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LRS Bianchi Type –I Magnetized Anisotropic Dark Energy Models with Variable Equation of State

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ABSTRACT

We discuss two dark energy models on LRS Bianchi Type-I magnetized anisotropic space –time with a variable equation of state (EoS). The EoS for dark energy ω is found to be time dependent and its existing range for derive models is in good agreement with the recent observations. Using the suitable condition, the anisotropic models approach to isotropic scenario. We also find that during the evolution of the universe, the EoS parameter for DE changes from $\omega > -1$ to $\omega = -1$ in first model whereas from $\omega > -1$ to $\omega < -1$ in second model which is consistent with recent observations. The cosmological constant Λ is found to be a positive decreasing function of time and it approaches a small positive value at late time (i.e. the present epoch) which is corroborated by results from recent supernovae Ia observations. The physical and geometric properties are also discussed.

Introduction

The nature of the dark energy component of the universe [1-3] remains one of the deepest mysteries of cosmology. There is certainly no lack of candidates: cosmological constant, quintessence [4-6], k-essence [7-9], phantom energy [10-12]. Modifications of the Friedmann equation such as Cardassian expansion [13,14] as well as what might be derived from brane cosmology [15-17] have also been used to explain the acceleration of the universe. A particular case of the linear Equation of state has used in the cosmological context by Xanthopuolos [18], he considered space-times with two hypersurface orthogonal, space-like, commuting killing fields.

The current standard model of cosmology implies the existence of dark energy which accounts for about 70% of the total energetic content of the universe, which ac-cording to the observations is spatially flat [19]. Several models have been proposed to explain dark energy [20-26]. An alternative consists of to consider a phenomenological decaying dark energy density with continuous creation of matter [26] or photons [27,28].The dark energy might decay slowly in the course of the cosmic evolution and thus provide the source term for matter and radiation. Different such models have been discussed and strong constraints come from accurate measurements of the CMB. Although some authors [29] have suggested cosmological model with anisotropic and viscous dark energy in order to explain an anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles.

Bianchi type models have been studied by several authors in an attempt to understand better the observed small amount of anisotropy in the universe. The same models have also been used to examine the role of certain anisotropic sources during the formation of the large-scale structures we see in the universe today. Some Bianchi cosmologies, for example, are natural hosts of large-scale magnetic fields and therefore, their study can shed light on the implications of cosmic magnetism for galaxy formation. The simplest Bianchi family that contains the flat FRW universe as a special case are the type-I space-times.

Measuring the equation of state for dark energy is one of the biggest efforts in observational cosmology today. The DE model has been characterized in a conventional manner by the equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ which is not necessarily constant, where ρ is the energy density and p is the fluid pressure [9]. The present data seem to slightly favour an evolving dark energy with EoS $\omega < -1$ around the present epoch and $\omega > -1$ in the near past. Obviously, ω cannot cross -1 for quintessence or phantom alone. Some efforts have been made to build a dark energy model whose EoS can cross the phantom divide. The simplest DE candidate is the vacuum energy ($\omega = -1$), which is mathematically equivalent to the cosmological constant (Λ). The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence ($\omega > -1$) [30], phantom energy ($\omega < -1$) [31] and quintom (that can across from phantom region to quintessence region as evolved) and have time dependent EoS parameter. Some other limits obtained from observational results coming from SNe Ia data [32] and combination of SNe Ia data with CMBR anisotropy and galaxy clustering statistics [33] are $-1.67 < \omega < -0.62$ and $-1.33 < \omega - 0.79$, respectively. The latest results in 2009, obtained after a combination of cosmological datasets coming from CMB anisotropies,

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luminosity distances of high redshift type Ia supernovae and galaxy clustering, constrain the dark energy EoS to $-1.44 < \omega < -0.92$ at 68% confidence level [34,35], However, it is not at all obligatory to use a constant value of ω. Due to lack of the observational evidence in making a distinction between constant and variable ω, usually the equation of state parameter is considered as a constant [36,37,4] with phase wise value $-1, 0, -\frac{1}{3}, and +1$ for vacuum fluid, dust fluid, radiation and stiff dominated universe respectively. But in general, ω is a function of time or redshift [38,39,40]. Some literature are also available on models with varying fields, such as cosmological models with variable EoS parameter in Kaluza-Klein metric and wormhole [41,42]. In recent years various form of time dependent ω have been used for variable Λ models by Mukhopadhyay et al. [43]. In well –known reviews on modified gravity [44,45],it is clearly indicated that any modified gravity may be represented as effective fluid with time dependent ω.

