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On a Symmetric Biderivations and Automorphismity of Prime and Semiprime Rings

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ABSTRACT

Our aim in this paper is to investigate the commuting property of the Traces of a Symmetric Biderivations on prime and semiprime ring \mathcal{R} . Furthermore, we look for the commutitivity of a ring \mathcal{R} under some suitable conditions.

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Keywords

Prime ring, Semiprime ring, Symmetric Biderivations, Trace of mapping, Commuting mappings.

1.Introduction

Throughout, \mathcal{R} will represent an associative ring with the center $Z(\mathcal{R})$. A ring \mathcal{R} satisfies that $u\mathcal{R} v=(0)$ leads either u=0 or v=0 is said to be prime ring, while \mathcal{R} is semiprime if whenever u \mathcal{R} u=(0) implies u=0. Moreover, \mathcal{R} has n-torsion free property if nu=0, u $\in \mathcal{R}$ implies that u=0 (see [1]). Recall that in the prime ring \mathcal{R} the statement \mathcal{R} is n-torsion free ring is equivalent to \mathcal{R} of characteristic different from n [2]. As usual, the symbol [u, v] is refer to the commutator uv – vu. We shall be frequently using the basic commutator identities $[u_{\omega}, v] = [u, v]_{\omega} + u[_{\omega}, v], [u, v_{\omega}] = [u, v]_{\omega} + v[u, \omega]$ (see [3]). A biadditive mapping $\mathcal{B}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is called Symmetric if $\mathcal{B}(u, v) = \mathcal{B}(v, u)$ for all pairs $u, v \in \mathcal{R}$ (see [4]). A mapping $\boldsymbol{\varphi}: \mathcal{R} \longrightarrow \mathcal{R}$ defined by $\varphi(u) = \mathcal{B}(u, u)$, where $\mathcal{B}(., .): \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is a Symmetric mapping will be called the Trace of \mathcal{B} (see [5]). It is obvious that in case the Symmetric mapping $\mathcal{B}(.,.):\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is also biadditive, the Trace of \mathcal{B} satisfies $\varphi(u+v) = \varphi(u) + 2 \mathcal{B}(u,v) + \varphi(v)$. A mapping $\mathcal{L}: \mathcal{R} \longrightarrow \mathcal{R}$ is said to be Centralizing on \mathcal{R} if $[\mathcal{L}(u), u] \in Z(\mathbb{R})$, for all $x \in \mathcal{R}$. In special case when $[\mathcal{L}(u), u] = 0$, for all $x \in \mathcal{R}$, the mapping \mathcal{L} is called Commuting on \mathcal{R} (see [6]). The centralizing and commuting mapping have been studied in many papers. It seems that the first result in this field was given by J. Vokman [7]. He prove that if d is a derivation of a prime ring \mathcal{R} of characteristic different from 2, such that the mapping $\mathcal{P}(u)$ = [d(u), u] then $\mathcal{P}=0$, that is d is commuting. In [8] Brezar generalized this result by showing that the same conclusion holds for any additive mapping. In [9] Brezar describe all commuting traces of biadditive mapping on certain prime rings. He prove that if the Characteristic of R different from 2, and \mathcal{R} does not satisfy S₄, then every such mapping (say \mathcal{P}) is of the form $\mathcal{P}(u) = \lambda u^2 + \mu(u) u + \nu(u)$ for all $x \in \mathcal{R}$, and $\lambda \in \mathcal{C}$ where \mathcal{C} is the extended centroid of \mathcal{R} (the center of the Martindale ring of quotient of \mathcal{R}) and $\mu: \mathcal{R} \longrightarrow \mathcal{C}$ is an additive mapping.

A biadditive mapping $\mathcal{D}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is called a Symmetric Biderivation if $\mathcal{D}(\mathbf{u}_{\omega}, \mathbf{y}) = \mathcal{D}(\mathbf{u}, \mathbf{v})_{\omega} + \mathbf{u}\mathcal{D}(\boldsymbol{\omega}, \mathbf{v})$ is fulfilled for all $\mathbf{u}, \mathbf{v}, \boldsymbol{\omega} \in \mathcal{R}$ [11]. On the other hand N. Argac in 2006 introduce the concept of Symmetric generalized Biderivation as follows: A symmetric biadditive mapping $\mathcal{Q}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is called a Symmetric generalized Biderivation if there exist a symmetric Biderivation \mathcal{D} such that $\mathcal{Q}(\mathbf{u}_{\omega}, \mathbf{y}) = \mathcal{Q}(\mathbf{u}, \mathbf{v}) \boldsymbol{\omega} + \mathbf{u}\mathcal{D}(\boldsymbol{\omega}, \mathbf{v})$ is fulfilled for all $\mathbf{u}, \mathbf{v}, \boldsymbol{\omega} \in \mathcal{R}$ (see [12).

The purpose of the present paper is looking for the commuting property of the Traces of a Symmetric Biderivations on prime and semiprime rings. Also, we present some results concerning with the commutitivity of rings.

2. Preliminary results

We facilitate our study with following known results which are necessary for developing the proof of our results. **Remarks (2.1):** [13]

Let \mathcal{R} be a prime ring, \mathcal{J} a nonzero ideal of \mathcal{R} . If a \mathcal{J} b=0, for a $\in \mathcal{R}$, it's easy to verify that either a=0 or b=0.

