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Construction of Balanced and Partially Balanced n-ary t-designs by Block Sum and Product (BSP) Methodology on 2-design

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ABSTRACT

In this paper we use 2-designs whose incidence matrix will take only binary values and construct a series of balanced and partially balanced n-ary t-designs by using a tool Block Sum and Product (BSP) Methodology. The simple 2- (6, 3, 2) design used as a parent 2-design for the procedure. This parent 2-design gives 36 new balanced n-ary t-designs and 3 partially balanced n-ary t-designs. We list out the parameters of newly constructed balanced and partially balanced n-ary t-designs.

Introduction

Initially, Tocher (1952) introduced balanced n-ary design. A number of authors had given method for construction of balanced n-ary design. Murthy and Das (1967) constructed balanced n-ary designs using a set of mutually orthogonal Latin squares. Saha and Dey (1973) constructed balanced n-ary designs using difference of sets as well as Agarwal and Das (1887) constructed balanced n-ary designs through BIB and two associate PBIB-triangular design. Agarwal and Sharma (1976) obtained a series of balanced n-ary designs by collapsing certain (n-1) tuplets of blocks suitably picked from the blocks of a BIBD. Saha (1975) gives the method of construction of balanced ternary (3-ary) design.

Definition 1.1 : A Balanced Incomplete Block Design (BIBD) is a set X of V (≥ 2) elements called treatments and a collection of B(> 0) subsets of X, called blocks, such that the following conditions are satisfied:

i) Each block contains exactly K treatments

ii) Each treatment appears in exactly R blocks

iii) Each pair of treatments appears simultaneously in exactly λ blocks.

Definition 1.2: A Partially Balanced Incomplete Block Design (PBIBD) (V, B, R, K $\lambda_1, \lambda_2, \dots \lambda_m$) with m associate classes is a design on a set X, such that the following conditions are satisfied:

i) Each block contains exactly K treatments

ii) Each treatment appears in exactly R blocks

iii) If x and y are the i^{th} associates for $1 \leq i \leq m$, then they occur together in λ_i blocks.

Definition 1.3: A t-(V, K, Λ_t) block design (abbreviated t-design) is an incidence structure of treatments and blocks such that the following holds:

i) There are V treatments

ii) Each block contains K treatments

iii) For any t treatments there are exactly Λ_t blocks that contain all these treatments.

Definition 1.4 : Balanced n-ary t-design is an arrangement of V treatments in B blocks such that:

i) i^{th} treatment occurs in the j^{th} block n_{ij} times;

$i=1,2,\dots,V;\ j=1,2,\dots,B$

ii) nij can take 0 or 1 or 2.....or (n-1) value.

iii) $\sum_{i=1}^{V} n_{ij} = K, \quad \sum_{j=1}^{B} n_{ij} = R, \quad \sum_{j=1}^{B} n_{lj} n_{mj} = \begin{cases} \Delta & if \quad l = m \\ \Lambda_2 & if \quad l \neq m \end{cases}$

iv) For any t treatments there are exactly Λ_t blocks that contain all these treatments.

Further let R_i be the number of blocks in which any treatment occurs i-times and K_i be the number of treatments which occurs i-times in each block. Then the following relations hold

$$\sum_{i=0}^{n-1} R_i = B, \qquad \sum_{i=0}^{n-1} K_i = V$$

$$\sum_{i=0}^{n-1} iR_i = R, \qquad \sum_{i=0}^{n-1} iK_i = K, \qquad \sum_{i=0}^{n-1} i^2 R_i = \Delta,$$

$$\Delta = RK - \Lambda_2 (V - 1)$$

Definition 1.5 : Partially Balanced n-ary t-design with m-associate classes is an arrangement of V treatments in B blocks such that:

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i) i^{th} treatment occurs in the j^{th} block n_{ij} times;

$i=1,2,\ldots,V; \quad j=1,2,\ldots,B$

ii) n_{ij} can take 0 or 1 or 2…..or (n-1) value.

iii) $\sum_{i=1}^{V} n_{ij} = K, \qquad \sum_{j=1}^{B} n_{ij} = R, \qquad \sum_{j=1}^{B} n_{ij}^2 = \Delta$

iv) With respect to any *(t-1)* treatment set *(i_1, i_2, ..., $i_{(t-1)}$)* the remaining *(V-t+1)* treatments can be classified in *m* groups such that the l^{th} group contain N_l treatments where,

$$\sum_{l=1}^{m} N_l = (V - t + 1)$$

If i_t is any treatment from l^{th} group then,

$$\sum_{J=1}^{B} n_{i_1 j} n_{i_2 j} \ldots n_{i_{(t-1)} j} n_{i_t j} = \Lambda_{tl} \qquad l = 1,2,\ldots,m$$

$$i_1 \neq i_2 \neq \ldots \neq i_{(t-1)} \neq i_t = 1,2,\ldots,V$$

- When m=1 we get the definition of balanced n-ary t-design
- When m=1 and n=2 we get the definition of classical t-design
- When m=1, n=2 and t=2 we get the definition of BIBD
- When n=2 and t=2 we get the definition of PBIBD with m-associate classes.

