

# On a Symmetric Biderivations and Automorphismity of Prime and Semiprime Rings 

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#### Abstract

Our aim in this paper is to investigate the commuting property of the Traces of a Symmetric Biderivations on prime and semiprime ring $\mathcal{R}$. Furthermore, we look for the commutitivity of a ring $\mathcal{R}$ under some suitable conditions.


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## Keywords

Prime ring,
Semiprime ring,
Symmetric Biderivations,
Trace of mapping,
Commuting mappings.

## 1.Introduction

Throughout, $\mathcal{R}$ will represent an associative ring with the center $\mathrm{Z}(\mathcal{R})$. A ring $\mathcal{R}$ satisfies that $u \mathcal{R} v=(0)$ leads either $u=0$ or $\mathrm{v}=0$ is said to be prime ring, while $\mathcal{R}$ is semiprime if whenever u $\mathcal{R} \mathrm{u}=(0)$ implies $\mathrm{u}=0$. Moreover, $\mathcal{R}$ has n -torsion free property if $n u=0, u \in \mathcal{R}$ implies that $u=0$ (see [1]). Recall that in the prime ring $\mathcal{R}$ the statement $\mathcal{R}$ is $n$-torsion free ring is equivalent to $\mathcal{R}$ of characteristic different from $n$ [2]. As usual, the symbol [ $u, v]$ is refer to the commutator $u v-v u$. We shall be frequently using the basic commutator identities $[\mathrm{u} \omega, \mathrm{v}]=[\mathrm{u}, \mathrm{v}] \omega+\mathrm{u}[\omega, \mathrm{v}],[\mathrm{u}, \mathrm{v} \omega]=[\mathrm{u}, \mathrm{v}] \omega+\mathrm{v}[\mathrm{u}, \omega]$ (see [3]). A biadditive mapping $\mathcal{B}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is called Symmetric if $\mathcal{B}(u, v)=\mathcal{B}(v, u)$ for all pairs $u, v \in \mathcal{R}$ (see [4]). A mapping $\varphi: \mathcal{R} \longrightarrow \mathcal{R}$ defined by $\varphi(\mathrm{u})=\mathcal{B}(\mathrm{u}, \mathrm{u})$, where $\mathcal{B}(.,):. \mathcal{R} \times \mathcal{R} \rightarrow \mathcal{R}$ is a Symmetric mapping will be called the Trace of $\mathcal{B}$ (see [5]). It is obvious that in case the Symmetric mapping $\mathcal{B}(.,):. \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is also biadditive, the Trace of $\mathcal{B}$ satisfies $\varphi(\mathrm{u}+\mathrm{v})=\varphi(\mathrm{u})+2 \mathcal{B}(\mathrm{u}, \mathrm{v})+\varphi(\mathrm{v})$. A mapping $\mathcal{L}: \mathcal{R} \rightarrow \mathcal{R}$ is said to be Centralizing on $\mathcal{R}$ if $[\mathcal{L}(\mathrm{u}), \mathrm{u}] \in \mathrm{Z}(\mathrm{R})$, for all $\mathrm{x} \in \mathcal{R}$. In special case when $[\mathcal{L}(\mathrm{u}), \mathrm{u}]=0$, for all $\mathrm{x} \in \mathcal{R}$, the mapping $\mathcal{L}$ is called Commuting on $\mathcal{R}$ (see [6]). The centralizing and commuting mapping have been studied in many papers. It seems that the first result in this field was given by J. Vokman [7]. He prove that if $d$ is a derivation of a prime ring $\mathcal{R}$ of characteristic different from 2, such that the mapping $\mathcal{P}(u)=\quad[d(u), u]$ then $\mathcal{P}=0$, that is $d$ is commuting. In [8] Bres̃ar generalized this result by showing that the same conclusion holds for any additive mapping. In [9] Brešar describe all commuting traces of biadditive mapping on certain prime rings. He prove that if the Characteristic of R different from 2 , and $\mathcal{R}$ does not satisfy $\mathrm{S}_{4}$, then every such mapping (say $\mathcal{P}$ ) is of the form $\mathcal{P}(\mathrm{u})=\lambda \mathrm{u}^{2}+\mu(\mathrm{u}) \mathrm{u}+v(\mathrm{u})$ for all $\mathrm{x} \in \mathcal{R}$, and $\boldsymbol{\lambda} \in \mathcal{C}$ where $\mathcal{C}$ is the extended centroid of $\mathcal{R}$ (the center of the Martindale ring of quotient of $\mathcal{R}$ ) and $\mu: \mathcal{R} \longrightarrow \mathcal{C}$ is an additive mapping.

A biadditive mapping $\mathcal{D}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is called a Symmetric Biderivation if $\mathcal{D}(u \omega, y)=\mathcal{D}(u, v) \omega+u^{\prime} \mathcal{D}(\omega$, v) is fulfilled for all $u, v, \omega \in \mathcal{R}$ [11]. On the other hand $N$. Argac in 2006 introduce the concept of Symmetric generalized Biderivation as follows: A symmetric biadditive mapping $\quad Q: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ is called a Symmetric generalized Biderivation if there exist a symmetric Biderivation $\mathcal{D}$ such that $Q(u \omega, y)=Q(u, v) \omega+u \mathcal{D}(\omega, v)$ is fulfilled for all $u, v, \omega \in \mathcal{R}$ (see [12).

The purpose of the present paper is looking for the commuting property of the Traces of a Symmetric Biderivations on prime and semiprime rings. Also, we present some results concerning with the commutitivity of rings.

## 2. Preliminary results

We facilitate our study with following known results which are necessary for developing the proof of our results.

## Remarks (2.1): [13]

Let $\mathcal{R}$ be a prime ring, $\mathcal{J}$ a nonzero ideal of $\mathcal{R}$. If a $\mathcal{J} \mathrm{b}=0$, for $\mathrm{a} \in \mathcal{R}$, it's easy to verify that either $\mathrm{a}=0$ or $\mathrm{b}=0$.
Lemma (2.2): [14]
Let $\mathcal{R}$ be a 2 -torision free prime ring and $\mathcal{J}$ be a nonzero ideal of $\mathcal{R}$. If $\mathcal{D}$ is a symmetric Biderivation such that $\mathcal{D}(\mathrm{x}, \mathrm{x})=0$, all $\mathrm{x} \in \mathcal{J}$.then either $\mathcal{D}=0$ or $\mathcal{R}$ is commutative.

