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Distance Closed Eccentric Domination Number of a Graph

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ABSTRACT

A subset D of the vertex set V(G) of a graph G is said to be a dominating set if every vertex not in D is adjacent to atleast one vertex in D. A dominating set D is said to be an eccentric dominating set if for every $v \in V–D$, there exists atleast one eccentric point of v in D. A subset D is a distance closed set of G if for each vertex $u \in D$ and for each $w \in V–D$, there exists atleast one vertex $v \in D$ such that $d_{<D>}(u, v) = d_G(u, w)$. An eccentric dominating set D of G is a distance closed eccentric dominating set if the induced subgraph <D> is distance closed. The minimum of the cardinalities of the distance closed eccentric dominating set is called as the distance closed eccentric domination number $\gamma_{dced}(G)$ of G. In this paper, bounds for $\gamma_{dced}(G)$, its exact value for some particular classes of graphs and some results on distance closed eccentric domination number are obtained.

1. Introduction

Let G be a finite, simple undirected graph on p vertices and q edges with vertex set V(G) and edge set E(G). For graph theoretic terminology refer Harary [3], Buckley and Harary [1].

The concept of distance in graphs plays a dominant role in the study of structural properties of a graph in various angles using related concept of eccentricity of vertices in graphs. The study of structural properties of graphs using distance and eccentricity started with the study of centre of tree and propagated in different directions in the study of structural properties of graph such as unique eccentric point graphs, K-eccentric point graphs, self centered graphs.

Let G be a connected graph and u be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus $e(v) = \max\{d(u, v); u \in V\}$. The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$, v is a central vertex if $e(v) = r(G)$. The center C(G) is the set of all central vertices. The central subgraph <C(G)> of a graph G is the subgraph induced by the center. v is a peripheral vertex if $e(v) = d(G)$. The periphery P(G) is the set of all peripheral vertices.

For a vertex v, each vertex at a distance e(v) from v is an eccentric vertex of v. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G) / d(u, v) = e(v)\}$. The open neighbourhood N(u) of a vertex u is the set of all vertices adjacent to v in V. $N[u] = N(u) \cup \{u\}$ is called the closed neighbourhood of v. For a vertex $v \in V(G)$, $N_i(u) = \{u \in V(G); d(u, v) = i\}$ is defined to be the i^{th} neighbourhood of v in G.

The concept of domination in graphs was introduced by Ore [8] and Cockayne et al. studied various bounds and results related to domination in [3].

A set $D \subseteq V$ is said to be a dominating set in G, if every vertex in V-D is adjacent to some vertex in D.

In 2010, T.N. Janakiraman, M. Bhanumathi and S. Muthamai defined eccentric domination in graphs [5] and studied eccentric domination in trees [1] and various bounds of eccentric domination in graphs [9]. Janakiraman, Alephonse and Sangeetha introduced the concept of distance closed domination in graphs [6], which mixes the concept of dominating set and distance preserving set. Motivated by these, we have defined the distance closed eccentric domination number of a graph, and studied its bounds.

A set $D \subseteq V(G)$ is an eccentric domianting set if D is a dominating set of G and for every $v \in V–D$, there exists atleast one eccentric point of v in D. An eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D' \subseteq D$ is an eccentric dominating set.

The eccentric domination number $\gamma_{ed}(G)$ of a graph G is the minimum cardinality of a eccentric dominating set.

A subset $S \subseteq V(G)$ is said to be a distance closed dominating set (D.C.D) set, if (i) <S> is distance closed (ii) S is a dominating set. The cardinality of a minimum D.C.D set of G is called the distance closed domination number of G and is denoted by $\gamma_{dcl}(G)$.

In this paper, we introduce a new dominating set called distance closed eccentric dominating set of a graph and we find upper and lower bounds for the new domination number in terms of various already known parameters. Also, we studied several interesting properties like Nordhaus-Gaddum type results relating the graphs and its complement.

2. Prior Results

The distance closed set of a graph G is defined as [6].