We used a tool Block Sum and Product (BSP) Methodology (2007) on a 2- (6, 3, 2) design and constructed new balanced and partially balanced n-ary t-designs. BSP Methodology gives number of designs, according to incidence matrix and parameters of newly constructed design it classified into balanced n-ary t-design or partially balanced n-ary t-design.

Construction of balanced and partially balanced n-ary t-designs by BSP Methodology on 2-(6, 3, 2) design

Let us consider a simple 2-design with parameters V=6, B=10, R=5, K=3, Λ_t=2. To apply BSP Methodology replace 1, 2,.....,6 treatments by X_1, X_2,....., X_6. Take block sum B_i then consider product of B_i.

Table 1. Parent design and the notation for BSP Methodology.

Block Number (i)	Treatment content in block i	Treatment replaced for BSP	Block sum (B_i) for BSP
1	1 2 3	X_1 X_2 X_3	$B_1=X_1+X_2+X_3$
2	1 2 4	X_1 X_2 X_4	$B_2=X_1+X_2+X_4$
3	1 3 6	X_1 X_3 X_6	$B_3=X_1+X_3+X_6$
4	1 4 5	X_1 X_4 X_5	$B_4=X_1+X_4+X_5$
5	1 5 6	X_1 X_5 X_6	$B_5=X_1+X_5+X_6$
6	2 3 5	X_2 X_3 X_5	$B_6=X_2+X_3+X_5$
7	2 4 6	X_2 X_4 X_6	$B_7=X_2+X_4+X_6$
8	2 5 6	X_2 X_5 X_6	$B_8=X_2+X_5+X_6$
9	3 4 5	X_3 X_4 X_5	$B_9=X_3+X_4+X_5$
10	3 4 6	X_3 X_4 X_6	$B_{10}=X_3+X_4+X_6$

$\prod_{i=1}^{10} B_i = B_1.B_2......B_{10}$ will be polynomial of degree 10 of variables X_1, X_2, …, X_6. This polynomial contains $K^B=3^{10}$ (=59049) terms. Similar types of terms of this polynomial are classified according to powers and new 39 designs are constructed.

Table 2. The description about the polynomial.

Design No.	Type of Power	Number of terms with all possible powers of the type in Col.(2)	Number of replications of terms in Col.(3)	Total number of terms
1	531100	150	1	150
2	532000	60	1	60
3	432100	150	10	1500
4	432100	60	12	720
5	422200	30	14	420
6	331111	15	115	1725
7	431110	60	16	960
8	422200	30	16	480
9	511111	6	13	78
10	333100	30	19	570
11	222220	6	156	936
12	322111	30	192	5760
13	332200	15	22	330
14	531100	30	2	60
15	332200	15	30	450
16	442000	30	2	60
17	222211	15	336	5040
18	322111	30	203	6090
19	441100	15	4	60
20	522100	90	2	180
21	441100	75	2	150
22	431110	60	20	1200
23	332200	60	25	1500
24	422110	30	28	840
25	522100	90	3	270
26	422110	60	30	1800
27	422110	60	34	2040
28	422110	30	36	1080
29	433000	30	4	120
30	332110	60	47	2820
31	521110	60	5	300
32	332110	30	50	1500
33	332110	30	56	1680
34	521110	60	6	360
35	432100	180	6	1080
36	332110	60	60	3600
37	421111	30	70	2100
38	322210	60	89	5340
39	322210	60	94	5640
			Total	**59049**

In the polynomial, there are 150 terms of the type of the power 531100 and each term is repeated 1 time. The powers of these 150 terms gives columns of the incidence matrix of design no.1 having V=6 and B=150. Likewise this polynomial gives 39 incidence matrices of new designs. According to incidence matrix and parameters of newly constructed design it classified into balanced n-ary t-design or partially balanced n-ary t-design.

Table 3. Type of design and the parameters of newly constructed design.

