-open maps but their composition g•f : (X, $\Im_1$) $\longrightarrow$ (Z, $\Im_3$) is not multi intuitionistic fuzzy rw-open as D is multi intuitionistic fuzzy open in X but (g•f)(D) = D is not multi intuitionistic fuzzy rw-open in Z.

2.4 Theorem

If f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) is a multi intuitionistic fuzzy open map and g : (Y, $\Im_2$) $\longrightarrow$ (Z, $\Im_3$) is a multi intuitionistic fuzzy rw-open map, then their composition g•f : (X, $\Im_1$) $\longrightarrow$ (Z, $\Im_3$) is a multi intuitionistic fuzzy rw-open map.

Proof

Let A be a multi intuitionistic fuzzy open set in (X, $\Im_1$). Since f is a multi intuitionistic fuzzy open map, f(A) is a multi intuitionistic fuzzy open set in (Y, $\Im_2$). Since g is a multi intuitionistic fuzzy rw-open map, g(f(A)) is a multi intuitionistic fuzzy rw-open set in (Z, $\Im_3$). But g(f(A)) = (g•f)(A). Thus g•f is a multi intuitionistic fuzzy rw-open map.

2.5 Remark

Every multi intuitionistic fuzzy w-open map is multi intuitionistic fuzzy rw-open but converse may not be true.

Proof

Consider the example. let X = { a, b }, Y = { x, y } and multi intuitionistic fuzzy sets A and B are defined as follows A = { < a, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < b, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) > }, B = { < x, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < y, (0.6, 0.6, 0.6), (0.3, 0.3, 0.3) >}. Then $\Im$ = { 0_X, 1_X, A} and σ = {0_Y, 1_Y, B**}** be multi intuitionistic fuzzy topologies on X and Y respectively. Then, the mapping f: (X, $\Im$) $\rightarrow$ (Y, σ) defined by f*(a)* = x and f*(b)*= y is multi intuitionistic fuzzy rw-open but it is not multi intuitionistic fuzzy w-open.

2.6 Theorem

A mapping f : (X, $\Im$)$\rightarrow$(Y, σ) is multi intuitionistic fuzzy rw-open if and only if for every multi intuitionistic fuzzy set A of X, f(mifint(A)) $\subseteq$ *mif*rwint(f(A)).

Proof

Let f be a multi intuitionistic fuzzy rw-open mapping and A be a multi intuitionistic fuzzy open set in X. Now mifint(A) $\subseteq$ A which implies that f(mifint(A)) $\subseteq$ f(A). Since f is a multi intuitionistic fuzzy rw-open mapping, f(mifint(A)) is a multi intuitionistic fuzzy rw-open set in Y such that f(mifint(A)) $\subseteq$ *f(*A*).* Therefore *f(mifint(*A*))*$\subseteq$ mifrwint *f(*A*).* For the converse, suppose that A is a multi intuitionistic fuzzy open set of X. Then *f*(A) = f(mifint(A)) $\subseteq$ mifrwint(*f(*A*)).* But mifwint(f(A)) $\subseteq$ *f*(A). Consequently f(A) = mifwint(A) which implies that *f(*A*)* is a multi intuitionistic fuzzy rw-open set of Y and hence *f* is a multi intuitionistic fuzzy rw-open map.

2.7 Theorem

If f : (X, $\Im$)$\rightarrow$(Y, σ) is a multi intuitionistic fuzzy rw-open map, then mifint(f^{-1}(A)) $\subseteq$ f^{-1}(mifrwint(A)) for every multi intuitionistic fuzzy set A of Y.

Proof

Let A be a multi intuitionistic fuzzy set of Y. Then mifint (f^{-1}(A)) is a multi intuitionistic fuzzy open set in X. Since *f* is multi intuitionistic fuzzy rw-open f(mifint (f^{-1}(A))) is multi intuitionistic fuzzy rw-open in Y and hence f(mifint (f^{-1}(A))) $\subseteq$ mifrwint(f(f^{-1}(A))) $\subseteq$ mifrwint (f(f^{-1}(A))) $\subseteq$ mifrwint(A). Thus mifint f^{-1}(A) $\subseteq$ f^{-1}(mifrwint(A)).

2.8 Theorem

A mapping *f* : (X, $\Im$) $\rightarrow$ (Y, σ) is multi intuitionistic fuzzy rw-open if and only if for each multi intuitionistic fuzzy set A of Y and for each multi intuitionistic fuzzy closed set U of X containing f^{-1}(A) there is a multi intuitionistic fuzzy rw-closed set V of Y such that A $\subseteq$ V and f^{-1}(V) $\subseteq$ U.