In Principle, once the metric is generalized to Bianchi types, the EoS parameter of the fluid can be generalized in a way conveniently to wield anisotropy with the considered metric . In such model, where both the metric and EoS parameter of the fluid are allowed to exhibit an anisotropic character, the universe can exhibit non-trivial isotropization histories and it can be examined whether the metric and/or the EoS parameter of fluid evolve toward isotropy. We may say about two main classes of such models; according to whether this anisotropization occurs at an early time or at late times of the universe, The former class can be related with the inflation field , which drives the inflation, which drives the late time acceleration of the universe. In the context of the former class, the generic inflationary model can be modified in a way to end inflation with a slightly anisotropic spatial geometry.

Bianchi type-I universe is the prime candidate for studying the possible effects of an anisotropy in the early universe on present-day observations, there are few other models (for example, $B-VI_0$), which describe an anisotropic space-time and generate interest among physicists.[46-49]. Pradhan and Bali [50] and Bali et al. [51] have studied $B-VI_0$ space time in connection with massive strings. Recently Amirhashchi et al.[52] presented dark energy models in an anisotropic $B-VI_0$ space –time by considering constant deceleration parameter. In this paper, we have investigated two new LRS Bianchi type-I magnetized dark energy models with variable equation of state . The discussion of the paper is as follows:

In section.2, the metric and the field equations are described. In section 3 deals with the solutions of the field equations in two different cases .Section 4 deals with physical and geometric behavior of the models. In section 5, we describe an other dark energy model and its physical aspects. Finally, conclusions are summarized in the last Section 6.

2. Metric and Field Equations:

We consider the LRS Bianchi Type-I metric in the following form

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 (dy^2 + dz^2) \tag{1}$$

where the scale factors a and b are functions of cosmic time only.

The energy momentum tensor for anisotropic dark energy with magnetic field is given by

$$T_j^i = {}^{DE}T_j^i + {}^{EM}T_j^i \tag{2}$$

where

$${}^{DE}T_j^i = (\rho_{ED} + p_{ED}) u_i u^j + p g_j^{\ i} \tag{3}$$

where u^i is the flow vector satisfying

$$g_{ij} u^i u^j = -1 \tag{4}$$

${}^{EM}T_j^i$ is the electromagnetic field tensor which is given by

$${}^{EM}T_{ij} = \frac{1}{4\pi}\left[F_{i\alpha}F_{j\beta}g^{\alpha\beta} - \frac{1}{4}g_{ij}F^{\alpha\beta}F_{\alpha\beta}\right] \tag{5}$$

where F_{ij} is the electromagnetic field tensor which satisfies the Maxwell equations

$$F_{[ij;\alpha]} = 0, \qquad (F^{ij}\sqrt{-g}) = 0 \tag{6}$$

In commoving coordinates, the incident magnetic field is taken along x-axis, with the help of Maxwell equations (6), the only non-vanishing component of F_{ij} is

$$F_{23} = const = M. \tag{7}$$

By preserving the diagonal form of the energy momentum tensor in a consistent way with the above metric, the simplest generalization of EoS parameter of perfect fluid may be to determine it separately on each spatial axis. Therefore the energy momentum tensor of perfect fluid is taken as

$$T_i^{\ j} = diag[T_0^0, T_1^1, T_2^2, T_3^3] \tag{8}$$

Thus, one may parameterize it as follows

$$T_i^{\ j} = dia[-\rho + \frac{M^2}{8\pi b^4}, p_x - \frac{M^2}{8\pi b^4}, p_y + \frac{M^2}{8\pi b^4}, p_z + \frac{M^2}{8\pi b^4}]$$

$$= dia[-(\rho + \frac{M^2}{8\pi b^4}), (\omega_x \rho - \frac{M^2}{8\pi b^4}), (\omega_y \rho + \frac{M^2}{8\pi b^4}), (\omega_z \rho + \frac{M^2}{8\pi b^4})]$$
$$= dia[-(\rho + \frac{M^2}{8\pi b^4}), (\omega\rho - \frac{M^2}{8\pi b^4}), (\omega + \delta)\rho + \frac{M^2}{8\pi b^4}, (\omega + \delta)\rho + \frac{M^2}{8\pi b^4}] \tag{9}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressures and $\omega_x, \omega_y, \omega_z$ are the directional EoS parameters along the x, y, z respectively, $\omega(t) = p/\rho$ is the deviation free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameter δ which is the deviation from ω along both y and z-axes. ω and δ are not necessarily constants and might be function of the cosmic time t.