Let S be a vertex subset of G. Then S is said to be distance closed set of G if for each vertex $u \in S$ and for each

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$w \in V-S$, there exists atleast one vertex $v \in S$ such that $d_{<S>}(u, v) = d_G(u, w)$.

Theorem 2.1. [7]

A vertex subset D of G is a distance closed set if and only if

(i) $e(u/<D>) \geq e(u/G)$ for $u \in D$;

(ii) Every non eccentric point of $<D>$ is a cut vertex.

Theorem 2.2. [7]

A graph G, which is not an odd path is distance closed if and only if

(i) G is a unique eccentric point graph and

(ii) Every vertex with eccentricity atmost $d-1$ is a cut vertex, where d is the diameter of G.

A graph with $\gamma_{dcl} = p$ is called a 0-distance closed dominating graph.

A ciliate $C_{p,q}$ is a graph obtained from p disjoint copies of the path P_{q-1} by linking together one end point of each in a cycle C_p. All these ciliates are the only graphs that are radius critical (graphs in which removal of every vertex changes the radius of the given graphs).

Theorem 2.3. [6]

A graph G is 0-distance closed graph if and only if G is one of the following

(i) G is P_{2n+1}

(ii) G is a ciliate.

Theorem 2.4. [6]

If T is a tree with number of vertices $p \geq 2$, then $\gamma_{dcl}(T) = p-k+2$, where k is the number of pendant vertices in T.

Theorem 2.5. [6]

For any connected graph G such that $\overline{G}$ is also connected, $\gamma_{dcl}(G) + \gamma_{dcl}(\overline{G}) \leq p+4$, where $\gamma_{dcl}(G)$ and $\gamma_{dcl}(\overline{G})$ are the cardinality of minimum distance closed dominating set of G and $\overline{G}$ respectively.

3. Main Results

In this paper, we define a new domination parameter, distance closed eccentric domination as follows.

3.1 Distance closed eccentric dominating sets in graphs

Definition 3.1.

A subset $S \subseteq V(G)$ is said to be a distance closed eccentric dominating (D.C.E.D) set if

(i) $<S>$ is distance closed;

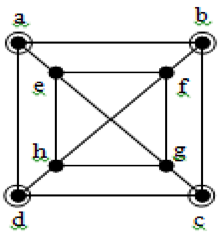
(ii) S is an eccentric dominating set.

The cardinality of a minimum D.C.E.D set of G is called the distance closed domination number of G and is denoted by $\gamma_{dced}(G)$.

Clearly, from the definition, $1 \leq \gamma_{dced} \leq p$ and graph with $\gamma_{dced} = p$ is called a 0-distance closed eccentric dominating graph. Also if S is a D.C.E.D set of G then the complement $V-S$ need not be a D.C.E.D set of G.

For any graph G, $\gamma_{ed}(G) \leq \gamma_{dced}(G)$.

Example 3.1.



Here,

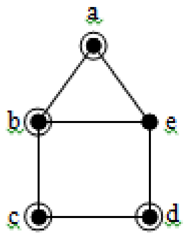
D = {a, b, c, d} is a minimum distance closed eccentric dominating set.

D′ = {a, b, c, d} is a minimum distance closed dominating set.

D′′ = {a, b, c, d} is a minimum eccentric dominating set.

Therefore, $\gamma_{dced}(G) = 4$, $\gamma_{ed}(G) = 4$, $\gamma_{ed}(G) = \gamma_{dced}(G)$.

Example 3.2.



Here,

D = {a, b, c, d} is a minimum distance closed eccentric dominating set.

D′ = {a, b, c, d} is a minimum distance closed dominating set.

D′′ = {a, b, c} is a minimum eccentric dominating set.

$\gamma_{dced}(G) = 4$, $\gamma_{dcl}(G) = 4$, $\gamma_{ed}(G) = 3$.

Therefore, $\gamma_{ed}(G) < \gamma_{cded}(G)$.

Obviously $\gamma(G) \leq \gamma_{ed}(G) \leq \gamma_{dced}(G)$.

Theorem 3.1.