Proof

Suppose that f is a multi intuitionistic fuzzy rw-open map. Let A be the multi intuitionistic fuzzy closed set of Y and U be a multi intuitionistic fuzzy closed set of X such that $f^{-1}(A) \subseteq U$. Then $V = (f^{-1}(U^c))^c$ is a multi intuitionistic fuzzy rw-closed set of Y such that $f^{-1}(V) \subseteq U$. For the converse, suppose that B is a multi intuitionistic fuzzy open set of X. Then $f^{-1}((f(B))^c \subseteq B^c$ and B^c is a multi intuitionistic fuzzy closed set in X. By hypothesis there is a multi intuitionistic fuzzy rw-closed set V of Y such that $(f(B))^c \subseteq V$ and $f^{-1}(V) \subseteq B^c$. Therefore $B \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(B) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(B) = V^c$. Since V^c is a multi intuitionistic fuzzy rw-open set of Y. Hence *f*(B) is multi intuitionistic fuzzy rw-open in Y and thus *f* is a multi intuitionistic fuzzy rw-open map.

2.9 Theorem

Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a multi intuitionistic fuzzy closed map. Then f is a multi intuitionistic fuzzy rw-closed map.

Proof

Let A be a multi intuitionistic fuzzy open set in $(X, \Im_1)$. Since f is a multi intuitionistic fuzzy closed map, f(A) is a multi intuitionistic fuzzy closed set in $(Y, \Im_2)$. Since every multi intuitionistic fuzzy closed set is multi intuitionistic fuzzy rw-closed, f(A) is a multi intuitionistic fuzzy rw-closed set in $(Y, \Im_2)$. Hence f is a multi intuitionistic fuzzy rw-closed map.

2.10 Remark

The converse of the above theorem 2.9 need not be true in general.

Proof

Consider the example. let X = [0, 1] and Y = [0, 1] and the multi intuitionistic fuzzy sets A of X and B of Y are defined as $\mu_A(x) = (0.5, 0.5, 0.5)$ if x = 2/3 and $\mu_A(x) = (1, 1, 1)$ if otherwise; $\gamma_A(x) = (0.5, 0.5, 0.5)$ if x = 2/3 and $\gamma_A(x) = (0, 0, 0)$ if otherwise; $\mu_B(x) = (0.7, 0.7, 0.7)$ if x = 2/3 and $\mu_B(x) = (1, 1, 1)$ if otherwise; $\gamma_B(x) = (0.3, 0.3, 0.3)$ if x=2/3 and $\gamma_B(x) = (0, 0, 0)$ if otherwise. Consider $\Im_1 = \{0_X, 1_X, A\}$ and $\Im_2 = \{0_Y, 1_Y, B\}$. Then $(X, \Im_1)$ and $(Y, \Im_2)$ are multi intuitionistic fuzzy topological spaces. Let $f: (X, \Im_1) \longrightarrow (Y, \Im_2)$ be the identity map. Then f is a multi intuitionistic fuzzy rw-closed map but it is not a multi intuitionistic fuzzy closed map, since the image of the multi intuitionistic fuzzy closed set A^c in X is not a multi intuitionistic fuzzy closed set in Y.

2.11 Theorem

The composition of two multi intuitionistic fuzzy rw-closed maps need not be a multi intuitionistic fuzzy rw-closed map.

Proof

Consider the multi intuitionistic fuzzy topological spaces $(X, \Im_1)$, $(Y, \Im_2)$ and $Z, \Im_3)$ and mappings defined in previous example. The maps f and g are multi intuitionistic fuzzy rw-closed but their composition is not multi intuitionistic fuzzy rw-closed, as A is a multi intuitionistic fuzzy closed set in X but $(g\bullet f)(A) = A$ is not multi intuitionistic fuzzy rw-closed in Z.

2.12 Theorem

If $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ and $g : (Y, \Im_2) \longrightarrow (Z, \Im_3)$ be two maps, then $g\bullet f : (X, \Im_1) \longrightarrow (Z, \Im_3)$ is multi intuitionistic fuzzy rw-closed map if f is multi intuitionistic fuzzy closed and g is multi intuitionistic fuzzy rw-closed.