The Einstein's field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij} \tag{10}$$

where the symbols have their usual meaning.

By adopting commoving coordinates, Einstein's field equation (10), for the Kontowski-Sachs space-time, the field equations take the form

$$\frac{2a_4 b_4}{a\,b} + \frac{{b_4}^2}{b^2} = 8\pi\rho + \frac{M^2}{b^4} \tag{11}$$

$$\frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{a_4 b_4}{ab} = -8\pi(\omega + \delta)\rho - \frac{M^2}{b^4} \tag{12}$$

$$\frac{2b_{44}}{b} + \frac{{b_4}^2}{b^2} = -8\pi\omega\rho + \frac{M^2}{b^4} \tag{13}$$

where a subscript 4 indicates differentiation with respect to t.

3. Solutions of the field equations

The spatial volume for the model (1) is given by

$$V = R^3 = ab^2 \tag{14}$$

where R is the mean scale factor. The mean Hubble parameter H is given as

$$H = \frac{R_4}{R} = \frac{1}{3}\frac{V_4}{V} = \frac{1}{3}\left(\frac{a_4}{a} + 2\frac{b_4}{b}\right) \tag{15}$$

The directional Hubble parameters in the directions of *x,y and z* respectively may be defined as

$$H_x = \frac{a_4}{a} \qquad and \quad H_y = H_z = \frac{b_4}{b} \tag{16}$$

The volumetric deceleration parameter q, the scalar expansion θ, shear scalar σ^2 and the average anisotropy parameter A_m are defined by

$$q = -RR_{44}/{R_4}^2 \tag{17}$$

$$\theta = \frac{a_4}{a} + 2\frac{b_4}{b} \tag{18}$$

$$\sigma^2 = 1/3\left(\frac{a_4}{a} - \frac{b_4}{b}\right)^2 \tag{19}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{20}$$

where $H_i (i = x, y, z)$ represents the directional Hubble parameter in the direction of x, y and z, respectively. $A_m = 0$ corresponds to isotropic expansion.

Initially we apply the law of variation for Hubble parameter for the LRS Bianchi Type –I metric may be given by

$$H = D(ab^2)^{-\frac{n}{3}} \tag{21}$$

where $D > 0$ *and* $n \geq 0$ are constants. Such type of relations have firstly been considered by Berman [53], Berman and Gomide [54] for solving FRW models. Letter on many authors have used this law to study FRW and Bianchi type models.

On integrating, after equating (15) and (21), we obtain

$$ab^2 = (nDt + c_1)^{\frac{3}{n}} \quad for \quad n \neq 0 \tag{22}$$

here c_1 is positive constants of integration. The values for the deceleration parameter for the mean scale factor as:

$$q = n - 1 \tag{23}$$

which is constant. The sign of q indicates where the model inflates or not. The positive sign of q i.e. $0 \leq n < 1$ indicates inflation. It is remarkable to mention here that through the current observations of SNe Ia and CMBR favors accelerating models $(q < 0)$, but both do not altogether rule out the deceleration ones which are also consistent with these observations [55]

Now we assume that the expansion (θ) is proportional to shear (σ), this condition lead to

$$\frac{a_4}{a} - \frac{b_4}{b} = \alpha_0 \sqrt{3}\left(\frac{a_4}{a} + 2\frac{b_4}{b}\right)$$

which yields to

$$\frac{a_4}{a} = m\frac{b_4}{b}$$

where $m = \dfrac{2\alpha_0\sqrt{3}+1}{1-\alpha_0\sqrt{3}}$ and α_0 are arbitrary constants. Above equation, after integration, reduces to

$$a = \beta b^m$$

where β is an integrating constant. Here , for simplicity and without any loss of generality, we assume $\beta = 1$. Hence we have

$$a = b^m \tag{24}$$

The motivation behind assuming this condition is explained with reference to Thorne[56], the observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent [57,58]. To put more precisely, red- shift studies place the limit

$$\frac{\sigma}{H} \leq 0.3,$$

On the ratio of shear σ to Hubble constant *H* in the neighborhood of our Galaxy today. Collins et al. [59] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\dfrac{\sigma}{\theta}$ is constant.