Lemma (2.3): [15]
Let $\mathcal{R}$ be a semiprime ring, $\mathcal{J}$ an ideal of $\mathcal{R}$. If $\mathcal{J}$ is a commutative as a ring, then $\mathcal{J} \subset Z(\mathcal{R})$. In addition if $\mathcal{R}$ is prime then $\mathcal{R}$ must be commutative.
Lemma (2.4): [16]
Let $\mathcal{R}$ be a semiprime ring, suppose that arb $+b r c=0$ holds for $a l l ~ r \in \mathcal{R}$ and some $a, b, c \in \mathcal{R}$. In this case $(a+c) r b=0$ is satisfied for all $r \in \mathcal{R}$.
Lemma (2.5): [17]
Let $\mathcal{R}$ be a semiprime ring suppose that there exissts $\mathrm{a} \in \mathcal{R}$ such that $\mathrm{a}[\mathrm{x}, \mathrm{y}]=0$ holds for all pairs $\mathrm{x}, \mathrm{y} \in \mathcal{R}$, then there exists an ideal U of $\mathcal{R}$ such that a $\in U \subset Z(\mathcal{R})$.
Lemma (2.6): [18]
Let $\mathcal{R}$ be a prime ring, and $\mathcal{J}$ be a nonzero left ideal of $\mathcal{R}$. If a $(\sigma, \boldsymbol{\tau})$-Biderivation $\mathcal{D}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ satisfies that $\mathcal{D}(\mathcal{J}, \mathcal{J})=0$, then $\mathcal{D}=0$.
Lemma (2.7): [19]
Let $\mathcal{d}$ be aderivation of a prime ring $\mathcal{R}$ and $\mathcal{J}$ a nonzero ideal of $\mathcal{R}$. For an element a of $\mathcal{R}$, if either (i) a $\mathcal{d}(\mathrm{x})=0$ for all $\mathrm{x} \in \mathcal{J}$ or (ii) $d(\mathrm{x}) \mathrm{a}=0$ for all $\mathrm{x} \in \mathcal{J}$. Then either $\mathrm{a}=0$ or $\mathbb{d}=0$.
Theorem (3.1):
Let $\mathcal{R}$ be a 2-torition free prime ring and U an ideal of $\mathcal{R}$. The existence of nonzero symmetric bi-derivation $\mathcal{F}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with commuting trace f on U , forces $\mathcal{R}$ to be commutative.
Proof:
Suppose f is commuting on U , that is:
$[f(u), u]=0$, for all $u \in U$.
The linearization of (1), using (1) gives:
$[\mathrm{f}(\mathrm{u}), \omega]+[\mathrm{f}(\omega), \mathrm{u}]+2[\mathcal{F}(\mathrm{u}, \omega), \omega]+2[\mathcal{F}(\mathrm{u}, \omega), \mathrm{u}]=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
Putting $2 \omega$ instead of $\omega$, comparing the above relation with the new one, we find:
$[f(u), \omega]+2[F(u, \omega), u]=0$, for all $u, \omega \in U$.
The substitution $\omega u$ for $\omega$ in (2), we get:
$[\mathrm{f}(\mathrm{u}), \omega] \mathrm{u}+\omega[\mathrm{f}(\mathrm{u}), \mathrm{u}]+2[\mathcal{F}(\mathrm{u}, \omega), \mathrm{u}] \mathrm{u}+2 \omega[\mathrm{f}(\mathrm{u}), \mathrm{u}]+2[\omega, \mathrm{u}] \mathrm{f}(\mathrm{u})=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
In view of relations (1), (2) and 2-toritionity free of $\mathcal{R}$, the last relation reduces to:
$[\omega, u] f(u)=0$, for all $u, \omega \in U$.
Let us write $z \omega$ for $\omega$ in (3), using (3) implies that:
$[\mathrm{z}, \mathrm{u}] \omega \mathrm{f}(\mathrm{u})=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
The linearization of (4) with respect to $u$, using (4) leads to:
$[\mathrm{z}, \mathrm{u}] \omega \mathrm{f}(\mathrm{v})+[\mathrm{z}, \mathrm{v}] \omega \mathrm{f}(\mathrm{u})+2[\mathrm{z}, \mathrm{u}] \omega \mathcal{F}(\mathrm{u}, \mathrm{v})+2[\mathrm{z}, \mathrm{v}] \omega \mathcal{F}(\mathrm{u}, \mathrm{v})=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
Replacing $u$ by -u , combining the relation so obtained with (5), we get because of the 2-toritionity free of $\mathcal{R}$ that:
$[\mathrm{z}, \mathrm{v}] \omega \mathrm{f}(\mathrm{u})+2[\mathrm{z}, \mathrm{u}] \omega \mathcal{F}(\mathrm{u}, \mathrm{v})=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
The substitution $f(u) \omega[z, v]$ for $\omega$ leads because of (3) to:
$[z, v] f(u) \omega[z, v] f(u)=0$, for all $u, \omega \in U$.
By remarks (2.1), we have:
$[\mathrm{z}, \mathrm{v}] \mathrm{f}(\mathrm{u})=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
Substituting $v \omega$ for $v$ in (6), we see:
$[\mathrm{z}, \mathrm{v}] \omega \mathrm{f}(\mathrm{u})=0$, for all $\mathrm{u}, \mathrm{v}, \omega \in \mathrm{U}$.
Using remarks (2.1) implies that either $f(u)=0$ or $[z, v]=0$ for all $u, v, z \in U$. If $f(u)=0$ for all $u \in U$, then $\mathcal{R}$ is commutative by lemma (2.2) (note that $\mathcal{F}$ is a nonzero mapping). Otherwise, $[\mathrm{z}, \mathrm{v}]=0$, for all $\mathrm{u}, \mathrm{v} \in \mathrm{U}$, this means that U is commutative ideal of $\mathcal{R}$.
Consequently, $\mathcal{R}$ is commutative by lemma (2.3).