Proof

Let A be a multi intuitionistic fuzzy closed set in $(X, \Im_1)$. Since f is a multi intuitionistic fuzzy closed map, f(A) is a multi intuitionistic fuzzy closed set in $(Y, \Im_2)$. Since g is a multi intuitionistic fuzzy rw-closed map, g(f(A)) is a multi intuitionistic fuzzy rw-closed set in $(Z, \Im_3)$. But $g(f(A)) = (g\bullet f)(A)$. Thus $g\bullet f$ is multi intuitionistic fuzzy rw-closed map.

2.13 Theorem

A map $f: X \longrightarrow Y$ is multi intuitionistic fuzzy rw-closed if for each multi intuitionistic fuzzy set D of Y and for each multi intuitionistic fuzzy open set E of X such that $E \supseteq f^{-1}(D)$, there is a multi intuitionistic fuzzy rw-open set A of Y such that $D \subseteq A$ and $f^{-1}(A) \subseteq E$.

Proof

Suppose that f is multi intuitionistic fuzzy rw-closed. Let D be a multi intuitionistic fuzzy subset of Y and E is a multi intuitionistic fuzzy open set of X such that $f^{-1}(D) \subseteq E$. Let $A = [\ f(E^c)\ ]^c$ is multi intuitionistic fuzzy rw-open set in a multi intuitionistic fuzzy topological space Y. Note that $f^{-1}(D) \subseteq E$ which implies $D \subseteq A$ and $f^{-1}(A) \subseteq E$. For the converse, suppose that E is a multi intuitionistic fuzzy closed set in X. Then $f^{-1}(f(E)^c) \subseteq E^c$ and E^c is multi intuitionistic fuzzy open. By hypothesis, there is a multi intuitionistic fuzzy rw-open set A of Y such that $[f(E)]^c \subseteq A$ and $f^{-1}(A) \subseteq E^c$. Therefore $E \subseteq [f^{-1}(A)]^c$. Hence $A^c \subseteq f(E) \subseteq f([f^{-1}(A)]^c) \subseteq A^c$ which implies $f(E) = A^c$. Since A^c is multi intuitionistic fuzzy rw-closed, f(E) is multi intuitionistic fuzzy rw-closed and thus f is multi intuitionistic fuzzy rw-closed.

2.14 Theorem

Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be multi intuitionistic fuzzy irresolute and A be multi intuitionistic fuzzy regular semi-open in Y. Then $f^{-1}(A)$ is multi intuitionistic fuzzy regular semi-open in X.

Proof

Let A be multi intuitionistic fuzzy regular semi-open in Y. To prove $f^{-1}(A)$ is multi intuitionistic fuzzy regular semi-open in X. That is to prove $f^{-1}(A)$ is both multi intuitionistic fuzzy semi-open and multi intuitionistic fuzzy semi-closed in X. Now A is multi intuitionistic fuzzy semi-open in Y. Since f is multi intuitionistic fuzzy irresolute, $f^{-1}(A)$ is multi intuitionistic fuzzy semi-open in X. Now A is multi intuitionistic fuzzy semi-closed in Y, as multi intuitionistic fuzzy regular semi-open set is multi intuitionistic fuzzy semi-closed. Then A^c is multi intuitionistic fuzzy semi-open in Y. Since f is multi intuitionistic fuzzy irresolute, $f^{-1}(A^c)$ is multi intuitionistic fuzzy semi-open in X. But $f^{-1}(A^c) = [f^{-1}(A)\ ]^c$ is multi intuitionistic fuzzy semi-open in X and so $f^{-1}(A)$ is multi intuitionistic fuzzy semi-closed in X. Thus $f^{-1}(A)$ is both multi intuitionistic fuzzy semi-open and multi intuitionistic fuzzy semi-closed in X and hence $f^{-1}(A)$ is multi intuitionistic fuzzy regular semi-open in X.

2.15 Theorem

If a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is a multi intuitionistic fuzzy irresolute and multi intuitionistic fuzzy rw-closed map and A is a multi intuitionistic fuzzy rw-closed set of X, then f(A) is a multi intuitionistic fuzzy rw-closed set in Y.

Proof

Let A be a multi intuitionistic fuzzy closed set of X. Let $f(A) \subseteq E$, where E is multi intuitionistic fuzzy regular semi-open in Y. Since f is multi intuitionistic fuzzy irresolute, $f^{-1}(E)$ is a multi intuitionistic fuzzy regular semi-open in X, by Theorem 2.14 and $A \subseteq f^{-1}(E)$. Since A is a multi intuitionistic fuzzy rw-closed set in X, $mifcl(A) \subseteq f^{-1}(E)$. Since f is multi intuitionistic fuzzy rw-closed, f(mifcl(A)) is a multi intuitionistic fuzzy rw-closed set contained in the multi intuitionistic fuzzy regular semi-open set E, which implies $mifcl(f(mifcl(A))) \subseteq E$ and hence $mifcl(f(A)) \subseteq E$. Therefore f(A) is a multi intuitionistic fuzzy rw-closed set in Y.