Using (24) (14) in (15), we obtain the expressions for metric function as follows

$$b = (nDt + c_1)^{\frac{1}{ml}} \tag{25}$$

$$a = (nDt + c_1)^{\frac{1}{l}} \tag{26}$$

where c_1 is an integrating constant

Hence the model (1) reduces to

$$ds^2 = -dt^2 + (nDt + c_1)^{\frac{2}{l}} dx^2 + (nDt + c_1)^{\frac{2}{ml}} (dy^2 + dz^2) \tag{27}$$

4. Physical aspects of dark energy model

The expressions for the Hubble parameter H, scalar of expansion θ ,shear scalar σ and the average anisotropy parameter A_m for the model (27) are given by

$$\theta = 3H = \frac{3L}{(nDt + c_1)} \tag{28}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{nD(m-1)}{ml}\right]^2 (nDt + c_1)^{-2} \tag{29}$$

$$A_m = \frac{(m^2+2)(nD)^2 - 2nDLml(m+2)}{3m^2l^2L^2} + 1 \tag{30}$$

Using equation (11) , the energy density of the fluid is obtain as

$$8\pi\rho=(2m+1)\frac{(nD)^2}{(ml)^2}(nDt+c_1)^{-2}-M^2(nDt+c_1)^{-4/ml} \tag{31}$$

Using equation (13) , the EoS parameter is obtained as

$$\omega=\frac{(2ml-3)\left(\frac{nD}{ml}\right)^2(nDt+c_1)^{-2}+M^2(nDt+c_1)^{-4/ml}}{(2m+1)\left(\frac{nD}{ml}\right)^2(nDt+c_1)^{-2}-M^2(nDt+c_1)^{-4/ml}} \tag{32}$$

Using equation (12), the skewness parameter δ are computed as

$$\delta=\frac{[2+l(m-1)-m(m+1)]\left(\frac{nD}{ml}\right)^2(nDt+c_1)^{-2}-2M^2(nDt+c_1)^{\frac{-4}{ml}}}{(2m+1)\left(\frac{nD}{ml}\right)^2(nDt+c_1)^{-2}-M^2(nDt+c_1)^{-4/ml}} \tag{33}$$

From (32), it is observed that the equation of state parameter ω is time dependent, it can be function of redshift z or scale factor R as well. The redshift dependence of ω can be linear like

$$\omega(z)=\omega_0+\omega' z \tag{34}$$

with $\omega'=\left.\frac{d\omega}{dz}\right|_{z=0}$ (see [60,61]) or nonlinear as

$$\omega(z)=\omega_0+\frac{\omega_1 z}{1+z} \tag{35}$$

[62,63]. So as for as the scale factor dependence of ω is concern. The parametrization

$$\omega(R)=\omega_0+\omega_R(1-R) \tag{36}$$

where ω_0 is the present value $(R=1)$ and ω_R is the measure of the time variation ω' is widely used in the literature [64]

So, if the present work is compare with experimental results [31,32,33,34], then one can conclude hat the limit of ω provided by (32) may accommodated with the acceptable range of EoS parameter . Also it is observed that at $t=t_c$, ω vanishes, where t_c is a critical time given by

$$t_c=\frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{2ml-3}{M^2}}\right)^{\frac{ml}{ml-2}}-\frac{c_1}{nD} \tag{37}$$

Thus, for this particular time , our model represents a dusty universe. We also note that earlier real matter at $t\le t_c$, where $\omega\ge 0$ later on at $t\ge t_c$, where $\omega>0$ converted to the dark energy dominated phase of universe.

For the value of ω to be in consistent with observation [31], we have the following general condition

$t_1<t<t_2$,

where

$$t_1=\frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+3.34\right)+0.67\right]}{2.67M^2}}\right)^{\frac{ml}{ml-2}}-\frac{c_1}{nD} \tag{38}$$

$$t_2=\frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+1.24\right)-0.38\right]}{1.62M^2}}\right)^{\frac{ml}{ml-2}}-\frac{c_1}{nD} \tag{39}$$

For this constrain, we obtain $-1.67 < \omega < -0.62$ which is in good agreement with the limit obtained from observational results coming from SNe Ia data [31].

For the value of ω to be in consistent with observation [12] , we have the following general condition

$$t_3 < t < t_4, \tag{40}$$

where

$$t_3 = \frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{3\left[m\left(\frac{2}{3}l + 2.66\right) + 0.33\right]}{2.33M^2}}\right)^{\frac{ml}{ml-2}} - \frac{c_1}{nD} \tag{41}$$

$$t_4 = \frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{3\left[m\left(\frac{2}{3}l + 1.58\right) - 0.21\right]}{1.79M^2}}\right)^{\frac{ml}{ml-2}} - \frac{c_1}{nD} \tag{42}$$

For this constrain, we obtain $-1.33 < \omega < -0.79$ which is in good agreement with the limit obtained from observational results coming from SNe Ia data [31] with CMB anisotropy and galaxy clustering statistics [32].