Lemma (2.2): [14]

Let \mathcal{R} be a 2-torision free prime ring and \mathcal{J} be a nonzero ideal of \mathcal{R} . If \mathcal{D} is a symmetric Biderivation such that $\mathcal{D}(\mathbf{x}, \mathbf{x}) = 0$, all $\mathbf{x} \in \mathcal{J}$ then either $\mathcal{D} = 0$ or \mathcal{R} is commutative.

Lemma (2.3): [15]

Let \mathcal{R} be a semiprime ring, \mathcal{J} an ideal of \mathcal{R} . If \mathcal{J} is a commutative as a ring, then $\mathcal{J} \subset Z(\mathcal{R})$. In addition if \mathcal{R} is prime then \mathcal{R} must be commutative.

Lemma (2.4): [16]

Let \mathcal{R} be a semiprime ring, suppose that arb+brc=0 holds for all $r \in \mathcal{R}$ and some a,b,c $\in \mathcal{R}$. In this case (a+c) r = 0 is satisfied for all $r \in \mathcal{R}$.

Lemma (2.5): [17]

Let \mathcal{R} be a semiprime ring suppose that there exists a $\in \mathcal{R}$ such that a[x, y] = 0 holds for all pairs $x, y \in \mathcal{R}$, then there exists an ideal U of \mathcal{R} such that a $\in U \subset Z(\mathcal{R})$.

Lemma (2.6): [18]

Let \mathcal{R} be a prime ring, and \mathcal{J} be a nonzero left ideal of \mathcal{R} . If a (σ, τ) -Biderivation $\mathcal{D}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ satisfies that $\mathcal{D}(\mathcal{J}, \mathcal{J}) = 0$, then $\mathcal{D} = 0$.

Lemma (2.7): [19]

Let d be aderivation of a prime ring \mathcal{R} and \mathcal{J} a nonzero ideal of \mathcal{R} . For an element a of \mathcal{R} , if either (i) ad(x)=0 for all $x \in \mathcal{J}$ or (ii) d(x)=0 for all $x \in \mathcal{J}$. Then either a=0 or d=0.

Theorem (3.1):

Let \mathcal{R} be a 2-torition free prime ring and U an ideal of \mathcal{R} . The existence of nonzero symmetric bi-derivation $\mathcal{F}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with commuting trace f on U, forces \mathcal{R} to be commutative.

Proof: Suppose f is commuting on U, that is: (1) [f(u), u] = 0, for all $u \in U$. The linearization of (1), using (1) gives: $[f(u), \boldsymbol{\omega}] + [f(\boldsymbol{\omega}), u] + 2[\boldsymbol{\mathcal{F}}(u, \boldsymbol{\omega}), \boldsymbol{\omega}] + 2[\boldsymbol{\mathcal{F}}(u, \boldsymbol{\omega}), u] = 0, \text{ for all } u, \boldsymbol{\omega} \in U.$ Putting 2_{ω} instead of $_{\omega}$, comparing the above relation with the new one, we find: $[f(u), \omega] + 2[F(u, \omega), u] = 0$, for all $u, \omega \in U$. (2) The substitution ωu for ω in (2), we get: $[f(u), \omega] u + \omega[f(u), u] + 2[\mathcal{F}(u, \omega), u]u + 2\omega[f(u), u] + 2[\omega, u] f(u) = 0$, for all $u, \omega \in U$. In view of relations (1), (2) and 2-toritionity free of \mathcal{R} , the last relation reduces to: $[\omega, u]$ f(u) =0, for all u, $\omega \in U$. (3) Let us write z_{ω} for ω in (3), using (3) implies that: $[z, u] \omega f(u) = 0$, for all $u, \omega \in U$. (4) The linearization of (4) with respect to u, using (4) leads to: (5) $[z, u] \omega f(v) + [z, v] \omega f(u) + 2[z, u] \omega \mathcal{F}(u, v) + 2[z, v] \omega \mathcal{F}(u, v) = 0$, for all $u, \omega \in U$. Replacing u by -u, combining the relation so obtained with (5), we get because of the 2-toritionity free of \mathcal{R} that:

 $[z, v] \omega$ f(u)+2[z, u] $\omega \mathcal{F}(u, v) = 0$, for all u, $\omega \in U$.

The substitution $f(u)_{\omega}[z, v]$ for ω leads because of (3) to:

 $[z, v] f(u) \omega [z, v] f(u) = 0$, for all $u, \omega \in U$.

By remarks (2.1), we have:

[z, v] f(u) = 0, for all $u, \omega \in U$.

Substituting v_{ω} for v in (6), we see:

 $[z, v] \omega f(u) = 0$, for all u,v, $\omega \in U$.

Using remarks (2.1) implies that either f(u) = 0 or [z, v] = 0 for all $u, v, z \in U$. If f(u) = 0 for all $u \in U$, then \mathcal{R} is commutative by lemma (2.2) (note that \mathcal{F} is a nonzero mapping). Otherwise, [z, v] = 0, for all $u, v \in U$, this means that U is commutative ideal of \mathcal{R} . Consequently, \mathcal{R} is commutative by lemma (2.3).

(6)

Theorem (3.2)

Let \mathcal{R} be a 2-torition free semiprime ring and α is an automorphisms of \mathcal{R} . If \mathcal{R} admitting a symmetric Biderivation \mathcal{F} : $\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with traces f satisfies that $[f(x)+\alpha(x), x] = 0$, for all $x \in \mathcal{R}$ then f is commuting on \mathcal{R} .