2.16 Theorem

If a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is multi intuitionistic fuzzy irresolute and multi intuitionistic fuzzy closed and A is a multi intuitionistic fuzzy rw-closed set in a multi intuitionistic fuzzy topological space X, then f(A) is a multi intuitionistic fuzzy rw-clsoed set in a multi intuitionistic fuzzy topological space Y.

Proof

The proof follows from the theorem 2.15 and the fact that every multi intuitionistic fuzzy closed map is a multi intuitionistic fuzzy rw-closed map.

2.17 Theorem

Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be two mappings such that $g \bullet f : X \longrightarrow Z$ is a multi intuitionistic fuzzy rw-closed map.Then

(i) if f is multi intuitionistic fuzzy continuous and surjective, then g is multi intuitionistic fuzzy rw-closed (ii) if g is multi intuitionistic fuzzy rw-irresolute and injective, then f is multi intuitionistic fuzzy rw-closed.

Proof

(i) Let E be a multi intuitionistic fuzzy closed set in Y. Since f is multi intuitionistic fuzzy continuous, $f^{-1}(E)$ is a multi intuitionistic fuzzy closed set in X. Since $g \bullet f$ is a multi intuitionistic fuzzy rw-closed map, $(g \bullet f)(f^{-1}(E))$ is a multi intuitionistic fuzzy rw-closed set in Z. But $(g \bullet f)(f^{-1}(E)) = g(E)$, as f is surjective. Thus g is multi intuitionistic fuzzy rw-closed.

(ii) Let B be a multi intuitionistic fuzzy closed set of X. Then $(g \bullet f)(B)$ is a multi intuitionistic fuzzy rw-closed set in Z, since $g \bullet f$ is a multi intuitionistic fuzzy rw-closed map. Since g is multi intuitionistic fuzzy rw-irresolute, $g^{-1}((g \bullet f)(B))$ is multi intuitionistic fuzzy rw-closed in Y. But $g^{-1}((g \bullet f)(B)) = f(B)$, as g is injective. Thus f is a multi intuitionistic fuzzy rw-closed map.

2.18 Theorem

If $f : (X, \Im) \rightarrow (Y, \sigma)$ is a multi intuitionistic fuzzy almost irresolute and multi intuitionistic fuzzy rw-closed map and A is a multi intuitionistic fuzzy w-closed set of X, then f(A) is multi intuitionistic fuzzy rw-closed.

Proof

Let $f(A) \subseteq O$ where O is a multi intuitionistic fuzzy regular semi-open set of Y. Since *f* is multi intuitionistic fuzzy almost irresolute therefore $f^{-1}(O)$ is a multi intuitionistic fuzzy semi-open set of X such that $A \subseteq f^{-1}(O)$. Since A is multi intuitionistic fuzzy w-closed set of X which implies that $mifcl(A) \subseteq f^{-1}(O)$ and hence $f(mifcl(A)) \subseteq O$ which implies that $mifcl(f(mifcl(A))) \subseteq O$ therefore $mifcl(f(A)) \subseteq O$ whenever $f(A) \subseteq O$ where O is a multi intuitionistic fuzzy regular semi-open set of Y. Hence *f(A)* is a multi intuitionistic fuzzy rw-closed set of Y.

2.19 Theorem

Every multi intuitionistic fuzzy homeomorphism is a multi intuitionistic fuzzy rw- homeomorphism.

Proof

Let a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a multi intuitionistic fuzzy homeomorphism. Then f and f^{-1} are multi intuitionistic fuzzy continuous. Since every multi intuitionistic fuzzy continuous map is multi intuitionistic fuzzy rw-continuous, f and f^{-1} are multi intuitionistic fuzzy rw-continuous. Therefore f is a multi intuitionistic fuzzy rw-homeomorphism.