## Theorem (3.2)

Let $\mathcal{R}$ be a 2 -torition free semiprime ring and $\alpha$ is an automorphisms of $\mathcal{R}$. If $\mathcal{R}$ admitting a symmetric Biderivation $\mathcal{F}$ : $\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with traces f satisfies that $[\mathrm{f}(\mathrm{x})+\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$ then f is commuting on $\mathcal{R}$.
Proof:
Suppose for any $\mathrm{x} \in \mathcal{R}$, we have:
$[\mathrm{f}(\mathrm{x})+\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$.
The linearization of (1) implies that:
$2[\mathcal{F}(\mathrm{x}, \mathrm{y}), \mathrm{x}]+[\mathrm{f}(\mathrm{y}), \mathrm{x}]+[\alpha(\mathrm{y}), \mathrm{x}]+2[\mathcal{F}(\mathrm{x}, \mathrm{y}), \mathrm{y}]+[\mathrm{f}(\mathrm{x}), \mathrm{y}]+[\alpha(\mathrm{x}), \mathrm{y}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Replacing x by -x , then comparing the above relation with the relation so obtained, we get:
$[\mathrm{f}(\mathrm{y}), \mathrm{x}]+[\alpha(\mathrm{y}), \mathrm{x}]+2[\mathcal{F}(\mathrm{x}, \mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{x}), \mathrm{y}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Putting $x y$ instead of $x$ in (2) yields:
$[\mathrm{f}(\mathrm{y}), \mathrm{x}] \mathrm{y}+\mathrm{x}[\mathrm{f}(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{y}), \mathrm{x}] \mathrm{y}+\mathrm{x}[\alpha(\mathrm{y}), \mathrm{y}]+2[\mathcal{F}(\mathrm{x}, \mathrm{y}), \mathrm{y}] \mathrm{y}+2[\mathrm{x}, \mathrm{y}] \mathrm{f}(\mathrm{y})+2 \mathrm{x}[\mathrm{f}(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{x}), \mathrm{y}] \alpha(\mathrm{y})+\alpha(\mathrm{x})[\alpha(\mathrm{y}), \mathrm{y}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Right multiplication of (2) by $y$, the subtracting the relation so obtained from above relation, we see:
$\mathrm{x}[\mathrm{f}(\mathrm{y}), \mathrm{y}]+\mathrm{x}[\alpha(\mathrm{y}), \mathrm{y}]+2[\mathrm{x}, \mathrm{y}] \mathrm{f}(\mathrm{y})+2 \mathrm{x}[\mathrm{f}(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{x}), \mathrm{y}] \alpha(\mathrm{y})+\alpha(\mathrm{x})[\alpha(\mathrm{y}), \mathrm{y}]-[\alpha(\mathrm{x}), \mathrm{y}] \mathrm{y}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Furthermore, the above relation reduces because of (1) to:
$2[\mathrm{x}, \mathrm{y}] \mathrm{f}(\mathrm{y})-2 \mathrm{x}[\alpha(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{x}), \mathrm{y}] \alpha(\mathrm{y})+\alpha(\mathrm{x})[\alpha(\mathrm{y}), \mathrm{y}]-[\alpha(\mathrm{x}), \mathrm{y}] \mathrm{y}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
That is
$\{\alpha(\mathrm{x})-2 \mathrm{x}\}[\alpha(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{x}), \mathrm{y}]\{\alpha(\mathrm{y})-\mathrm{y}\}+2[\mathrm{x}, \mathrm{y}] \mathrm{f}(\mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Putting yx for x in (3) leads to:
$\mathrm{y}\{\alpha(\mathrm{x})-2 \mathrm{x}\}[\alpha(\mathrm{y}), \mathrm{y}]+\{\alpha(\mathrm{y})-\mathrm{y}\} \alpha(\mathrm{x})[\alpha(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{y}), \mathrm{y}] \alpha(\mathrm{x})\{\alpha(\mathrm{y})-\mathrm{y}\}+\alpha(\mathrm{y})[\alpha(\mathrm{x}), \mathrm{y}]\{\alpha(\mathrm{y})-\mathrm{y}\}+2 \mathrm{y}[\mathrm{x}, \mathrm{y}] \mathrm{G}(\mathrm{y}, \mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y}$ $\in \mathcal{R}$.
Left multiplying of (3) by y , we get:
$\{\alpha(\mathrm{x})-2 \mathrm{x}\}[\alpha(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{x}), \mathrm{y}]\{\alpha(\mathrm{y})-\mathrm{y}\}+2[\mathrm{x}, \mathrm{y}] \mathrm{G}(\mathrm{y}, \mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Subtracting the last relation from (4) leads to:
$\{\alpha(\mathrm{y})-\mathrm{y}\} \alpha(\mathrm{x})[\alpha(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{y}), \mathrm{y}] \alpha(\mathrm{x})\{\alpha(\mathrm{y})-\mathrm{y}\}+\alpha(\mathrm{y})[\alpha(\mathrm{x}), \mathrm{y}]\{\alpha(\mathrm{y})-\mathrm{y}\}-\mathrm{y}[\alpha(\mathrm{x}), \mathrm{y}]\{\alpha(\mathrm{y})-\mathrm{y}\}=0$,
That is
$\{\alpha(\mathrm{y})-\mathrm{y}\} \alpha(\mathrm{x})[\alpha(\mathrm{y}), \mathrm{y}]+[\alpha(\mathrm{y}), \mathrm{y}] \alpha(\mathrm{x})\{\alpha(\mathrm{y})-\mathrm{y}\}+\{\alpha(\mathrm{y})-\mathrm{y}\}[\alpha(\mathrm{x}), \mathrm{y}]\{\alpha(\mathrm{y})-\mathrm{y}\}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Now, define $\mu(\mathrm{y})=\alpha(\mathrm{y})-\mathrm{y}$, then
$[\alpha(\mathrm{y}), \mathrm{y}]=[\alpha(\mathrm{y})-\mathrm{y}, \mathrm{y}]=[\mu(\mathrm{y}), \mathrm{y}]$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
According to (6), the relation (5) can be written as:
$\mu(\mathrm{y}) \alpha(\mathrm{x})[\mu(\mathrm{y}), \mathrm{y}]+[\mu(\mathrm{y}), \mathrm{y}] \alpha(\mathrm{x}) \mu(\mathrm{y})+\mu(\mathrm{y})[\alpha(\mathrm{x}), \mathrm{y}] \mu(\mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Equivalently
$\mu(\mathrm{y}) \alpha(\mathrm{x}) \mu(\mathrm{y}) \mathrm{y}-\mu(\mathrm{y}) \alpha(\mathrm{x}) \mathrm{y} \mu(\mathrm{y})+\mu(\mathrm{y}) \mathrm{y} \alpha(\mathrm{x}) \mu(\mathrm{y})-\mathrm{y} \mu(\mathrm{y}) \alpha(\mathrm{x}) \mu(\mathrm{y})+\mu(\mathrm{y}) \alpha(\mathrm{x}) \mathrm{y} \mu(\mathrm{y})-\mu(\mathrm{y}) \mathrm{y} \alpha(\mathrm{x}) \mu(\mathrm{y})=0$.
That is
$-\mathrm{y} \mu(\mathrm{y}) \alpha(\mathrm{x}) \mu(\mathrm{y})+\mu(\mathrm{y}) \alpha(\mathrm{x}) \mu(\mathrm{y}) \mathrm{y}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Setting $\mathrm{a}=-\mathrm{y} \mu(\mathrm{y}), \mathrm{b}=\mu(\mathrm{y})$ and $\mathrm{c}=\mu(\mathrm{y}) \mathrm{y}$, then an application of Lemma (2.4) on above relation, we find:
$(\mu(\mathrm{y}) \mathrm{y}-\mathrm{y} \mu(\mathrm{y})) \alpha(\mathrm{x}) \mu(\mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
That is
$[\mu(\mathrm{y}), \mathrm{y}] \alpha(\mathrm{x}) \mu(\mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Putting $\alpha(\mathrm{x}) \mathrm{y}$ instead of $\alpha(\mathrm{x})$ in (8) yields:
$[\mu(\mathrm{y}), \mathrm{y}] \alpha(\mathrm{x}) \mathrm{y} \mu(\mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Right multiplication of (7) by $y$, then subtracting relation (8) from the relation so obtained, we find:
$[\mu(\mathrm{y}), \mathrm{y}] \alpha(\mathrm{x})[\mu(\mathrm{y}), \mathrm{y}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Since is an automorphisms on $\mathcal{R}$, then the semiprimeness of $\mathcal{R}$ leads to:
$[\mu(\mathrm{y}), \mathrm{y}]=0$, for all $\mathrm{y} \in \mathcal{R}$.
This means because of (6) that:
$[\alpha(\mathrm{y}), \mathrm{y}]=0$, for all $\mathrm{y} \in \mathcal{R}$.
An application of (10) on the relation (1), we arrive at:
$[\mathrm{f}(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$.
In case $\mathcal{R}$ is prime ring, we have because of theorem (3.1) the following corollary.
Corollary (3.3)
Let $\mathcal{R}$ be a 2 -torition free prime ring and $\alpha$ is antomorphisms of $\mathcal{R}$. If $\mathcal{R}$ admitting a symmetric Biderivation $\mathcal{F}$ : $\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with traces f satisfies that $[\mathrm{f}(\mathrm{x})+\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$, then $\mathcal{R}$ is commutative.