Proof: Suppose for any $x \in \mathcal{R}$, we have: $[f(x) + \alpha(x), x] = 0$, for all $x \in \mathcal{R}$. The linearization of (1) implies that: $2[\mathcal{F}(x, y), x] + [f(y), x] + [\alpha(y), x] + 2[\mathcal{F}(x, y), y] + [f(x), y] = 0$, for all $x, y \in \mathcal{R}$. (1)

Replacing x by -x, then comparing the above relation with the relation so obtained, we get:

$[f(y), x] + [\alpha(y), x] + 2[\mathcal{F}(x, y), y] + [\alpha(x), y] = 0, \text{ for all } x, y \in \mathcal{R}.$	(2)
$[f(y), x]y + x[f(y), y] + [\alpha(y), x]y + x[\alpha(y), y] + 2[\mathcal{F}(x, y), y]y + 2[x, y]f(y) + 2x[f(y), y] + [\alpha(x), y]\alpha(y) + \alpha(x)[\alpha(y), y] = \frac{1}{2}$	0, for all
x, $y \in \mathcal{R}$. Right multiplication of (2) by y, the subtracting the relation so obtained from above relation, we see: $x[f(y), y] + x[\alpha(y), y] + 2[x, y] f(y) + 2x[f(y), y] + [\alpha(x), y]\alpha(y) + \alpha(x)[\alpha(y), y] - [\alpha(x), y]y=0$, for all x, $y \in \mathcal{R}$.	
Furthermore, the above relation reduces because of (1) to: $2[x, y] f(y) - 2x[\alpha(y), y] + [\alpha(x), y] \alpha(y) + \alpha(x)[\alpha(y), y] - [\alpha(x), y] y=0$, for all x, y $\in \mathcal{R}$.	
That is $\{\alpha(x)-2x\}[\alpha(y), y] + [\alpha(x), y] \{\alpha(y)-y\} + 2[x, y] f(y)=0$, for all x, y $\in \mathcal{R}$.	(3)
Putting yx for x in (3) leads to: $y\{\alpha(x)-2x\}[\alpha(y), y] + \{\alpha(y)-y\} \alpha(x) [\alpha(y), y] + [\alpha(y), y] \alpha(x)\{\alpha(y)-y\} + \alpha(y)[\alpha(x), y] \{\alpha(y)-y\} + 2y[x, y] G(y, y)=0$, fo $\in \mathcal{R}$.	r all x, y (4)
Left multiplying of (3) by y, we get: $\{\alpha(x)-2x\}[\alpha(y), y] + [\alpha(x), y] \{\alpha(y)-y\} + 2[x, y] G(y, y)=0$, for all x, y $\in \mathcal{R}$.	
Subtracting the last relation from (4) leads to: $\{\alpha(y)-y\} \alpha(x) [\alpha(y), y] + [\alpha(y), y] \alpha(x) \{\alpha(y)-y\} + \alpha(y)[\alpha(x), y] \{\alpha(y)-y\} - y[\alpha(x), y] \{\alpha(y)-y\} = 0,$ That is	
$\{\alpha(y)-y\} \alpha(x) [\alpha(y), y] + [\alpha(y), y] \alpha(x) \{\alpha(y)-y\} + \{\alpha(y)-y\} [\alpha(x), y] \{\alpha(y)-y\} = 0, \text{ for all } x, y \in \mathcal{R}.$	(5)
Now, define $\mu(y) = \alpha(y) - y$, then $[\alpha(y), y] = [\alpha(y) - y, y] = [\mu(y), y]$, for all x, y $\in \mathcal{R}$.	(6)
According to (6), the relation (5) can be written as:	(0)
$\mu(\mathbf{y}) \alpha(\mathbf{x}) [\mu(\mathbf{y}), \mathbf{y}] + [\mu(\mathbf{y}), \mathbf{y}] \alpha(\mathbf{x}) \mu(\mathbf{y}) + \mu(\mathbf{y})[\alpha(\mathbf{x}), \mathbf{y}] \mu(\mathbf{y}) = 0, \text{ for all } \mathbf{x}, \mathbf{y} \in \mathcal{R}.$	(7)
Equivalently $\mu(\mathbf{y})\alpha(\mathbf{x}) + \mu(\mathbf{y})\alpha(\mathbf{x})\mathbf{y} + \mu(\mathbf{y})\mathbf{y}\alpha(\mathbf{x})\mu(\mathbf{y}) - \mathbf{y}\mu(\mathbf{y})\alpha(\mathbf{x}) + \mu(\mathbf{y})\alpha(\mathbf{x})\mathbf{y}\mu(\mathbf{y}) - \mu(\mathbf{y})\mathbf{y}\alpha(\mathbf{x})\mu(\mathbf{y}) = 0.$	
That is	
$-y\mu(y) \alpha(x) \mu(y)+\mu(y)\alpha(x) \mu(y)y = 0$, for all x, $y \in \mathcal{R}$.	
Setting $a = -y \mu(y)$, $b = \mu(y)$ and $c = \mu(y)y$, then an application of Lemma (2.4) on above relation, we find:	
$(\mu(y)y - y\mu(y)) \alpha(x) \mu(y) = 0$, for all x, $y \in \mathcal{R}$. That is	
$[\mu(\mathbf{y}), \mathbf{y}] \alpha(\mathbf{x}) \mu(\mathbf{y}) = 0, \text{ for all } \mathbf{x}, \mathbf{y} \in \mathcal{R}.$	(8)
Putting $\alpha(x)$ y instead of $\alpha(x)$ in (8) yields: $[\mu(y), y] \alpha(x) y_{\mu}(y) = 0$, for all x, y $\in \mathbb{R}$.	(9)
Right multiplication of (7) by y, then subtracting relation (8) from the relation so obtained, we find: $[\mu(y), y] \alpha(x) [\mu(y), y]=0$, for all x, $y \in \mathcal{R}$.	
Since is an automorphisms on \mathcal{R} , then the semiprimeness of \mathcal{R} leads to: [μ (y), y]=0, for all y $\in \mathcal{R}$.	
This means because of (6) that: $[\alpha(y), y] = 0$, for all $y \in \mathcal{R}$.	(10)
An application of (10) on the relation (1), we arrive at: $[f(x), x] = 0$, for all $x \in \mathcal{R}$.	
In case \mathcal{R} is prime ring, we have because of theorem (3.1) the following corollary. Corollary (3.3) Let \mathcal{R} be a 2-torition free prime ring and α is an automorphisms of \mathcal{R} . If \mathcal{R} admitting a symmetric Bideriva	ation T :
$\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with traces f satisfies that $[f(x)+\alpha(x), x]=0$, for all $x \in \mathcal{R}$, then \mathcal{R} is commutative.	····· ·
Corollary (3.4) Let \mathcal{R} be a non-commutative prime ring of characteristic different from 2 and α is an automorphisms of R. If $\mathcal{F}: \mathcal{R}$ >	$\times \mathcal{R} \rightarrow$
\mathcal{R} is a symmetric Biderivation with traces f satisfies that $[f(x)+\alpha(x), x] = 0$, for all $x \in \mathcal{R}$. In this case \mathcal{F} is zero on \mathcal{R} . Proof:	
According to theorem (3.2), we have: $[f(x), x] = 0$, for all $x \in \mathcal{R}$.	(1)
Now, using similar techniques as used in theorem (3.1) to get (3) from (1), we arrive at: $\begin{bmatrix} x & y \end{bmatrix} f(x) = 0$ for all $x \neq \sigma \mathcal{P}$	(2)
An application of lemma (2.7) on (3) one can conclude that for any x $\not\in Z(\mathcal{R})$, we have $f(x) = 0$ (note that for any fixed x	(∠) s ∈R the
additive mapping $f_x(y)=[x, y]$ is a derivation).	