For the value of ω to be in consistent with observation [33,34] , we have the following general condition

$$t_5 < t < t_6,$$

$$t_5 = \frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{3\left[m\left(\frac{2}{3}l + 2.88\right) + 0.44\right]}{2.44M^2}}\right)^{\frac{ml}{ml-2}} - \frac{c_1}{nD} \tag{43}$$

$$t_6 = \frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{3\left[m\left(\frac{2}{3}l + 1.84\right) - 0.08\right]}{1.92M^2}}\right)^{\frac{ml}{ml-2}} - \frac{c_1}{nD} \tag{44}$$

For this constrain, we obtain $-1.33 < \omega < -0.79$ which is in good agreement with the limit obtained from observational results [33,34].

From (31), we note that energy density of the fluid $\rho(t)$ is a decreasing function of time and $\rho \geq 0$ when

$$t \leq \frac{1}{nD}\left(\frac{nD}{ml}\sqrt{\frac{2m+1}{M^2}}\right)^{\frac{ml}{ml-2}} - \frac{c_1}{nD} \tag{45}$$

Here ρ is a positive decreasing function of time and it approaches to zero as $t \to \infty$.

In absence of any curvature, matter energy density Ω_m and dark energy Ω_Λ are related by the equation

$$\Omega_m + \Omega_\Lambda = 1 \tag{46}$$

Where $\Omega_m = \frac{\rho}{3H^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H^2}$. Thus , (46) reduces to

$$\frac{\rho}{3H^2} + \frac{\Lambda}{3H^2} = 1 \tag{47}$$

Using (28) and (31) in (47), the cosmological constant is obtained as

$$\Lambda = \left[3L^2 - \frac{1}{8\pi}(2m+1)\left(\frac{nD}{ml}\right)^2\right](nDt + c_1)^{-2} + \frac{M^2}{8\pi}(nDt + c_1)^{-\frac{4}{ml}} \tag{48}$$

From (48), we observe that Λ is decreasing function of time and it is always positive when

$$t > \frac{1}{nD}\left[M^2(24\pi L^2 - (2m+1)\left(\frac{nD}{ml}\right)^2) \right]^{\frac{ml}{2(ml-2)}} + \frac{c_1}{nD} \quad (43)$$

We observe that cosmological parameter is decreasing function of time and it approaches a small positive value at late time (i.e. At present epoch). Recent cosmological observations[65,2]; [3,66,67] suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda\left(Gh/c^3\right) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ- term. Thus, the nature of Λ in our derive DE model is supported by recent observations.

From (28)-(29) , it can be seen that all the kinematical parameters $H, \theta, and\ \sigma$ diverge at the initial singularity. There is a Point Type singularity [48] at $t = -\frac{c_1}{nD}$ in the model . The mean anisotropic parameter is constant and it increases. Thus, the dynamics of the mean anisotropy parameter depends on the value of n . Since $\frac{\sigma^2}{\theta^2} = cons\tan t,$ the models does not approach isotropy through the whole evolution of the universe.

5. Other dark energy model

Now we take the following ansatz for the scale factor, where the increase in terms of time evolution is

$$R(t) = te^t \quad (50)$$

By the above choice of scale factor yields a time dependent deceleration parameter. We define the deceleration parameter q as usual,

$$q = -\frac{R_{44}R}{R_4{}^2} = -\frac{R_{44}}{RH^2} \quad (51)$$

Using (50) into (51), we find

$$q = -1 + \frac{1}{(1+t^2)} \quad (52)$$

Using (24) and (51) in (15), we obtain the expressions for metric functions as follows

$$a = \left(te^t\right)^{\frac{3m}{2+m}} \quad (53)$$

$$b = (te^t)^{3/2+m} \quad (54)$$

Hence the model (1) reduces to

$$ds^2 = -dt^2 + (te^t)^{\frac{6m}{m+2}} dx^2 + (te^t)^{\frac{6}{m+2}} (dy^2 + dz^2) \quad (55)$$

The expressions for the Hubble parameter H , scalar of expansion θ, shear σ and the average anisotropy parameter A_m for the model (55) are given by

$$\theta = 3H = 3\left(\frac{R_4}{R}\right) = 3\left(\frac{t+1}{t}\right) \quad (56)$$

$$\sigma^2 = \frac{(m-1)^2}{m^2r^2}\left(\frac{t+1}{t}\right)^2 \quad (57)$$

$$A_m = \frac{(m^2+2) - 2mr(m+2)}{3mr^2} + 1 \quad (58)$$

Where $r = \frac{m+2}{3m}$. Since $\frac{\sigma^2}{\theta^2} \neq 0$ for all values of m except for $m = 1$, hence the model is anisotropic except for $m = 1$.