## Corollary (3.4)

Let $\mathcal{R}$ be a non-commutative prime ring of characteristic different from 2 and $\alpha$ is antomorphisms of R . If $\mathcal{F}: \mathcal{R} \times \mathcal{R} \longrightarrow$ $\mathcal{R}$ is a symmetric Biderivation with traces f satisfies that $[\mathrm{f}(\mathrm{x})+\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$. In this case $\mathcal{F}$ is zero on $\mathcal{R}$.
Proof:
According to theorem (3.2), we have:
$[\mathrm{f}(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$.
Now, using similar techniques as used in theorem (3.1) to get (3) from (1), we arrive at:
$[x, y] f(x)=0$, for all $x, y \in \mathcal{R}$.
An application of lemma (2.7) on (3) one can conclude that for any $x \notin Z(\mathcal{R})$, we have $f(x)=0$ (note that for any fixed $x \in \mathcal{R}$ the additive mapping $f_{x}(y)=[x, y]$ is a derivation $)$.

Now, if $\mathrm{x} \in \mathrm{Z}(\mathcal{R})$, then $\mathrm{x}+\mathrm{y} \notin \mathrm{Z}(\mathcal{R})$ and $\mathrm{x}-\mathrm{y} \notin \mathrm{Z}(\mathcal{R})$ for any $\mathrm{y} \notin \mathrm{Z}(\mathcal{R})$, so we have:
$0=\mathrm{f}(\mathrm{x}+\mathrm{y})=\mathrm{f}(\mathrm{x})+2 \mathcal{F}(\mathrm{x}, \mathrm{y})$
$0=\mathrm{f}(\mathrm{x}-\mathrm{y})=\mathrm{f}(\mathrm{x})-2 \mathcal{F}(\mathrm{x}, \mathrm{y})$
Combining the relations (3) and (4) leads because of characteristic different from 2 of $\mathcal{R}$ that $f(x)=0$. Therefore we have proved that:
$\mathrm{f}(\mathrm{x})=0$, for all $\mathrm{x} \in \mathcal{R}$.
Hence $\mathcal{F}=0$ by lemma (2.2).