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Now, if $x \in Z(\mathcal{R})$, then $x+y \notin Z(\mathcal{R})$ and $x-y \notin Z(\mathcal{R})$ for any $y \notin Z(\mathcal{R})$, so we have:	
$0 = f(x+y) = f(x) + 2 \mathcal{F}(x, y)$	(3)
$0 = f(x-y) = f(x) - 2 \mathcal{F}(x, y) $ (4)	4)
Combining the relations (3) and (4) leads because of characteristic different from 2 of \mathcal{R} that $f(x)=0$. Therefore we have pr	oved
that:	
$f(x)=0$, for all $x \in \mathcal{R}$.	
Hence $\mathcal{F}=0$ by lemma (2.2).	
Theorem (3.5)	
Let \mathcal{R} be a 2-tortion free prime ring and U a nonzero ideal of \mathcal{R} . If G: $\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is a symmetric generalized Bideriva	ation
associated with Biderivation D satisfies that the Trace of G is centralizing on U, then either G of D has commuting Traces on D .	υ.
Proof: Suppose that: $[g(u), u) = \overline{Z(\mathcal{P})}$ for all $u \in U$	(1)
By linearization of (1) we obtain that:	(1)
$[g(\mathbf{u}), \boldsymbol{\omega}] + [g(\boldsymbol{\omega}), \mathbf{u}] + 2[G(\mathbf{u}, \boldsymbol{\omega}), \boldsymbol{\omega}] + 2[G(\mathbf{u}, \boldsymbol{\omega}), \mathbf{u}] \in \mathbb{Z}(\mathbb{R}), \text{ for all } \mathbf{u}, \boldsymbol{\omega} \in \mathbb{U}.$	
Putting - u instead of u, comparing the above relation with the new one, we find:	
$[g(u), \omega] + 2[G(u, \omega), u] \in \mathbb{Z}(\mathcal{R})$, for all $u, \omega \in \mathbb{U}$.	(2)
The substitution ω u for ω in (2), we get:	
$([g(u), \mathbf{\omega}] u + 2[G(u, \mathbf{\omega}), u]) u + 2[\mathbf{\omega}, u] d(u) + \mathbf{\omega}[g(u), u] + 2\mathbf{\omega}[d(u), u] \in \mathbb{Z}(\mathbb{R}), \text{ for all } u, \mathbf{\omega} \in \mathbb{U}.$	
Therefore we have:	
$[([g(u), \boldsymbol{\omega}] u + 2[G(u, \boldsymbol{\omega}), u]) u + 2[\boldsymbol{\omega}, u] d(u) + \boldsymbol{\omega}[g(u), u] + 2\boldsymbol{\omega}[d(u), u], u] = 0, \text{ for all } u, \boldsymbol{\omega} \in U.$	
The above relation can be written because of (1) and (2) as:	
$[\omega, u] [g(u), u] + 2[[\omega, u], u] d(u) + 4[\omega, u] [d(u), u] + 2\omega[[d(u), u], u]=0$, for all u, $\omega \in U$.	(3)
The substitution $g(u)_{\omega}$ for ω in (3)	
$[g(u), u] \ \omega \ [g(u), u] + g(u)[\omega, u] \ [g(u), u] + 4[g(u), u] \ [\omega, u] \ d(u) + 2g(u)[[\omega, u], u] \ d(u) + 4[g(u), u] \ \omega$	
$[d(u), u] + 4g(u)[\omega, u] [d(u), u] + 2g(u) \omega [[d(u), u], u] = 0$, for all $u, \omega \in U$.	
According to (3), the above relation reduces to: $[g(y), y] = [g(y), y] + 4[g(y), y] + 4[g(y), y] + 4[g(y), y] [g_y, y] d(y) = 0$ for all y, as C U	
$[g(u), u] \bigoplus [g(u), u] + 4[g(u), u] \bigoplus [u(u), u] + 4[g(u), u] [((u), u] u(u)=0, for all u, ((u))=0.$	
$\left[g(u), u\right]^2 + 4 \omega[g(u), u] [d(u), u] + 4[g(u), u] [\omega, u] d(u)=0$, for all $u, \omega \in U$.	(4)
Left multiplication of (4) by g(u) vields that:	
$g(u)\omega[g(u), u]^2 + 4g(u)\omega[g(u), u] [d(u), u] + 4g(u)[g(u), u] [\omega, u] d(u)=0$, for all u, $\omega \in U$.	(5)
Again, replacing ω by $g(u)\omega$ in (4), we get because of (1) that:	
$g(u)\omega[g(u), u]^2 + 4g(u)\omega[g(u), u] [d(u), u] + 4g(u)[g(u), u] [\omega, u]d(u) + 4[g(u), u] \omega [g(u), u] d(u) = 0.$	
By subtracting the relation (5) from the above relation leads because of 2-torisionity free of \mathcal{R} to:	
$[g(u), u] \omega [g(u), u] d(u)=0$, for all u, $\omega \in U$.	
Left multiplication of the last relation by d(u), we see:	
$[g(u), u]d(u) \omega [g(u), u] d(u)=0$, for all $u, \omega \in U$.	
Using remark (2.1) leads us to: $[g(u), u] d(u)=0$, for all $u \in U$.	
Left multiplication by r, we get:	
$[g(u), u] r d(u)=0$, for all $u \in U$ and $r \in \mathcal{R}$.	(6)
Putting ru instead of r in (6), we obtain: $[g(u), u] r ud(u)=0$, for all $u \in U$ and $r \in \mathcal{R}$.	(7)
Right multiplication of (6) by u, subtracting the relation (7) from the relation so obtained, we find: $[g(u), u] r [d(u), u]=0$, for all $u \in U$ and $r \in \mathcal{R}$.	
By the primeness of \mathcal{R} , we get the assertion of this result.	
Theorem (3.6) Let \mathcal{D} be a 2 torision free combining and α is an automorphisms of \mathbf{P} . If \mathcal{D} admitting a summatric \mathbf{P} definition	n T .
Let κ be a 2-ionsion nee semiprine ring and α is an automorphisms of κ . If κ admitting a symmetric Biderivation	п Ъ:
$\mathcal{K} \times \mathcal{K} \longrightarrow \mathcal{K}$ with traces 1 satisfies that $[1(x)x + x\alpha(x), x] = 0$, for all $x \in \mathcal{K}$, then \mathcal{K} contains a central ideal.	
Suppose for any $x \in \mathcal{R}$, we have:	