(i) $\gamma_{dced}(K_n) = 2$, for $n \geq 3$

(ii) $\gamma_{dced}(K_{1,n}) = 3$, for $n \geq 3$

(iii) $\gamma_{dced}(K_{m,n}) = 4$, for $m, n \geq 3$

(iv) $\gamma_{dced}(W_n) = 4$, for $n \geq 3$

Proof.

(i) When $G = K_n$, radius = diameter = 1. Let D = {u, v}. Here u or v dominate other vertices and is also an eccentric point of other vertices. Since $e(u) = e(v) = 1$. Clearly D will form the distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = 2$.

(ii) When $G = K_{1,n}$. Let D = {u, v, w}. Here, let u be a central vertex of degree n in G. Clearly $r(G) = 1$ and diameter of G is 2 and also u dominates all vertices in $V-D$ and every point of $V-D$ has an eccentric point in D. Here any two non adjacent vertices, v, w of eccentricity 2 together with u will form a distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = 3$.

(iii) When $G = K_{m,n}$, for $m, n \geq 2$. $V(G) = V_1 \cup V_2$. $|V_1| = m$ and $|V_2| = n$ such that each element of V_1 is adjacent to every vertex of V_2 and vice versa.

Here G is a graph with radius 2 and diameter 2, and every vertices of G has eccentricity equal to 2.

Let D = {u, v, x, y}, $u, x \in V_1$ and $v, y \in V_2$, u dominate all the vertices of V_2 and it is eccentric to elements of $V_1 - \{u\}$. Similarly v dominate all the vertices of V_1 and it is eccentric to elements of $V_2 - \{v\}$.

Hence D is a minimum distance closed eccentric dominating set and hence $\gamma_{dced}(G) = 4$.

(iv) When $G = W_n$. Let D = {u, v, x, w} where u and v are any three adjacent non central vertices and w is the central vertex. Clearly $r(G) = 1$ and diameter of G is 2. Hence D is the distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = 4$.

Next, we obtain some bounds for the dced-number of graphs.

Theorem 3.2.

Let n be an even integer. Let G be obtained from the complete graph K_n by deleting edges of a linear factor then $\gamma_{dced}(G) = n$.

Proof.

Let D be a γ_{dced}-set of G. We know that $\gamma_{dced}(G) \leq n$. If $\gamma_{dced}(G) \neq n$, then $\gamma_{dced}(G) < n$.

This implies that $\exists$ u $\in$ D such that e(u / <D>) < e(u/G). Therefore D is not a distance closed set which is a contradiction to our assumption. Hence $\gamma_{dced}(G) = n$.

Theorem 3.3.

If T is a tree with number of vertices $p \geq 2$ then $\gamma_{dced}(T) = p-k+2$, where k is the number of pendant vertices in T.

Proof.

We have by Theorem 2.4, $\gamma_{dcl}(T) = p-k+2$. Therefore

$$p-k+2 \leq \gamma_{dced}(T) \quad (1)$$

Also, the set of all non-pendent vertices with any two pendent vertices at distance d to each other forms a d.c.e.d set. Therefore

$$\gamma_{dced}(T) \leq p-k+2 \quad (2)$$

From equations (1) and (2)

$\gamma_{dced}(T) = p-k+2$

Theorem 3.4.

If G is a wounded spider with atleast two non-wounded legs then $\gamma_{dced}(G) = s + 2$ where s is the number of support vertices which are adjacent to non wounded legs.

Proof.

Let G be a wounded spider. Let u be the vertex of maximum degree $\Delta(G)$, and S be the set of support vertices which are adjacent to non wounded legs. The set $S \cup \{u\} \cup \{v\}$ where e(v) = 4 is a minimum distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = s + 2$ where s = |S|.

Theorem 3.5.

If G is a 2-self centered graph with a dominating edge which is not in a triangle then $\gamma_{dced}(G) = 4$.

Proof.

Let uv $\in$ E(G) be a dominating edge of G which is not in a triangle and let wu and vx $\in$ E(G) then D = {u, v, w, x} is a minimum distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = 4$.

Theorem 3.6.