## Theorem (3.5)

Let $\mathcal{R}$ be a 2-torition free prime ring and U a nonzero ideal of $\mathcal{R}$. If G : $\times \mathcal{R} \longrightarrow \mathcal{R}$ is a symmetric generalized Biderivation associated with Biderivation $\mathcal{D}$ satisfies that the Trace of $G$ is centralizing on U , then either G or $\mathcal{D}$ has commuting Traces on U .
Proof: Suppose that:
$[\mathrm{g}(\mathrm{u}), \mathrm{u})] \in \mathrm{Z}(\mathcal{R})$, for all $u \in \mathrm{U}$.
By linearization of (1), we obtain that:
$[g(u), \omega]+[g(\omega), u]+2[G(u, \omega), \omega]+2[G(u, \omega), u] \in Z(R)$, for all $u, \omega \in U$.
Putting - $u$ instead of $u$, comparing the above relation with the new one, we find:
$[g(u), \omega]+2[G(u, \omega), u] \in Z(\mathcal{R})$, for all $u, \omega \in U$.
The substitution $\omega u$ for $\omega$ in (2), we get:
$([g(u), \omega] u+2[G(u, \omega), u]) u+2[\omega, u] d(u)+\omega[g(u), u]+2 \omega[d(u), u] \in Z(R)$, for all $u, \omega \in U$.
Therefore we have:
$[([g(u), \omega] u+2[G(u, \omega), u]) u+2[\omega, u] d(u)+\omega[g(u), u]+2 \omega[d(u), u], u]=0$, for all $u, \omega \in U$.
The above relation can be written because of (1) and (2) as:
$[\omega, \mathrm{u}][\mathrm{g}(\mathrm{u}), \mathrm{u}]+2[[\omega, \mathrm{u}], \mathrm{u}] \mathrm{d}(\mathrm{u})+4[\omega, \mathrm{u}][\mathrm{d}(\mathrm{u}), \mathrm{u}]+2 \omega[[\mathrm{~d}(\mathrm{u}), \mathrm{u}], \mathrm{u}]=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
The substitution $g(u) \omega$ for $\omega$ in (3)
$[g(u), u] \omega[g(u), u]+g(u)[\omega, u][g(u), u]+4[g(u), u][\omega, u] d(u)+2 g(u)[[\omega, u], u] d(u)+4[g(u), u] \omega$
$[d(u), u]+4 g(u)[\omega, u][d(u), u]+2 g(u) \omega[[d(u), u], u]=0$, for all $u, \omega \in U$.
According to (3), the above relation reduces to:
$[g(u), u] \omega[g(u), u]+4[g(u), u] \omega[d(u), u]+4[g(u), u][\omega, u] d(u)=0$, for all $u, \omega \in U$.
In view of (1), the last relation can be given as:
$\omega[g(u), u]^{2}+4 \omega[g(u), u][d(u), u]+4[g(u), u][\omega, u] d(u)=0$, for all $u, \omega \in U$.
Left multiplication of (4) by $g(u)$ yields that:
$\mathrm{g}(\mathrm{u}) \omega[\mathrm{g}(\mathrm{u}), \mathrm{u}]^{2}+4 \mathrm{~g}(\mathrm{u}) \omega[\mathrm{g}(\mathrm{u}), \mathrm{u}][\mathrm{d}(\mathrm{u}), \mathrm{u}]+4 \mathrm{~g}(\mathrm{u})[\mathrm{g}(\mathrm{u}), \mathrm{u}][\omega, \mathrm{u}] \mathrm{d}(\mathrm{u})=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
Again, replacing $\omega$ by $g(u) \omega$ in (4), we get because of (1) that:
$\mathrm{g}(\mathrm{u}) \omega[\mathrm{g}(\mathrm{u}), \mathrm{u}]^{2}+4 \mathrm{~g}(\mathrm{u}) \omega[\mathrm{g}(\mathrm{u}), \mathrm{u}][\mathrm{d}(\mathrm{u}), \mathrm{u}]+4 \mathrm{~g}(\mathrm{u})[\mathrm{g}(\mathrm{u}), \mathrm{u}][\omega, \mathrm{u}] \mathrm{d}(\mathrm{u})+4[\mathrm{~g}(\mathrm{u}), \mathrm{u}] \omega[\mathrm{g}(\mathrm{u}), \mathrm{u}] \mathrm{d}(\mathrm{u})=0$.
By subtracting the relation (5) from the above relation leads because of 2-torisionity free of $\mathcal{R}$ to:
$[g(u), u] \omega[g(u), u] d(u)=0$, for all $u, \omega \in U$.
Left multiplication of the last relation by $d(u)$, we see:
$[\mathrm{g}(\mathrm{u}), \mathrm{u}] \mathrm{d}(\mathrm{u}) \omega[\mathrm{g}(\mathrm{u}), \mathrm{u}] \mathrm{d}(\mathrm{u})=0$, for all $\mathrm{u}, \omega \in \mathrm{U}$.
Using remark (2.1) leads us to:
$[g(u), u] d(u)=0$, for all $u \in U$.
Left multiplication by r , we get:
$[g(u), u] r d(u)=0$, for all $u \in U$ and $r \in \mathcal{R}$.
Putting ru instead of $r$ in (6), we obtain:
$[\mathrm{g}(\mathrm{u}), \mathrm{u}] \mathrm{r} u d(\mathrm{u})=0$, for all $\mathrm{u} \in \mathrm{U}$ and $\mathrm{r} \in \mathcal{R}$.
Right multiplication of (6) by $u$, subtracting the relation (7) from the relation so obtained, we find:
$[g(u), u] r[d(u), u]=0$, for all $u \in U$ and $r \in \mathcal{R}$.
By the primeness of $\mathcal{R}$, we get the assertion of this result.
Theorem (3.6)
Let $\mathcal{R}$ be a 2-torision free semiprime ring and $\alpha$ is antomorphisms of R . If $\mathcal{R}$ admitting a symmetric Biderivation $\mathcal{F}$ :
$\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with traces f satisfies that $[\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$, then $\mathcal{R}$ contains a central ideal.
Proof:
Suppose for any $\mathrm{x} \in \mathcal{R}$, we have:
$[\mathrm{f}(\mathrm{x}), \mathrm{x}] \mathrm{x}=\mathrm{x}[\alpha(\mathrm{x}), \mathrm{x}]$.
The linearization of (1), we find:
$[\mathrm{f}(\mathrm{y}) \mathrm{x}+\mathrm{y} \alpha(\mathrm{y})+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{y}, \mathrm{x}]+[\mathrm{f}(\mathrm{x}) \mathrm{y}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{x}+\mathrm{f}(\mathrm{y}) \mathrm{y}+\mathrm{x} \alpha(\mathrm{y})+\mathrm{y} \alpha(\mathrm{x}), \mathrm{x}]+$
$[\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x})+\mathrm{f}(\mathrm{y}) \mathrm{x}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{y}, \mathrm{y}]+[2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{x}+\mathrm{f}(\mathrm{x}) \mathrm{y}+\mathrm{x} \alpha(\mathrm{y})+\mathrm{y} \alpha(\mathrm{x}), \mathrm{y}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Putting -y instead of $y$, combining the relation so obtained with the above relation, we see:
$[\mathrm{f}(\mathrm{x}) \mathrm{y}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{x}+\mathrm{f}(\mathrm{y}) \mathrm{y}+\mathrm{x} \alpha(\mathrm{y})+\mathrm{y} \alpha(\mathrm{x}), \mathrm{x}]+[\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x})+\mathrm{f}(\mathrm{y}) \mathrm{x}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{y}, \mathrm{y}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Also, the substitution 2 y for y and comparing the new relation with the last one gives:
$[\mathrm{f}(\mathrm{x}) \mathrm{y}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{x}, \mathrm{x}]+[\mathrm{x} \alpha(\mathrm{y})+\mathrm{y} \alpha(\mathrm{x}), \mathrm{x}]+[\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x}), \mathrm{y}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Replacing y by yx in (2) gives:
$[\mathrm{f}(\mathrm{x}) \mathrm{y}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{x}, \mathrm{x}] \mathrm{x}+2[\mathrm{y}, \mathrm{x}](\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x}))-[\mathrm{yx} \alpha(\mathrm{x}), \mathrm{x}]+\mathrm{x}[\alpha(\mathrm{y}) \alpha(\mathrm{x}), \mathrm{x}]+[\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x}), \mathrm{y}] \mathrm{x}=0$
The above relation can be written because of (2) as:
$2[\mathrm{y}, \mathrm{x}](\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x}))-[\mathrm{yx} \alpha(\mathrm{x}), \mathrm{x}]+\mathrm{x}[\alpha(\mathrm{y}) \alpha(\mathrm{x}), \mathrm{x}]-[\mathrm{x} \alpha(\mathrm{y})+\mathrm{y} \alpha(\mathrm{x}), \mathrm{x}] \mathrm{x}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Using the commutator's identities, we get:
$\mathrm{x}[\alpha(\mathrm{y}), \mathrm{x}](\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x} \alpha(\mathrm{y})[\alpha(\mathrm{x}), \mathrm{x}]-\mathrm{y}[\alpha(\mathrm{x}), \mathrm{x}] \mathrm{x}+2[\mathrm{y}, \mathrm{x}] \mathrm{f}(\mathrm{x}) \mathrm{x}-\mathrm{yx}[\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Putting xy for y in (3) gives:
$\mathrm{x}[\alpha(\mathrm{x}), \mathrm{x}] \alpha(\mathrm{y})(\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x} \alpha(\mathrm{x})[\alpha(\mathrm{y}), \mathrm{x}](\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x} \alpha(\mathrm{x}) \alpha(\mathrm{y})[\alpha(\mathrm{x}), \mathrm{x}]-\mathrm{xy}[\alpha(\mathrm{x}), \mathrm{x}] \mathrm{x}+$
$2 \mathrm{x}[\mathrm{y}, \mathrm{x}] \mathrm{f}(\mathrm{x}) \mathrm{x}-\mathrm{xyx}[\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Left multiplication of (3) by $x$, we see:
$\mathrm{x}^{2}[\alpha(\mathrm{y}), \mathrm{x}] \alpha(\mathrm{y})(\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x}^{2} \alpha(\mathrm{y})[\alpha(\mathrm{x}), \mathrm{x}]-\mathrm{xy}[\alpha(\mathrm{x}), \mathrm{x}] \mathrm{x}+2 \mathrm{x}[\mathrm{y}, \mathrm{x}] \mathrm{f}(\mathrm{x}) \mathrm{x}-\mathrm{xyx}[\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Subtracting (5) from (4), putting y for $\alpha(\mathrm{y})$, we find:
$\mathrm{x}[\alpha(\mathrm{x}), \mathrm{x}] \mathrm{y}(\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x})[\mathrm{y}, \mathrm{x}](\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x}) \mathrm{y}[\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Using the commutator's identities, the above relation reduces to:
$-\mathrm{x}^{2}(\alpha(\mathrm{x})-\mathrm{x}) \mathrm{y}(\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x}) \mathrm{y}(\alpha(\mathrm{x})-\mathrm{x}) \mathrm{x}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Putting yx instead of $y$ in the above relation, we see:

- $\mathrm{x}^{2}(\alpha(\mathrm{x})-\mathrm{x})$ y $\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x})+\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x})$ y $\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x}) \mathrm{x}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.

An application of Lemma (2.1) on (6) leads to:
$\left(\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x}) \mathrm{x}-\mathrm{x}^{2}(\alpha(\mathrm{x})-\mathrm{x})\right)$ y $\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Equivalently
$\mathrm{x}[\alpha(\mathrm{x})-\mathrm{x}, \mathrm{x}]$ y $\mathrm{x}(\alpha(\mathrm{x})-\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Now, in a relation (7), the substitution yx for y once and right multiplication by x in another implies that:
$\mathrm{x}[\alpha(\mathrm{x})-\mathrm{x}, \mathrm{x}]$ y $\mathrm{x}^{2}(\alpha(\mathrm{x})-\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
$\mathrm{x}[\alpha(\mathrm{x})-\mathrm{x}, \mathrm{x}] \mathrm{y} \mathrm{x}(\alpha(\mathrm{x})-\mathrm{x}) \mathrm{x}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Subtracting the relation (8) from (9) yields that:
$\mathrm{x}[\alpha(\mathrm{x})-\mathrm{x}, \mathrm{x}] \mathrm{y} \mathrm{x}[\alpha(\mathrm{x})-\mathrm{x}, \mathrm{x}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Equivalently
$\mathrm{x}[\alpha(\mathrm{x}), \mathrm{x}]$ y $\mathrm{x}[\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Using the semiprimeness of $\mathcal{R}$, we have:
$\mathrm{x}[\alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Combining the relations (1) and (10) implies that:
$[\mathrm{f}(\mathrm{x}), \mathrm{x}] \mathrm{x}=0$, for all $\mathrm{x} \in \mathcal{R}$.
Now, using similar techniques on the relation (11) as used to get (2) from (1) we arrive at:
$[\mathrm{f}(\mathrm{x}), \mathrm{y}] \mathrm{x}+[\mathrm{f}(\mathrm{x}), \mathrm{x}] \mathrm{y}+2[\mathcal{F}(\mathrm{x}, \mathrm{y}), \mathrm{x}] \mathrm{x}=0$, for all $\mathrm{x}, \mathrm{y} \in \mathcal{R}$.
Replacing $y$ by $y x$ in (12), using the relation (12) and the commutator's identities, we get:
$[f(x), y] x^{2}+[f(x), x] y x+2[\mathcal{F}(x, y), x] x^{2}+2[y, x] f(x) x+2 y[f(x), x] x=0$, for all $x, y \in \mathcal{R}$.
Right multiplication of (12) by $x$, then subtracting the relation so obtained from (13) leads because of (11) and the 2-torisionity free of R to:
$[y, x] f(x) x=0$, for all $x, y \in \mathcal{R}$.
Putting zy for y in (14), using (14) implies that:
$[z, x]$ y $f(x) x=0$, for all $x, y, z \in \mathcal{R}$.
The linearization of the relation (15) with respect x gives:
$[\mathrm{z}, \boldsymbol{\omega}] \mathrm{y} \mathrm{f}(\boldsymbol{\omega}) \omega+[\mathrm{z}, \boldsymbol{\omega}] \mathrm{y}(\mathrm{x}) \omega+2[\mathrm{z}, \boldsymbol{\omega}]$ y $\mathcal{F}(\boldsymbol{\mathcal { F }}, \mathrm{x}) \omega+[\mathrm{z}, \boldsymbol{\omega}] \mathrm{y} \mathrm{f}(\boldsymbol{\omega}) \mathrm{x}+[\mathrm{z}, \boldsymbol{\omega}] \mathrm{y} \mathrm{f}(\mathrm{x}) \mathrm{x}+2[\mathrm{z}, \boldsymbol{\omega}]$ y $\mathcal{F}(\boldsymbol{\omega}, \mathrm{x}) \mathrm{x}$
$[\mathrm{z}, \mathrm{x}]$ y $\mathrm{f}(\boldsymbol{\omega}) \omega+[\mathrm{z}, \mathrm{x}]$ y $\mathrm{f}(\mathrm{x}) \boldsymbol{\omega}+2[\mathrm{z}, \mathrm{x}]$ y $\mathcal{F}(\boldsymbol{\mathcal { F }}, \mathrm{x}) \omega+[\mathrm{z}, \mathrm{x}]$ y $\mathrm{f}(\boldsymbol{\omega}) \mathrm{x}+[\mathrm{z}, \mathrm{x}]$ y $\mathrm{f}(\mathrm{x}) \mathrm{x}+2[\mathrm{z}, \mathrm{x}]$ y $\mathcal{F}(\boldsymbol{\omega}, \mathrm{x}) \mathrm{x}=0$
Replacing $x$ by $-x$, combining the above relation with the new one, we get:
$[\mathrm{z}, \boldsymbol{\omega}] \mathrm{y} \mathrm{f}(\boldsymbol{\omega}) \mathrm{x}+2[\mathrm{z}, \boldsymbol{\omega}] \mathrm{y} \boldsymbol{\mathcal { F }}(\boldsymbol{\mathcal { L }}, \mathrm{x}) \mathrm{x}+[\mathrm{z}, \boldsymbol{\omega}] \mathrm{y} \mathrm{f}(\mathrm{x}) \mathrm{x}+[\mathrm{z}, \mathrm{x}] \mathrm{y} \mathrm{f}(\boldsymbol{\omega}) \boldsymbol{\omega}+[\mathrm{z}, \mathrm{x}] \mathrm{y} \mathrm{f}(\mathrm{x}) \omega+2[\mathrm{z}, \mathrm{x}] \mathrm{y} \boldsymbol{\mathcal { F }}(\boldsymbol{\omega}, \mathrm{x}) \mathrm{x}=0$
Putting $2 \omega$ for $\omega$, comparing the above relation with the relation so obtained, we arrive at:
$[z, \omega]$ y $f(x) x+[z, x]$ y $f(x) \omega+2[z, x]$ y $\mathcal{F}(\omega, x) x=0$, for all $x, y, z, \omega \in \mathcal{R}$.
The substitution $f(x) x$ y $[z, \omega]$ for $y$ leads because of (14) to:
$[\mathrm{z}, \omega] \mathrm{f}(\mathrm{x}) \mathrm{x}$ y $[\mathrm{z}, \omega] \mathrm{f}(\mathrm{x}) \mathrm{x}=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \omega \in \mathcal{R}$.