The dynamics of the mean anisotropic parameter depends on the value of m. The mean anisotropic parameter is constant. We observed that when $m = 0,\ A_m \to \infty$ and for $m = 1,\ A_m = 0$. Thus, the observed isotropy of the universe can be achieved in phantom model.

The energy density of the fluid can be find by using (53) and (54) in (11)

$$8\pi\rho = \frac{(2m+1)}{m^2r^2}\left(\frac{t+1}{t}\right)^2 - M^2(te^t)^{\frac{-4}{rm}} \quad (59)$$

Using (53) , (54) and (59) in (13) , the EoS parameter ω is obtained as

$$\omega=-\frac{\left[\frac{3(t+1)^2-2mr}{t^2m^2r^2}-M^2(te^t)^{-\frac{4}{rm}}\right]}{\frac{(2m+1)}{r^2m^2}\left(\frac{t+1}{t}\right)^2-M^2(te^t)^{-\frac{4}{rm}}} \tag{60}$$

Using (53), (54), (59) and (60) in (12), the skew parameter δ are computed as

$$\delta=-\frac{\left[\frac{(m^2+m-2)}{m^2r^2}\left(\frac{t+1}{t}\right)^2+\frac{(1-m)}{mrt^2}+2M^2(te^t)^{\frac{-4}{rm}}\right]}{\frac{(2m+1)}{r^2m^2}\left(\frac{t+1}{t}\right)^2-M^2\left(te^t\right)^{-\frac{4}{rm}}} \tag{61}$$

So, if the present work is compared with experimental results [31,32,33,34], then one can conclude that the limit of ω provided by (60) may accommodated with the acceptable range of EoS parameter. Also it is observed that at $t=t_c$, ω *vanishes* , where t_c is a critical time given by the following relation

$$\frac{3(t+1)^2-2mr}{m^2r^2t_c{}^2}-M^2(t_ce^{t_c})^{-\frac{4}{rm}}=0 \tag{62}$$

Thus for this particular time, our model represents a dusty universe, We also note that the earlier real matter at $t\le t_c$, where $\omega\ge 0$ later on at $t>t_c$, where $\omega<0$ converted to the dark energy dominated phase of universe.

From (59), we note that energy density of the fluid $\rho(t)$ is a decreasing function of time and $\rho\ge 0$ when

$$(te^t)^{4/rm}\left(\frac{t+1}{t}\right)^2\ge\frac{M^2m^2r^2}{2m+1} \tag{63}$$

Here ρ is a positive decreasing function of time and it approaches to zero as $t\to\infty$.

Using (56),(59) in (47), the cosmological constant is obtained as

$$\Lambda=\left[3-\frac{9(2m+1)}{8\pi(m+2)^2}\right]\left(\frac{t+1}{t}\right)^2-\frac{M^2}{8\pi}(te^t)^{\frac{-4}{rm}} \tag{64}$$

From (64), we observe that Λ is a decreasing function of time and it is always positive when

$$\left(\frac{t+1}{t}\right)\left(te^t\right)^{\frac{12}{m+2}}\ge\frac{M^2(m+2)^2}{24\pi(m-1)^2} \tag{65}$$

We observe that cosmological parameter is decreasing function of time and it approaches a small positive value at late time. Thus ,the nature of Λ in this derived DE model is also in good agreement with recent observations [1,2,68,69,70].

6. Conclusion

On getting motivated from increasing evidence for the need of a geometry that resembles Bianchi morphology to explain the observed anisotropy in the Wilkinson microwave anisotropy probe (WMAP) data, we have discussed some features of the Bianchi type- VI_0 universes in the presence of a fluid that wields an anisotropic EoS parameter in general relativity. A new class of anisotropic LRS Bianchi type –I magnetized Dark energy models with variable EoS parameter ω has been investigated by using time dependent deceleration parameter. In literature it is a plebeian practice to consider constant deceleration parameter. Now for a universe which was decelerating in past and acceleration at present epoch, the DP must show signature flipping as already discussed in Section 2 . Therefore our consideration of DP to be variable is physically justified.