 $[f(x), x] x = x[\alpha(x), x].$

The linearization of (1), we find:

 $[f(y)x+y\alpha(y)+2\ {\boldsymbol{\mathcal F}}(x,\,y)y,\,x]+[f(x)y+2\ {\boldsymbol{\mathcal F}}(x,\,y)x+f(y)y+x\alpha(y)+y\alpha(x),\,x]+$

(1)

$ [f(x)x + x\alpha(x) + f(y)x + 2 \mathcal{F}(x, y)y, y] + [2 \mathcal{F}(x, y)x + f(x)y + x\alpha(y) + y\alpha(x), y] = 0, \text{ for all } x, y \in \mathcal{R}. $ Putting -y instead of y, combining the relation so obtained with the above relation, we see: $ [f(x)y + 2 \mathcal{F}(x, y)x + f(y)y + x\alpha(y) + y\alpha(x), x] + [f(x)x + x\alpha(x) + f(y)x + 2 \mathcal{F}(x, y)y, y] = 0, \text{ for all } x, y \in \mathcal{R}. $	
Also, the substitution 2y for y and comparing the new relation with the last one gives: $[f(x)y + 2 \mathcal{F}(x, y)x, x] + [x\alpha(y) + y\alpha(x), x] + [f(x)x + x\alpha(x), y] = 0$, for all x, y $\in \mathcal{R}$.	(2)
Replacing y by yx in (2) gives: $[f(x)y + 2 \mathcal{F}(x, y)x, x]x + 2 [y, x](f(x)x + x\alpha(x)) - [yx\alpha(x), x] + x[\alpha(y)\alpha(x), x] + [f(x)x + x\alpha(x), y]x=0$	
The above relation can be written because of (2) as: 2 [y, x]($f(x)x + x\alpha(x)$) - [$yx\alpha(x), x$] + x[$\alpha(y)\alpha(x), x$] - [$x\alpha(y) + y\alpha(x), x$]x=0, for all x,y $\in \mathbb{R}$.	
Using the commutator's identities, we get: $x[\alpha(y), x](\alpha(x)-x) + x\alpha(y)[\alpha(x), x] - y[\alpha(x), x]x + 2 [y, x] f(x)x - yx[\alpha(x), x]=0$, for all x, $y \in \mathcal{R}$.	(3)
Putting xy for y in (3) gives: $x[\alpha(x), x] \alpha(y) (\alpha(x)-x) + x\alpha(x)[\alpha(y), x](\alpha(x)-x) + x\alpha(x)\alpha(y) [\alpha(x), x] - xy[\alpha(x), x]x + 2x [y, x] f(x)x - xyx[\alpha(x), x]=0$, for all x, y $\in \mathcal{R}$.	(4)
Left multiplication of (3) by x, we see: $x^{2}[\alpha(y), x]\alpha(y)(\alpha(x)-x) + x^{2}\alpha(y)[\alpha(x), x] - xy[\alpha(x), x]x + 2x [y, x] f(x)x - xyx[\alpha(x), x] = 0$, for all x,y $\in \mathbb{R}$.	(5)
Subtracting (5) from (4), putting y for $\alpha(y)$, we find: $x[\alpha(x), x] y (\alpha(x)-x) + x(\alpha(x)-x)[y, x] (\alpha(x)-x) + x(\alpha(x)-x) y[\alpha(x), x]=0$, for all x,y $\in \mathcal{R}$.	
Using the commutator's identities, the above relation reduces to: - $x^2(\alpha(x)-x) y (\alpha(x)-x) + x(\alpha(x)-x) y (\alpha(x)-x) x=0$, for all x,y $\in \mathbb{R}$.	
Putting yx instead of y in the above relation, we see: - $x^2(\alpha(x)-x) y x(\alpha(x)-x) + x(\alpha(x)-x) y x(\alpha(x)-x) x=0$, for all x,y $\in \mathcal{R}$.	(6)
An application of Lemma (2.1) on (6) leads to: ($x(\alpha(x)-x)x - x^2(\alpha(x)-x)$) y $x(\alpha(x)-x) = 0$, for all x, y $\in \mathcal{R}$.	
Equivalently $x[\alpha(x)-x, x] y x(\alpha(x)-x) = 0$, for all $x, y \in \mathcal{R}$.	(7)