2.20 Remark: The converse of the above theorem need not be true.

Proof: Consider the example, let X = Y = { 1, 2, 3 } and the multi intuitionistic fuzzy sets A, B, C are defined as A = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) > }, C ={ < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) > }. Consider $\Im_1 = \{ 0_X, 1_X, A, C \}$ and $\Im_2 = \{ 0_Y, 1_Y, B \}$. Then $(X, \Im_1)$ and $(Y, \Im_2)$ are multi intuitionistic fuzzy topological spaces. Define a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ by f(1) = 1, f(2) = 3 and f(3) = 2. Here the function f is a multi intuitionistic fuzzy rw-homeomorphism but it is not a multi intuitionistic fuzzy homeomorphism, as the image of a multi intuitionistic fuzzy open set A in $(X, \Im_1)$ is A which is not a multi intuitionistic fuzzy open set in $(Y, \Im_2)$.

2.21 Theorem

Let X and Y be multi intuitionistic fuzzy topological spaces and $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a bijective map. Then the following statements are equivalent.

(a) f^{-1} is a multi intuitionistic fuzzy rw-continuous map

(b) f is a multi intuitionistic fuzzy rw-open map

(c) f is a multi intuitionistic fuzzy rw-closed map.

Proo

(a) $\Longrightarrow$(b). Let A be any multi intuitionistic fuzzy open set in X. Since f^{-1} is multi intuitionistic fuzzy rw-continuous, $(f^{-1})^{-1}(A) = f(A)$ is multi intuitionistic fuzzy rw-open in Y. Hence f is a multi intuitionistic fuzzy rw-open map.

(b) $\Longrightarrow$(c). Let A be any multi intuitionistic fuzzy closed set in X. Then A^c is multi intuitionistic fuzzy rw-open in X. Since f is a multi intuitionistic fuzzy rw-open map, $f(A^c)$ is multi intuitionistic fuzzy rw-open in Y. But $f(A^c) = [f(A)]^c$, as f is a bijection map. Hence f(A) is multi intuitionistic fuzzy rw-closed in Y. Therefore f is multi intuitionistic fuzzy rw-closed.

(c)$\Longrightarrow$(a). Let A be any multi intuitionistic fuzzy closed set in X. Then f(A) is a multi intuitionistic fuzzy rw-closed set in Y. But $(f^{-1})^{-1}(A) = f(A)$. Therefore f^{-1} is a multi intuitionistic fuzzy rw-continuous map.

2.22 Theorem

Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be a bijection and multi intuitionistic fuzzy rw-continuous map. Then the following statements are equivalent.

(a) f is a multi intuitionistic fuzzy rw-open map

(b) f is a multi intuitionistic fuzzy rw-homeomorphism

(c) f is a multi intuitionistic fuzzy rw-closed map.

Proof

(a) $\Longrightarrow$(b) By hypothesis and assumption f is a multi intuitionistic fuzzy rw-homeomorphism.

(b) $\Longrightarrow$(c) Since f is a multi intuitionistic fuzzy rw-homeomorphism; it is multi intuitionistic fuzzy rw-open. So by the above theorem 5.2.17, it is a multi intuitionistic fuzzy rw-closed map.

(c) $\Longrightarrow$(a) Let B be a multi intuitionistic fuzzy open set in X, so that B^c is a multi intuitionistic fuzzy closed set and f being multi intuitionistic fuzzy rw-clsoed, $f(B^c)$ is multi intuitionistic fuzzy rw-closed in Y. But $f(B^c) = [f(B)]^c$ thus f(B) is multi intuitionistic fuzzy rw-open in Y. Therefore f is a multi intuitionistic fuzzy rw-open map.

2.23 Theorem

Every multi intuitionistic fuzzy rwc-homeomorphism is multi intuitionistic fuzzy rwc-homeomorphism but not conversely.

Proof: The proof follows from the fact that every multi intuitionistic fuzzy rw-irresolute map is multi intuitionistic fuzzy rw-continuous but not conversely.

2.24 Theorem

Let $(X, \Im_1)$, $(Y, \Im_2)$ and $(Z, \Im_3)$ be multi intuitionistic fuzzy topological spaces and $f : (X, \Im_1)\longrightarrow (Y, \Im_2)$, $g : (Y, \Im_2) \longrightarrow (Z, \Im_3)$ be multi intuitionistic fuzzy rwc-homeomorphisms. Then their composition $g\bullet f : (X, \Im_1) \longrightarrow (Z, \Im_3)$ is a multi intuitionistic fuzzy rwc-homeomorphism.