By the semiprimeness of $\mathcal{R}$, we have:
$[\mathrm{z}, \omega] \mathrm{f}(\mathrm{x}) \mathrm{x}=0$, for all $\mathrm{x}, \mathrm{z}, \omega \in \mathcal{R}$.
An application of Lemma (2.7) on the above relation we conclude that
$[\mathrm{z}, \omega] \mathrm{f}(\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{z}, \omega \in \mathcal{R}$.
According to Lemma (2.5), there exists an ideal U such that $\mathrm{f}(\mathrm{x}) \in \mathrm{U} \subset \mathrm{Z}(\mathcal{R})$.
An immediate consequence of the above theorem, we have the following corollary:
Corollary (3.7)
Let $\mathcal{R}$ be a 2 -torision free prime ring and $\alpha$ is antomorphisms of $\mathcal{R}$. If $\mathcal{R}$ admitting a symmetric Biderivation $\mathcal{F}$ : $\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ with traces f satisfies that $[\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \alpha(\mathrm{x}), \mathrm{x}]=0$, for all $\mathrm{x} \in \mathcal{R}$, then $\mathcal{R}$ is commutative.

## Theorem (3.8)

Let $\mathcal{R}$ be a 2-torition free prime ring and U is an ideal of $\mathcal{R}$. If $\mathcal{R}$ admitting a symmetric Biderivations $\mathcal{F}, \mathrm{G}: \mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ such that there traces f , g respectively satisfies that $\mathrm{f}(\mathrm{x}) \mathrm{x}+\mathrm{x} \mathrm{g}(\mathrm{x})=0$, for all $\mathrm{x}, \in \mathrm{U}$, then $\mathcal{R}$ is commutative or both $\mathcal{F}$ and G are zeros on $\mathcal{R}$.
Proof:
In view of our hypothesis, we have:
$f(x) x+x g(x)=0$, for all $x \in U$.
The linearization of (1) and using (1), we see:
$\mathrm{f}(\mathrm{x}) \mathrm{y}+\mathrm{f}(\mathrm{y}) \mathrm{x}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{x}+2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{y}+\mathrm{xg}(\mathrm{y})+\mathrm{y} \mathrm{g}(\mathrm{x})+2 \mathrm{xG}(\mathrm{x}, \mathrm{y})+2 \mathrm{yG}(\mathrm{x}, \mathrm{y})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathrm{U}$.
Putting 2x for x , then comparing the relation so obtained with the above relation, we get:
$2 \mathcal{F}(\mathrm{x}, \mathrm{y}) \mathrm{x}+\mathrm{f}(\mathrm{x}) \mathrm{y}+2 \mathrm{xG}(\mathrm{x}, \mathrm{y})+\mathrm{y} \mathrm{g}(\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y} \in \mathrm{U}$.
The substitution yx instead of $y$ in (2), gives:
$2 \mathcal{F}(x, y) x^{2}+2 y f(x) x+f(x) y x+2 x G(x, y) x+2 x y g(x)+y x g(x)=0$, for all $x, y \in U$.
Right multiplication of (2) by $x$, we get:
$2 \mathcal{F}(x, y) x^{2}+f(x) y x+2 x G(x, y) x+y g(x) x=0$, for all $x, y \in U$.
Subtracting the above relation from (3) gives:
$2 y f(x) x+2 x y g(x)+y x g(x)-y g(x) x=0$, for all $x, y \in U$.
According to (1), the last relation reduces to:
$y f(x) y x+2 x y g(x)-y f(x) x=0$, for all $x, y \in U$.
Now, by adding $\pm \operatorname{yxg}(x)$ and using (1), the relation (6) can be written as:
$[y, x] g(x)+[y g(x), x]=0$, for all $x, y \in U$.
That is
$2[y, x] g(x)+y[g(x), x]=0$, for all $x, y \in U$.
Replacing y by zy in above relation implies that:
$2 z[y, x] g(x)+2[z, x]$ y $g(x)+z y[g(x), x]=0$, for all $x, y, z \in U$.
The above relation reduces because of (7) and the 2-toritionity free of $\mathcal{R}$ to:
$[\mathrm{z}, \mathrm{x}]$ y $\mathrm{g}(\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{U}$.
The linearization of the relation (8) with respect to $x$ gives:
$[z, \omega]$ y $g(x)+2[z, \omega]$ y $G(\omega, x)+[z, x] y g(\omega)+2[z, x]$ y $G(\omega, x)=0$, for all $x, y, \omega, z \in U$.
Putting $2 \omega$ for $\omega$, comparing the above relation with the relation so obtained, we arrive at:
$[z, \omega] y \mathrm{~g}(\mathrm{x})+2[\mathrm{z}, \mathrm{x}]$ y $\mathrm{G}(\omega, \mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \omega \in \mathrm{U}$.
The substitution $\mathrm{yg}(\mathrm{x}) \mathrm{r}$ for y leads because of (8) to:
$[\mathrm{z}, \omega] \mathrm{y} \mathrm{g}(\mathrm{x}) \mathrm{rg}(\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{u}, \omega \in \mathrm{U}$ and $\mathrm{r} \in \mathcal{R}$
Again, replace $r$ by $r[z, \omega] y$ in the last relation, we find:
$[\mathrm{z}, \omega] \mathrm{g}(\mathrm{x}) \mathrm{r}[\mathrm{z}, \omega] \mathrm{y} \mathrm{g}(\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{u}, \omega \in \mathrm{U}$ and $\mathrm{r} \in \mathcal{R}$
Using the primeness of $\mathcal{R}$, we find:
$[z, \omega]$ y $g(x)=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \omega \in \mathrm{U}$.
An application of Remark (2.1), either $[z, \omega]=0$ or $g(x)=0$, for all $x, z, \omega \in U$. If $[z, \omega]=0$, for all $\omega, z \in U$ then $U$ is commutative.