Now, in a relation (7), the substitution yx for y once and right multiplication by x in another implies that: x[α (x)- x, x] y x ² (α (x)-x) =0, for all x,y $\in \mathcal{R}$.	(8)
$x[\alpha(x)-x, x] y x(\alpha(x)-x) x=0$, for all $x, y \in \mathcal{R}$.	(9)
Subtracting the relation (8) from (9) yields that: $x[\alpha(x)-x, x] y x[\alpha(x)-x, x]=0$, for all $x, y \in \mathcal{R}$.	
Equivalently $x[\alpha(x), x] = 0$, for all x, $y \in \mathcal{R}$.	
Using the semiprimeness of \mathcal{R} , we have: x[$\alpha(x)$, x]=0, for all x,y $\in \mathcal{R}$.	(10)
Combining the relations (1) and (10) implies that: [f(x), x] $x = 0$, for all $x \in \mathcal{R}$.	(11)
Now, using similar techniques on the relation (11) as used to get (2) from (1) we arrive at: $[f(x), y] x + [f(x), x] y + 2[\mathcal{F}(x, y), x] x = 0$, for all $x, y \in \mathcal{R}$.	(12)
Replacing y by yx in (12), using the relation (12) and the commutator's identities, we get: $[f(x),y] x^2 + [f(x),x] yx + 2[\mathcal{F}(x, y), x]x^2 + 2[y, x] f(x)x + 2 y[f(x),x]x = 0$, for all x,y $\in \mathcal{R}$.	(13)
Right multiplication of (12) by x, then subtracting the relation so obtained from (13) leads because of (11) and the 2 free of R to:	-torisionity
$[y, x] f(x)x=0, \text{ for all } x, y \in \mathcal{R}.$	(14)
Putting zy for y in (14), using (14) implies that: $[z, x] y f(x)x=0$, for all x,y,z $\in \mathbb{R}$.	(15)
The linearization of the relation (15) with respect x gives: $[z, \omega] y f(\omega)\omega + [z, \omega] y f(x)\omega + 2[z, \omega] y \mathcal{F}(\omega, x)\omega + [z, \omega] y f(\omega)x + [z, \omega] y f(x)x + 2[z, \omega] y \mathcal{F}(\omega, x)x$	
$[z, x] y f(\boldsymbol{\omega})\boldsymbol{\omega} + [z, x] y f(x)\boldsymbol{\omega} + 2[z, x] y \mathcal{F}(\boldsymbol{\omega}, x)\boldsymbol{\omega} + [z, x] y f(\boldsymbol{\omega})x + [z, x] y f(x)x + 2[z, x] y \mathcal{F}(\boldsymbol{\omega}, x)x = 0$	
Replacing x by - x, combining the above relation with the new one, we get: $[z, \omega] y f(\omega)x + 2[z, \omega] y \mathcal{F}(\omega, x)x + [z, \omega] y f(x)x + [z, x] y f(\omega)\omega + [z, x] y f(x)\omega + 2[z, x] y \mathcal{F}(\omega, x)x=0$	
Putting 2_{ω} for $_{\omega}$, comparing the above relation with the relation so obtained, we arrive at:	
$[z, \boldsymbol{\omega}] \text{ y } f(x)x + [z, x] \text{ y } f(x)\boldsymbol{\omega} + 2[z, x] \text{ y } \mathcal{F}(\boldsymbol{\omega}, x)x = 0, \text{ for all } x, y, z, \boldsymbol{\omega} \in \mathcal{R}.$	
The substitution $f(x)x y [z, \omega]$ for y leads because of (14) to:	
$[z, \omega] f(x)x y [z, \omega] f(x)x = 0$, for all x,y,z, $\omega \in \mathcal{R}$.	