Let G be a 2-self centered graph then $\gamma_{dced}(G) \leq 2\delta(G) + 1$.

Proof.

Let v $\in$ V(G) such that deg v = $\delta(G)$. Let S = {v} $\cup$ N(v) is an eccentric dominating set.

Consider x $\in$ N(v) such that x has no eccentric vertex in N(v). Then x has eccentric vertex y in $N_2(v)$. Let $S \subseteq N_2(v)$ such that vertices in N(v) have their eccentric vertices in S. Thus D = {v} $\cup$ N(v) $\cup$ S form a distance closed eccentric dominating set. Hence, $\gamma_{dced}(G) \leq 1 + \delta(G) + \delta(G) = 1 + 2\delta(G)$.

Theorem 3.7.

If G is a graph and radius two $\delta(G) > 2$ and u is a central vertex with <V – N[u]> is connected then $\gamma_{dced}(G) \leq p - \delta(G)/2$.

Proof.

Let u $\in$ V(G) such that e(u) = 2 and deg(u) > 2. Clearly V – N(u) is a dominating set of G. But vertices in N(u) may have their eccentric vertices in N(u) only. Let S be the subset of N(u) such that vertices in N(u) – S have their eccentric vertices in S. Then [V – N(u)] $\cup$ S is an eccentric dominating set of G. Let v $\in$ N(u) and is also in a diametral path passing through u. Then [V – N(u)] $\cup$ S $\cup$ {v} is a D.C.E.D set.

$$\gamma_{dced}(G) \leq p - \deg u + \frac{\deg u}{2} + 1$$
$$= p - \deg(u)/2 + 1$$
$$\gamma_{dced}(G) \leq p - \frac{\delta(G)}{2} + 1.$$

Theorem 3.8.

Let G be a graph of order p. If r(G) = 1, then $\gamma_{dced}(G) \leq \frac{n-t+4}{2}$.

Proof.

Let u $\in$ V(G) such that e(u) = 1. Let t be the number of vertices with eccentricity 1. E_1 = {v $\in$ V(G) / e(v) = 1}. Therefore $|E_1|$ = t, u dominates all other vertices, and for t–1 vertices u is an eccentric vertex.

Let E_2 = {v $\in$ V(G) / e(v) = 2}. Therefore $|E_2|$ = p – t and vertices of E_2 have their eccentric vertices in E_2 only. Let $S \subset E_2$ such that S contains eccentric vertices of elements of E_2 – S. Then D = {u} $\cup$ {S} is an eccentric dominating set of G. If S contains two non adjacent vertices then D is also a distance closed set otherwise let v $\in$ E_2 such that v is not adjacent to atleast one element of S. Then D $\cup$ {v} is a distance closed eccentric domianting set. Hence $\gamma_{dced}(G) \leq 1 + 1 + \frac{n-t}{2} = \frac{n-t+4}{2}$.

Theorem 3.9.

Let G be a graph with radius r(G) > 2. Then $\gamma_{dced}(G) \leq p - \Delta(G) + 2$.

Proof.

Let u $\in$ V(G) such that deg u = $\Delta(G)$. Consider V – N(u). Since r(G) > 2, vertices in N(u) have their eccentric vertices in V – N(u) only. Hence V – N(u) is an eccentric dominating set of G.

Consider N(u), if N(u) is complete, (V – N(u)) $\cup$ {v}, where v $\in$ N(u) and degree $\geq$ 2 is a distance closed eccentric dominating set. If N(u) is not complete, let x, y $\in$ N(u) such that x and y are not adjacent in N(u). Then (V – N(u)) $\cup$ {x, y} is a distance closed eccentric dominating set. Hence $\gamma_{dced}(G) \leq p - \Delta(G) + 2$.

Remark 3.1.

(i) $\gamma_{dced}(G) = 2$ if and only if G is complete.

(ii) If $G \neq K_p$ then $\gamma_{dced}(G) \geq 3$.

Theorem 3.10.

If r(G) = 1, d(G) = 2 and $\gamma_{ed}(G) = 2$, then $\gamma_{dced}(G) = 3$.