Our Power law solution represents the singular model where the spatial scale factors and volume scalar vanish at $t=-\frac{c_2}{nD}$. All the physical parameters are infinite at this initial epoch and tend to zero as $t\to\infty$. There is a point Type singularity [48] at $t=-\frac{c_2}{nD}$ in the model.

It is observed that, in early stage, the EoS parameter ω is positive i.e. the universe was matter dominated in early stage but in late time, the universe is evolving with negative values i. e. the present epoch . DE model presents the dynamics of EoS parameter ω provided by (32) whose range is in good agreement with the acceptable range by the recent observations [31-34].

The existence of DE, in whatever form , is needed to reconcile the measured geometry of space with the total amount of matter in the universe, DE models present the dynamics of EoS parameter ω provided by (32) and (60) whose range is in good agreement with the acceptable range by the recent observations [31-34] .It can be easily seen that in both DE models . Thus , our both anisotropic parameter vanishes at $m=1.$ Thus, our both anisotropic models approach to isotropy at $m=1.$ It is already discussed in previous section , we obtain cosmological constant dominated universe, quintessence and phantom fluid dominated universe [45], representing different phases of the universe through- out the evolving process.

Our DE model is of great importance in the sense that the nature of decaying vacuum energy $\Lambda(t)$ is supported by recent cosmological observations. Though there are many suspects (candidates) such as cosmological constant, vacuum energy, scalar field, brane world, cosmological nuclear-energy, etc. as reported in the vast literature for DE, the proposed model in this paper favors EoS parameter as a possible suspect for the DE.

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References

[1] Riess A. G., et al., "Observational Evidence from supernovae for an accelerating universe and cosmological constant," The Astronomical Journal, Vol. 116, No. 3, p. 1009 (1998),.

[2] Perlmutter S., *et al.*, "Measurements of Ω and from 42 High-Redshift Supernovae," The Astronomical Journal, Vol. 517, No. 2, p. 565 (1999).

[3] Sahni V., "Dark Matter and Dark Energy," Cornell University Library, Ithaca, (2004).

[4] Ratra B. and Peebles P. J. E., "Cosmological Consequences of a Rolling Homogeneous Scalar Field," Physical Review D, Vol. 37, No. 12, pp. 3406-3427 (1988).

[5] Caldwell R. R., Dave R. and Steinhardt P. J., "Cosmo-logical Imprint of an Energy Component with General equation of state," Physical Review Letters, Vol. 80, No. 8, pp. 1582-1585 (1998).

[6] Barreiro T., " Quinte Sence Arinsing from Exponential Potential" *Review D*, Vol. 61, No. 12, pp. 127301-127305 (2000).

[7] Armendariz-Picon C., Damour T. and Mukhanov V., "K -Inflation," Physical Letters B, Vol. 458pp. 209-218.

[8] Armendariz-Picon C., Mukhanov V. and Steinhardt P. J., "Essentials of K-Essence," Physical Review D, Vol No. 10, pp. 103510-103523 (2001).

[9] Gonzalez-Diaz P. F., "K-Essential Phantom Energy :Dooms-day around the Corner?" Physical L 1-2, pp. 1-4(2004).

[10] Caldwell R. R., "A Phantom Menace? Cosmological Con-sequences of Dark Energy Component with Super Negative Equation of State," Physical Letters B, Vol. 545, pp. 23-29 (2002).

[11] Carroll S. M., Hoffman M. and Trodden M., "Can the Dark Energy Equation-of-State Parameter be less than–1?" Physical Letters D, Vol. 68, No. 2, pp. 23509- 23520 (2003).

[12] Elizalde E., Nojiri S.and Odintsov S. D., "Late-Time Cosmology in a (Phantom) scalar-Tensor theory:Dark Energy and the Cosmic Speed up," Physical Letters D, Vol. 70, No. 4, pp. 043539-043559 (2004).

[13] Freese K. and Lewis M., "Cardassian Expansion Model in which the Universe is Flat, Matter Dominated and Accelerating," Physical Letters B, Vol. 540, No. 1-2, pp. 1-8 (2002).

[14] Gondolo P. and Freese K., "Fluid Interpretation of Car-dassian Expansion," Physical Letters D, Vol. 68, N, pp. 063509-063519 (2003).

[15] Deffayet C., Dval G. R., "Accelerated Universe from Gravity Leaking to Extra Dimensions" Physical Letters D, Vol. 65, No. 4, pp. 044023- 044032 (2002).

[16] Dvali G., Gabadadze G. and Porrati M., "4D Gravity on a Brane in 5D Minkowski Space," Physical L 485, No. 1-3, pp. 208-214 (2000).