By the semiprimeness of \mathcal{R} , we have: [z, ω] f(x)x =0, for all x,z, $\omega \in \mathcal{R}$.	
An application of Lemma (2.7) on the above relation we conclude that $[z, \omega] f(x)=0$, for all $x, z, \omega \in \mathcal{R}$.	
According to Lemma (2.5), there exists an ideal U such that $f(x) \in U \subset Z(\mathcal{R})$.	
An immediate consequence of the above theorem, we have the following corollary: Corollary (3.7) Let \mathcal{D} be a 2 terrision free prime ring and σ is an automorphisms of \mathcal{D} . If \mathcal{D} admitting a symmetric Piderivation	n T .
Let \mathcal{R} be a 2-torision free prime ring and α is an automorphisms of \mathcal{R} . If \mathcal{R} admitting a symmetric Biderivation $\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with traces f satisfies that $[f(x)x + x\alpha(x), x] = 0$, for all $x \in \mathcal{R}$, then \mathcal{R} is commutative.	n j -:
Theorem (3.8) Let \mathcal{R} be a 2-torition free prime ring and U is an ideal of \mathcal{R} . If \mathcal{R} admitting a symmetric Biderivations \mathcal{F} . G: $\mathcal{R} \times \mathcal{R}$	$\rightarrow \mathcal{R}$
such that there traces f, g respectively satisfies that $f(x)x + x g(x) = 0$, for all $x, \in U$, then \mathcal{R} is commutative or both \mathcal{F} and \mathcal{G}	G are
zeros on \mathcal{R} .	
In view of our hypothesis, we have:	
$f(x)x+x g(x)=0$, for all $x \in U$.	(1)
The linearization of (1) and using (1), we see: $f(x)y+f(y)x+2 \mathcal{F}(x, y)x+2 \mathcal{F}(x, y)y + x g(y)+y g(x)+2xG(x, y)+2yG(x, y)=0$, for all x,y $\in U$.	
Putting 2x for x, then comparing the relation so obtained with the above relation, we get:	
$2 \mathcal{F}(x, y)x + f(x)y + 2xG(x, y) + y g(x) = 0, \text{ for all } x, y \in U.$	(2)
The substitution yx instead of y in (2), gives: $2\pi T(x) = \frac{2}{3} + 2\pi T(x) = \frac{2}{$	(2)
2 $\mathcal{F}(x, y)x + 2y f(x)x + f(x)yx + 2xG(x, y)x + 2xyg(x) + yx g(x) = 0$, for all x, y $\in \mathbb{U}$.	(3)
2 $\mathcal{F}(x, y)x^2 + f(x)yx + 2xG(x, y)x + y g(x)x = 0$, for all x,y $\in U$.	(4)
Subtracting the above relation from (3) gives: $2y f(x)x + 2xyg(x) + yx g(x) - y g(x)x = 0$, for all x, y $\in U$.	(5)
According to (1), the last relation reduces to: $yf(x)yx + 2xyg(x) - yf(x)x=0$, for all x,y $\in U$.	(6)
Now, by adding \pm yxg(x) and using (1), the relation (6) can be written as: [y, x] g(x)+ [yg(x), x]=0, for all x,y \in U.	
That is	
$2[y, x] g(x) + y[g(x), x] = 0$, for all x, $y \in U$.	(7)
Replacing y by zy in above relation implies that: $2z[y, x] g(x) + 2[z, x] y g(x) + zy[g(x), x]=0$, for all x,y,z $\in U$.	
The above relation reduces because of (7) and the 2-toritionity free of \mathcal{R} to: [<i>z</i> , <i>x</i>] y g(x)=0, for all x,y,z \in U.	(8)
The linearization of the relation (8) with respect to x gives: $[z, \alpha] \neq g(x) + 2[z, \alpha] \neq G(\alpha, x) + [z, x] \neq g(\alpha) + 2[z, x] \neq G(\alpha, x) = 0$, for all x, y, α , $z \in U$.	
Putting 2_{0} for $_{0}$, comparing the above relation with the relation so obtained, we arrive at:	
$[z, \omega] \neq g(x) + 2[z, x] \neq G(\omega, x)=0$, for all x,y,z, $\omega \in U$.	
The substitution $yg(x)r$ for y leads because of (8) to: $[\pi_{x} + h] = g(x) - 0$ for all $x + y - y = 0$. For all $x + y - y = 0$.	
$[z, \omega]$ y $g(x)$ f $g(x) = 0$, for all x,y,z,u, $\omega \in 0$ and f $\in \mathcal{R}$	
Again, replace r by $r[z, \omega]$ in the last relation, we find: [z = v] u g(x) = [z = v] u g(x) = 0 for all x u z u v c L and z c .	
Using the primeness of \mathcal{P} , we find:	
$[z, \omega]$ y g(x) =0, for all x,y,z, $\omega \in U$.	
An application of Remark (2.1), either $[z, \omega]=0$ or $g(x)=0$, for all $x, z, \omega \in U$. If $[z, \omega]=0$, for all $\omega, z \in U$ then U is commutative commutation.	ative.
Consequently, \mathcal{R} is commutative by Lemma (2.3). Otherwise $g(x)=0$, for all $x \in U$.	(9)
The linearization of the relation (9) gives: $g(x)+g(u)+2G(x, u)=0$, for all x,u \in U.	
In view of (9) and 2-torition free of R, the above becomes: G(x, u)=0, for all $x, u \in U$.	(10)
Hence G is zeros on \mathcal{R} by Lemma (2.6). On the other hand, according to (9), the relation (1) reduces to:	. *