Design No.	Type of Design	Parameters of newly constructed design											
1	Balanced 6-ary 4-disgin	R_0	R_1	R_2	R_3	R_4	R_5	B	R	Δ	Λ_{31}	N_1	Λ_4
		50	50	0	25	0	25	150	250	900	252	2	150
		K_0	K_1	K_2	K_3	K_4	K_5	V	K	Λ_2	Λ_{32}	N_2	
		2	2	0	1	0	1	6	10	320	318	2	
2	Partially Balanced 6-ary 3-disgin	R_0	R_1	R_2	R_3	R_4	R_5	B	R	Δ	Λ_{31}	N_1	
		30	0	10	10	0	10	60	100	380	0	2	
		K_0	K_1	K_2	K_3	K_4	K_5	V	K	Λ_2	Λ_{32}	N_2	
		3	0	1	1	0	1	6	10	124	180	2	
3	Balanced 5-ary 4-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_{31}	N_1	Λ_4	
		50	25	20	35	20	150	250	740	336	2	246	
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2	Λ_{32}	N_2		
		2	1	1	1	1	6	10	352	426	2		

4	Balanced 5-ary 4-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_{31}	N_1	Λ_4
		20	10	10	10	10	30	100	300	84	2	96
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2	Λ_{32}	N_2	
		2	1	1	1	1	6	10	140	216	2	

5	Balanced 5-ary 4-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_{31}	N_1	Λ_4
		10	0	15	0	5	30	50	140	72	2	64
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2	Λ_{32}	N_2	
		2	0	3	0	1	6	10	72	96	2	

6	Balanced 4-ary 6-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_3	Λ_4	Λ_5	Λ_6
		0	10	0	5	15	25	55	57	79	105	135
		K_0	K_1	K_2	K_3	V	K	Λ_2				
		0	4	0	2	6	10	39				

7	Balanced 5-ary 5-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_{31}	N_1	Λ_4	Λ_5
		10	30	0	10	10	60	100	280	156	2	172	120
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2	Λ_{32}	N_2		
		1	3	0	1	1	6	10	144	192	2		

8	Balanced 5-ary 4-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_{31}	N_1	Λ_4
		10	0	15	0	5	30	50	140	72	2	64
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2	Λ_{32}	N_2	
		2	0	3	0	1	6	10	72	96	2	

9	Balanced 6-ary 6-disgin	R_0	R_1	R_2	R_3	R_4	R_5	B	R	Δ	Λ_3	Λ_4	Λ_5	Λ_6
		0	5	0	0	0	1	6	10	30	18	22	26	30
		K_0	K_1	K_2	K_3	K_4	K_5	V	K	Λ_2				
		0	5	0	0	0	1	6	10	14				

10	Balanced 4-ary 4-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4
		10	5	0	15	30	50	140	54	2	54
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2	
		2	1	0	3	6	10	72	108	2	

11	Balanced 3-ary 5-disgin	R_0	R_1	R_2	B	R	Δ	Λ_3	Λ_4	Λ_5
		1	0	5	6	10	20	24	32	32
		K_0	K_1	K_2	V	K	Λ_2			
		1	0	5	6	10	16			

12	Balanced 4-ary 6-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4	Λ_5	Λ_6
		0	15	10	5	30	50	100	120	2	182	260	360
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2			
		0	3	2	1	6	10	80	126	2			

13	Balanced 4-ary 4-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4
		5	0	5	5	15	25	65	36	2	36
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2	
		2	0	2	2	6	10	37	54	2	

14	Balanced 6-ary 4-disgin	R_0	R_1	R_2	R_3	R_4	R_5	B	R	Δ	Λ_{31}	N_1	Λ_4
		10	10	0	5	0	5	30	50	180	24	2	30
		K_0	K_1	K_2	K_3	K_4	K_5	V	K	Λ_2	Λ_{32}	N_2	
		2	2	0	1	0	1	6	10	64	90	2	

15	Balanced 4-ary 4-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4
		5	0	5	5	15	25	65	36	2	36
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2	
		2	0	2	2	6	10	37	54	2	

16	Partially Balanced 5-ary 3-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_{31}	N_1
		15	0	5	0	10	30	50	180	0	2
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2	Λ_{32}	N_2
		3	0	1	0	2	6	10	64	96	2

17	Balanced 3-ary 6-disgin	R_0	R_1	R_2	B	R	Δ	Λ_3	Λ_4	Λ_5	Λ_6
		0	5	10	15	25	45	66	104	160	240
		K_0	K_1	K_2	V	K	Λ_2				
		0	2	4	6	10	41				

18	Balanced 4-ary 6-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4	Λ_5	Λ_6
		0	15	10	5	30	50	100	120	2	182	260	360
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2			
		0	3	2	1	6	10	80	126	2			

19	Balanced 5-ary 4-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_{31}	N_1	Λ_4
		5	5	0	0	5	15	25	85	12	2	16
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2	Λ_{32}	N_2	
		2	2	0	0	2	6	10	33	48	2	