Proof.

$\gamma_{ed}(G) = 2$. Let D = {x, y} be a γ_{ed}-set, E_1 = {v $\in$ V(G) / e(v) = 1} and E_2 = {v $\in$ V(G) / e(v) = 2}.

Case (i): e(x) = e(y) = 1. This is not possible.

Case (ii): e(x) = 1, e(y) = 2.

In this case, y is eccentric to all other vertices of E_2. Let w $\in$ E_2, w is not adjacent to y. Let S = {x, y, w}. This is a minimum distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = 3$.

Case (iii): e(x) = 2 = e(y).

All the other vertices of E_2 are adjacent to either x or y. If there is an z $\in$ E_2 such that it is adjacent to both x and y then x, y are not eccentric to z. Hence D is not a γ_{ed}-set which is a contradiction. Therefore, there is no z in E_2 which is adjacent to both x and y.

Subcase (i)

x and y are adjacent in G. Take S = {u, x, y} where e(u) = 1. S is a minimum distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = 3$.

Subcase (ii)

x and y are not adjacent in G. In this case take S = {x, y, z}, e(z) = 2. S is a minimum distance closed eccentric dominating set. Hence $\gamma_{dced}(G) = 3$.

Thus, we see that, when G is of radius one and diameter 2, $\gamma_{dced}(G) = 3$ if $\gamma_{ed}(G) = 2$.

Corollary 3.1.

(i) If r(G) = 1 and G has a pendent vertex then $\gamma_{dced}(G) = 3$.

(ii) If r(G) = 1 and E_2 is independent then $\gamma_{dced}(G) = 3$.

Proof.

(i) Let u $\in$ V(G) with e(v) = 1. Let v be the pendent vertex of G. Then e(v) = 2 also v is eccentric to all other vertices of eccentricity 2. Therefore {u, v} is an eccentric dominating set. Let w $\in$ V(G) such that e(w) = 2. Then D = {u, v, w} is a distance close eccentric dominating set. Therefore $\gamma_{dced}(G) = 3$.

(ii) Let u $\in$ V(G) such that e(u) = 1. Let v, w $\in E_2$. D = {u, v, w} form a distance closed eccentric dominating set. Therefore $\gamma_{dced}(G) = 3$.

Theorem 3.11.

If G is a ciliate on p vertices then $\gamma_{dced}(G) = p$.

Proof

Since $\gamma_{dcl}(G) = p$, $\gamma_{dced}(G)$ is also equal to p. Therefore $\gamma_{dced}(G) = p$.

A graph with $\gamma_{dcl} = p$ is called a 0-distance closed dominating graph.

Theorem 3.12.

There is no graph G such that both G and $\overline{G}$ are 0-distance closed eccentric dominating graphs.

Proof.

Since all the 0-D.C.D graphs are with diameter ≥ 3 and there is no graph for which both G and $\overline{G}$ are with diameter ≥ 3 we have the result.

Theorem 3.13.

If $G = \overline{K}_2 + K_1 + K_1 + \overline{K}_2$, then $\gamma(G) = 2$, $\gamma_{ed}(G) = 4$, $\gamma_{dcl}(G) = 4$, $\gamma_{dced}(G) = 4$.

If $G = K_n + K_1 + K_1 + K_n$, n > 2 then $\gamma(G) = 2$, $\gamma_{ed}(G) = 2$, $\gamma_{dcl}(G) = 4$, $\gamma_{dced}(G) = 4$.

Proof.

Proof is obvious.

Theorem 3.14.

For a bicentral tree T with radius 2, $\gamma_{dced}(T) \leq 4$.

Proof.

All the four vertices of a diametral path form a distance closed eccentric dominating set. Hence the theorem follows.

Theorem 3.15.

For a bicentral tree T, with radius two which is not a path

(i) $\gamma_{dced}(T) = \gamma_{ed}(T) + 1$, if there exists atleast one peripheral vertex with support vertex S such that deg S = 2.