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$f(x)x=0$, for all x, $\in U$.		(11)
The linearization (11), using	g (11) implies that:	
$f(x)_{\omega} + f(\omega)x + 2 \mathcal{F}(x, \omega)x$	+ 2 $\mathcal{F}(x, \omega) \omega = 0$, for all x, $\in U$.	(12)
Putting - ω for ω in (12), the second s	hen comparing the relation so obtained with (12), we find:	
$f(\boldsymbol{\omega})\mathbf{x} + 2 \boldsymbol{\mathcal{F}}(\mathbf{x}, \boldsymbol{\omega})\boldsymbol{\omega} = 0, \text{ for } \mathbf{x}$	or all x, _ω ∈U.	(13)
Replacing x by _w x in (13)	leads because of (11) to:	
$f(\boldsymbol{\omega})\mathbf{x}\boldsymbol{\omega} + 2\boldsymbol{\omega} \boldsymbol{\mathcal{F}}(\mathbf{x}, \boldsymbol{\omega})\boldsymbol{\omega} = 0$	0, for all x, ω ∈ U.	(14)
Left multiplication of (13) b	by w gives:	
$\omega f(\omega)x + 2\omega \mathcal{F}(x, \omega)\omega =$	=0, for all $x, \omega \in U$.	(15)
Subtracting the relation (14)) from (15) leads to:	
$\omega f(\omega)x - f(\omega)x_{\omega} = 0$, for a	ll x,ω ∈U.	
Replacing x by $x_{\omega}d(\omega)$ in	the last relation implies because of (11) that:	
ω f(ω)x ω f(ω)=0, for all	x, ω ∈ U.	
Using the primeness of \mathcal{R} , w	we find:	
$ω$ f($ω$)=0, for all $ω \in U$.		(16)
Now, right multiplication of	f (13) by f(ω yields because of (16) that:	
$f(\omega) \ge f(\omega) = 0$, for all x, ω	€U.	(17)
By remark (2.1), we find:		
$f(\boldsymbol{\omega})=0$, for all $\boldsymbol{\omega} \in U$.		(18)
Using similar arguments on $\mathcal{F}(\boldsymbol{\omega}, \mathbf{u})=0$, for all $\boldsymbol{\omega}, \mathbf{u} \in \mathbf{U}$	(18) as used to get (10) from (9), we arrive at:	
Therefore \mathcal{F} is zeros on \mathcal{R} b	oy Lemma (2.6).	•
Theorem (3.9)		
Let \mathcal{R} be a 2-torition	free prime ring and U is an ideal of \mathcal{R} . The existence of no	onzero symmetric Biderivations \mathcal{F} , G:
$\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ such that then	te traces f, g respectively satisfies that $f(x)x - x g(x)=0$ holds for	all $x \in U$ forces \mathcal{R} to be commutative.
Proof:		
If $\mathcal{F} = G$, then g is commuti	ng on U , consequently, \mathcal{R} is commutative by theorem (3.1). So	we assume that $\mathcal{F} \neq G$. In view of our
hypothesis, we have: $f(x) = x g(x)$ for all $x \in I$		(1)
$I(x)x = x g(x)$, for all $x \in U$	J.	(1)
Using similar arguments as 2y f(x)x - 2xyg(x) + yx g(x)	used to get (5) from (1), we find: $f_{1} = y g(x)x$ for all $x y \in U$	
Equivalently		
2y f(x)x = 2xyg(x) + y [x, g]	(x) , for all $x, y \in U$.	(2)
Replacing y by $g(x)y$ in (2)	gives:	
2 g(x)y f(x)x = 2xg(x)yg(x)	$+ g(x)y[x, g(x)], \text{ for all } x, y \in U.$	(3)
Left multiplication of (2) by	y $g(x)$ and subtracting the relation so obtained from the relation	n (3), we arrive because of 2-toritionity
free of \mathcal{R} to:		
[x, g(x)] y g(x) = 0, for all x	$y \in U$	(4)
The substitution yx for y in $[x, g(x)] y xg(x) = 0$, for all	(4), we find: x,y $\in U$.	(5)
Now, Right multiplication of $[x, g(x)] y [x, g(x)] = 0$, for	of (4) by x, then subtracting the relation so obtained from (5) im all $x, y \in U$.	plies that:
According to remark (2.1),	we get:	
$[x, g(x)] = 0, \text{ for all } x \in U.$		
Hence \mathcal{R} is commutative by Reference	theorem (3.1). \blacksquare	

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