Consequently, $\mathcal{R}$ is commutative by Lemma (2.3). Otherwise
$g(x)=0$, for all $x \in U$.
The linearization of the relation (9) gives:
$\mathrm{g}(\mathrm{x})+\mathrm{g}(\mathrm{u})+2 \mathrm{G}(\mathrm{x}, \mathrm{u})=0$, for all $\mathrm{x}, \mathrm{u} \in \mathrm{U}$.
In view of (9) and 2-torition free of R , the above becomes:
$\mathrm{G}(\mathrm{x}, \mathrm{u})=0$, for all $\mathrm{x}, \mathrm{u} \in \mathrm{U}$.
Hence G is zeros on $\mathcal{R}$ by Lemma (2.6).
On the other hand, according to (9), the relation (1) reduces to:
$f(x) x=0$, for all $x, \in U$.
The linearization (11), using (11) implies that:
$\mathrm{f}(\mathrm{x}) \omega+\mathrm{f}(\omega) \mathrm{x}+2 \mathcal{F}(\mathrm{x}, \omega) \mathrm{x}+2 \mathcal{F}(\mathrm{x}, \omega) \omega=0$, for all $\mathrm{x}, \in \mathrm{U}$.
Putting - $\omega$ for $\omega$ in (12), then comparing the relation so obtained with (12), we find:
$\mathrm{f}(\omega) \mathrm{x}+2 \mathcal{F}(\mathrm{x}, \boldsymbol{\omega}) \omega=0$, for all $\mathrm{x}, \omega \in \mathrm{U}$.
Replacing $x$ by $\omega x$ in (13) leads because of (11) to:
$\mathrm{f}(\omega) \mathrm{x} \omega+2 \omega \mathcal{F}(\mathrm{x}, \omega) \omega=0$, for all $\mathrm{x}, \omega \in \mathrm{U}$.
Left multiplication of (13) by $\omega$ gives:
$\omega \mathrm{f}(\omega) \mathrm{x}+2 \omega \mathcal{F}(\mathrm{x}, \omega) \omega=0$, for all $\mathrm{x}, \omega \in \mathrm{U}$.
Subtracting the relation (14) from (15) leads to:
$\omega f(\omega) \mathrm{x}-\mathrm{f}(\omega) \mathrm{x} \omega=0$, for all $\mathrm{x}, \omega \in \mathrm{U}$.
Replacing x by $\mathrm{x} \omega \mathrm{d}(\boldsymbol{\omega})$ in the last relation implies because of (11) that:
$\omega \mathrm{f}(\omega) \mathrm{x} \omega \mathrm{f}(\omega)=0$, for all $\mathrm{x}, \omega \in \mathrm{U}$.
Using the primeness of $\mathcal{R}$, we find:
$\omega \mathrm{f}(\omega)=0$, for all $\omega \in \mathrm{U}$.
Now, right multiplication of (13) by $f(\omega$ yields because of (16) that:
$f(\omega) \times f(\omega)=0$, for all $x, \omega \in U$.
By remark (2.1), we find:
$f(\omega)=0$, for all $\omega \in U$.
Using similar arguments on (18) as used to get (10) from (9), we arrive at:
$\mathcal{F}(\omega, \mathrm{u})=0$, for all $\omega, \mathrm{u} \in \mathrm{U}$.
Therefore $\mathcal{F}$ is zeros on $\mathcal{R}$ by Lemma (2.6).
Theorem (3.9)
Let $\mathcal{R}$ be a 2 -torition free prime ring and $U$ is an ideal of $\mathcal{R}$. The existence of nonzero symmetric Biderivations $\mathcal{F}$, G : $\mathcal{R} \times \mathcal{R} \longrightarrow \mathcal{R}$ such that there traces $\mathrm{f}, \mathrm{g}$ respectively satisfies that $\mathrm{f}(\mathrm{x}) \mathrm{x}-\mathrm{x} \mathrm{g}(\mathrm{x})=0$ holds for all $\mathrm{x} \in U$ forces $\mathcal{R}$ to be commutative. Proof:
If $\mathcal{F}=\mathrm{G}$, then g is commuting on $\boldsymbol{U}$, consequently, $\mathcal{R}$ is commutative by theorem (3.1). So we assume that $\mathcal{F} \neq \mathrm{G}$. In view of our hypothesis, we have:
$\mathrm{f}(\mathrm{x}) \mathrm{x}=\mathrm{xg}(\mathrm{x})$, for all $\mathrm{x} \in U$.
Using similar arguments as used to get (5) from (1), we find:
$2 \mathrm{y} f(\mathrm{x}) \mathrm{x}=2 \mathrm{xyg}(\mathrm{x})+\mathrm{yx} \mathrm{g}(\mathrm{x})-\mathrm{yg}(\mathrm{x}) \mathrm{x}$, for all $\mathrm{x}, \mathrm{y} \in U$.
Equivalently
$2 \mathrm{y} f(\mathrm{x}) \mathrm{x}=2 \mathrm{xyg}(\mathrm{x})+\mathrm{y}[\mathrm{x}, \mathrm{g}(\mathrm{x})]$, for all $\mathrm{x}, \mathrm{y} \in U$.
Replacing $y$ by $g(x) y$ in (2) gives:
$2 \mathrm{~g}(\mathrm{x}) \mathrm{y} \mathrm{f}(\mathrm{x}) \mathrm{x}=2 \mathrm{xg}(\mathrm{x}) \mathrm{yg}(\mathrm{x})+\mathrm{g}(\mathrm{x}) \mathrm{y}[\mathrm{x}, \mathrm{g}(\mathrm{x})]$, for all $\mathrm{x}, \mathrm{y} \in U$.
Left multiplication of (2) by $g(x)$ and subtracting the relation so obtained from the relation (3), we arrive because of 2-toritionity free of $\mathcal{R}$ to:
$[\mathrm{x}, \mathrm{g}(\mathrm{x})] \mathrm{y} \mathrm{g}(\mathrm{x})=0$, for all $\mathrm{x}, \mathrm{y} \in U$.
The substitution $y x$ for $y$ in (4), we find:
$[x, g(x)]$ y $x g(x)=0$, for all $x, y \in U$.
Now, Right multiplication of (4) by $x$, then subtracting the relation so obtained from (5) implies that:
$[\mathrm{x}, \mathrm{g}(\mathrm{x})] \mathrm{y}[\mathrm{x}, \mathrm{g}(\mathrm{x})]=0$, for all $\mathrm{x}, \mathrm{y} \in U$.
According to remark (2.1), we get:
$[\mathrm{x}, \mathrm{g}(\mathrm{x})]=0$, for all $\mathrm{x} \in U$.
Hence $\mathcal{R}$ is commutative by theorem (3.1).

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