20	Balanced 6-ary 4-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**R_5**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	
		30	15	30	0	0	15	90	150	510	144	2	120	
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**K_5**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**		
		2	1	2	0	0	1	6	10	198	252	2		
21	Balanced 5-ary 4-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**		
		25	25	0	0	25	75	125	425	132	2	80		
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		2	2	0	0	2	6	10	165	168	2			
22	Balanced 5-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**	
		10	30	0	10	10	60	100	280	156	2	172	120	
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		1	3	0	1	1	6	10	144	192	2			
23	Balanced 4-ary 4-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**B**	**R**	**Δ**	**Λ_3**	**Λ_4**				
		20	0	20	20	60	100	260	180	144				
		K_0	**K_1**	**K_2**	**K_3**	**V**	**K**	**Λ_2**						
		2	0	2	2	6	10	148						
24	Balanced 5-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**	
		5	10	10	0	5	30	50	130	84	2	104	80	
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		1	2	2	0	1	6	10	74	108	2			
25	Balanced 6-ary 4-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**R_5**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	
		30	15	30	0	0	15	90	150	510	144	2	120	
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**K_5**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**		
		2	1	2	0	0	1	6	10	198	252	2		
26	Balanced 5-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**	
		10	20	20	0	10	60	100	260	180	2	208	160	
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		1	2	2	0	1	6	10	148	204	2			
27	Balanced 5-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**	
		10	20	20	0	10	60	100	260	180	2	208	160	
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		1	2	2	0	1	6	10	148	204	2			
28	Balanced 5-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**	
		5	10	10	0	5	30	50	130	84	2	104	80	
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		1	2	2	0	1	6	10	74	108	2			
29	Partially Balanced 5-ary 3-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**			
		15	0	0	10	5	30	50	170	0	2			
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		3	0	0	2	1	6	10	66	108	2			
30	Balanced 4-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**		
		10	20	10	20	60	100	240	180	2	228	180		
		K_0	**K_1**	**K_2**	**K_3**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**				
		1	2	1	2	6	10	152	228	2				
31	Balanced 6-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**R_5**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**
		10	30	10	0	0	10	60	100	320	144	2	148	100
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**K_5**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**		
		1	3	1	0	0	1	6	10	136	168	2		
32	Balanced 4-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**		
		5	10	5	10	30	50	120	96	2	114	90		
		K_0	**K_1**	**K_2**	**K_3**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**				
		1	2	1	2	6	10	76	108	2				
33	Balanced 4-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**		
		5	10	5	10	30	50	120	96	2	114	90		
		K_0	**K_1**	**K_2**	**K_3**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**				
		1	2	1	2	6	10	76	108	2				
34	Balanced 6-ary 5-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**R_5**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**	**Λ_5**
		10	30	10	0	0	10	60	100	320	144	2	148	100
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**K_5**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**		
		1	3	1	0	0	1	6	10	136	168	2		
35	Balanced 5-ary 4-disgin	**R_0**	**R_1**	**R_2**	**R_3**	**R_4**	**B**	**R**	**Δ**	**Λ_{31}**	**N_1**	**Λ_4**		
		60	30	30	30	30	180	300	900	312	2	288		
		K_0	**K_1**	**K_2**	**K_3**	**K_4**	**V**	**K**	**Λ_2**	**Λ_{32}**	**N_2**			
		2	1	1	1	1	6	10	420	588	2			

36	Balanced 4-ary 5-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4	Λ_5	
		10	20	10	20	60	100	240	180	2	228	180	
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2			
		1	2	1	2	6	10	152	228	2			
37	Balanced 5-ary 6-disgin	R_0	R_1	R_2	R_3	R_4	B	R	Δ	Λ_3	Λ_4	Λ_5	Λ_6
		0	20	5	0	5	30	50	120	108	146	190	240
		K_0	K_1	K_2	K_3	K_4	V	K	Λ_2				
		0	4	1	0	1	6	10	76				
38	Balanced 4-ary 5-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4	Λ_5	
		10	10	30	10	60	100	220	216	2	272	240	
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2			
		1	1	3	1	6	10	156	228	2			
39	Balanced 4-ary 5-disgin	R_0	R_1	R_2	R_3	B	R	Δ	Λ_{31}	N_1	Λ_4	Λ_5	
		10	10	30	10	60	100	220	216	2	272	240	
		K_0	K_1	K_2	K_3	V	K	Λ_2	Λ_{32}	N_2			
		1	1	3	1	6	10	156	228	2			

Discussion

The aim of this paper has been introduced a tool Block Sum and Product Methodology (BSP) for construction of design. In this paper we used BSP Methodology tool on 2-(6, 3, 2) design as a parent 2-design and obtain a 36 balanced n-ary t-designs and 3 partially balanced n-ary t-designs. Here it is difficult to give incidence matrices of newly constructed designs so we list out the parameters of newly constructed designs. BSP Methodology tool can be used on any 2-design, according to incidence matrix and parameters of newly constructed design it can classified into balanced or partially balanced n-ary t-designs.

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