(ii) $\gamma_{dced}(T) = \gamma_{ed}(T)$, if degree of every support vertex is greater than two.

Proof.

(i) Let T be a bicentral tree. In this case, diam(T) = 3, $T = \overline{K}_n + K_1 + K_1, \overline{K}_m$, for n, m $\geq$ 1, when n, m = 1.

$\gamma_{ed}(T) = 3$, $\gamma_{dcl}(T) = 4$. Hence $\gamma_{dced}(T) = \gamma_{ed}(T) + 1$.

(ii) When n, m > 1, $\gamma_{dcl}(T) = 4$, $\gamma_{ed}(T) = 4$. Hence $\gamma_{dced}(T) = \gamma_{ed}(T)$.

Theorem 3.16.

For a unicentral tree T, with radius two which is not a path.

(i) $\gamma_{dced}(T) \leq \gamma_{ed}(T) + 2$, if there exists atleast one peripheral vertex with support vertex s such that deg s = 2.

(ii) $\gamma_{dced}(T) \leq \gamma_{ed}(T) + 1$, if degree of all the support vertices are greater than two.

Proof.

In this case, diam(T) = 4, and let $T \neq P_5$. Let S be the set of all support vertices of T and let v be the central vertex of T.

Case (i)

Subcase (a)

If there exists only one peripheral vertex with degree of its support = 2. Let x be a peripheral vertex and it is eccentric to all other vertices except its support. Let y be the peripheral vertex with degree of its support greater than 2. Then S $\cup$ {x} is a γ_{ed}-set of T and S $\cup$ {x} $\cup$ {v, x} is a γ_{dced}-set of T. Hence $\gamma_{dced}(T) \leq \gamma_{ed}(T) + 2$.

Subcase (b)

If there exists two peripheral vertices with degree of their support = 2, then the degree of v is greater than or equal to two. Let x and y be two peripheral vertices and u, w be their supports such that deg u = deg w > 2. x or y is eccentric to each other vertices. Then {v} $\cup$ {x, y} is a γ_{ed}-set of T and S $\cup$ {v} $\cup$ {x, y} is a γ_{dced}-set of T. Hence $\gamma_{dced}(T) \leq \gamma_{ed}(T) + 2$.

Case (ii)

If degree of all support vertices are greater than two then S $\cup$ {x, y} is a γ_{ed}-set of T, where x and y are two peripheral vertices at distance 4 to each other, and S $\cup$ {x, y} $\cup$ {v} be the γ_{dced}-set of T. Here v is the central vertex of T. Hence $\gamma_{dced}(T) \leq \gamma_{ed}(T) + 1$.

Theorem 3.17.

If H is any self-centered unique eccentric point graph with m vertices and $G = H \circ 2K_1$, then $\gamma_{dced}(G) = 2n/3$ where n = 3m.

Proof.

If H is any self centered unique eccentric point graph, then every vertex of H is an eccentric vertex. Hence m is even and G has 3m vertices. Let $v_1, v_2, \ldots, v_m$ represent the vertices of H and $\{v_i', v_i''\}$ for i = 1, 2, ..., m be the vertices of m copies of $2K_1$. Then in G, v_i', v_i'' are adjacent to v_i and if v_j is the eccentric vertex of v_i in H, then v_i', v_i'' are eccentric vertices of v_j in G and v_i', v_i'' are eccentric vertices of v_i. It is clear that $v_1, v_2, ..., v_m\} \cup \{x_1, x_2, ..., x_m\}$ where $x_i = v_i'$ or v_i'' are minimum distance closed eccentric dominating sets of G. Hence $\gamma_{dced}(G) = 2n/3$.

Remark 3.2.

If H is self centered but not a unique eccentric point graph then $\gamma_{dced}(G)$ need not be 2n/3 where $G = H \circ 2K_1$. For example, when $G = C_5 \circ 2K_1$, $\gamma_{dced}(G) = 8 < 2n/3 = 10$.

Conclusion

Here we have evaluated the distance closed eccentric domination number of some families of graphs and also studied some bounds for the distance closed eccentric domination number of graph.

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