Proof

Let A be a multi intuitionistic fuzzy rw-open set in $(Z, \Im_3)$. Since g is a multi intuitionistic fuzzy rw-irrresolute, $g^{-1}(A)$ is a multi intuitionistic fuzzy rw-open set in $(Y, \Im_2)$. Since f is multi intuitionistic fuzzy rw-irresolute, $f^{-1}(g^{-1}(A))$ is a multi intuitionistic fuzzy rw-open set in $(X, \Im_1)$. But $f^{-1}(g^{-1}(A)) = (g\bullet f)^{-1}(A)$. Therefore $g\bullet f$ is multi intuitionistic fuzzy rw-irresolute. To prove that $(g\bullet f)^{-1}$ is multi intuitionistic fuzzy rw-irresolute. Let B be a multi intuitionistic fuzzy rw-open set in $(X, \Im_1)$. Since f^{-1} is multi intuitionistic fuzzy rw-irresolute, $(f^{-1})^{-1}(B)$ is a multi intuitionistic fuzzy rw-open set in $(Y, \Im_2)$. Also $(f^{-1})^{-1}(B) = f(B)$. Since g^{-1} is multi intuitionistic fuzzy rw-irresolute, $((g^{-1})^{-1})(f(B))$ is a multi intuitionistic fuzzy rwopen set in $(Z, \Im_3)$.That is $((g^{-1})^{-1})(f(B)) = g(f(B)) = (g\bullet f)(B) = ((g\bullet f)^{-1})^{-1}(B)$. Therefore $(g\bullet f)^{-1}$ is multi intuitionistic fuzzy rw-irresolute. Thus $g\bullet f$ and $(g\bullet f)^{-1}$ are multi intuitionistic fuzzy rw-irresolute. Hence $g\bullet f$ is a multi intuitionistic fuzzy rwc-homoeomorphism.

2.25 Theorem

The set MIFRWC-H$(X, \Im)$ is a group under the composition of maps.

Proof

Define a binary operation*: MIFRWC-H$(X, \Im)\times$MIFRWC-H$(X, \Im) \longrightarrow$ MIFRWC-H$(X, \Im)$ by $f*g = g\bullet f$, for all f, $g\in$MIFRWC-H$(X, \Im)$ and $\bullet$ is the usual operation of composition of maps. Then by theorem 2.24, $g\bullet f\in$MIFRWC-H$(X, \Im)$. We know that the composition of maps is associative and the identity map I: $(X, \Im)\longrightarrow (X, \Im)$ belonging to MIFRWC-H$(X, \Im)$ serves as the identity element. If $f\in$MIFRWC-H$(X, \Im)$, then $f^{-1}\in$MIFRWC-H$(X, \Im)$ such that $f\bullet f^{-1} = f^{-1}\bullet f = I$ and so inverse exists for each element of MIFRWC-H$(X, \Im)$. Therefore (MIFRWC-H$(X, \Im)$, $\bullet$) is a group under the operation of composition of maps.

2.26 Theorem

Let $f : (X, \Im_1)\longrightarrow (Y, \Im_2)$ be a multi intuitionistic fuzzy rwc-homeomorphism. Then f induces an isomorphism from the group MIFRWC-H$(X, \Im_1)$ onto the group MIFRWC-H$(Y, \Im_2)$.

Proof

Using the map f, we define a map Ψ_f : MIFRWC-H$(X, \Im_1)\longrightarrow$MIFRWC-H$(Y, \Im_2)$ by $\Psi_f(h) = f\bullet h\bullet f^{-1}$, for every $h\in$MIFRWC-H$(X, \Im_1)$. Then Ψ_f is a bijection. Further, for all $h_1, h_2\in$MIFRWC-H$(X, \Im_1)$, $\Psi_f(h_1\bullet h_2) = f\bullet(h_1\bullet h_2)\bullet f^{-1} = (f\bullet h_1\bullet f^{-1})\bullet(f\bullet h_2\bullet f^{-1}) = \Psi_f(h_1) \bullet\Psi_f(h_2)$. Therefore Ψ_f is a homeomorphism and so it is an isomorphism induced by f.

Conclusion

In this paper, we have introduced the concepts of multi intuitionistic fuzzy rw-open maps and multi intuitionistic fuzzy rw-closed maps in the multi intuitionistic fuzzy topological spaces. We have proved that the composition of two multi intuitionistic fuzzy rw-open maps need not be multi intuitionistic fuzzy rw-open map and the composition of two multi intuitionistic fuzzy rw-closed maps need not be multi intuitionistic fuzzy rw-closed map and have studied some of their properties. We can extend this concept into the bipolar valued fuzzy and bipolar valued intuitionistic fuzzy topological spaces.

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