[17] Dvali G. and Turner M. S., "Dark Energy as a Modification of the Friedmann Equation," Cornell University Library, Ithaca (2003)

[18] Xanthopuolos B. C., "Perfect Fluids Satisfying a less than Extremely Relativistic Equation of State" Mathematical Physics, Vol. 28, No. 4, pp. 905-913 (1987).

[19] Spergel D. N., *et al*., "Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology," The Astrophysical Journal Supplement Series, Vol. 170, No. 2, pp. 377-408 (2007).

[20] Peebles J.E., "Cosmological constant and Dark Energy," Reviews of Modern Physics, Vol. 75, No. 32, pp. 559-606 (2003).

[21] Padmanabhan T., "Cosmological Constant—The weight the Vacuum," Physics Reports, Vol. 380, No. 5-6, pp. 235-320 (2003).

[22] Tortora E. and Demianski M., "Two Viable Quintessence Models of the Universe: Confrontations of Theoretical Prediction with Observational Data," Astronomy & Astro-physics, Vol. 431, No. 1, pp. 27-44 (2005).

[23] Cardone V. F., *et al*., "Some Astrophysical Implications of Dark Matter and Gas Profiles in a New Galaxy Cluster Model," Astronomy & Astrophysics, Vol. 429, No. 1, pp. 49-64 (2005).

[24] Peebles P. J. E. and Rathra B., "Cosmology with a Time- Variable Cosmological 'Constant'," Astrophysical Journal, Part 2 Letters, Vol. 325, No. 2, pp. L17-L20 (1988).

[25] Sahni V. and A. A. S " The case for a Positive cosmological Λ-Term," International Journal of Modern Physics D, Vol. 9, No.4, pp. 373-443 (2000).

[26] Ma Y.-Z., "Variable cosmological Constant Model: The Reconstuction Equations and Constraints from Current Observational Data," Nuclear Physics B, Vol. 804, No. 1-2, pp. 262-285 (2008).

[27] Lima J. A. S., *et al*., "Is the Radiation Temperature- Red- shift Relation of the Standard Cosmology in Accordance with the Data?" Monthly Notices of the Royal Astronomical Society, Vol. 312, No.4,.page No.747-752 (2000).

[28] Lima J. A. S. and Alcaniz J. S., "Angular size in quintessence cosmology," Astronomy & Astrophysics, Vol. 348, No. 1, pp. 1-5 (1999).

[29]Koivisto .T. and Mota D. F., "Accelerating Cosmologies with an Anisotropic Equation of State," Astrophysical Journal , Vol. 679, No. 1, pp. 1-5 (2008).

[30].Steinhardt, P.J., Wang, L.M., Zlatev, I: "Cosmological Tracking Solution", Phys. Rev. D59, 023504 (1999)

[31]. Knop, R.K., et al.: “New Constariants : Ω_n ,$\Omega_m and\ \omega$ from an Independent Set of Unique Redshift Supernovae Observed with the Hubble Space Telescope”, Atrophys. J. 598, 102 (2003)
[32]. Tegmark,M., et al.: “ The Three Dimensional Power Spectrum of Galaxies from the Sloan Digital Sky Survey”, Astrophys. J. 606, 702 (2004b)
[33]. Hinshaw, G.,et al.: (WMAP Collaboration): Five Year Wilkinson Microwave Anisotropy Probe (WMAP) Observation: Data Processing, SkybMaps and Basic Results”, Astrophys. J. Suppl. Ser. 180, 225 (2009)
[34]. Komatsu, E., et al.: ”Five –Year, Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation “,Astrophys, J. Suppl. Ser. 180, 330(2009)
[35]. Kujat, J., et al.: “ Prospects for Determinig the Equation of State of the Dark Energy: What can be Learned from Multiple Observable?”, Astrophys. J. 572, 1(2002)
[36]. Bartelmann, M., et al.:” Equation of Dark Matter Haloes in a Variety of Dark- Energy Cosmologies”, New Astron. Rev. 49, 199(2005)
[37]. Yadav, A.K.: “Some Anisotropic Dark Energy Models in Bianchi Type-V Space Time”, Astrophys. Space Sci. (2011) doi:10.1007/s10509-011-0745-3.
[38].Jimenez, R..: “ The Value of the Equation of State of Dark Energy”, New Astron. Rev. 47, 761 (2003)
[39]. Das, A., et al.:” Cosmological with Decaying Tachyon Matter”, Phys. Rev. D72, 043528(2005)
[40]. Rahaman, F., Bhui, B. C.:”,Cosmological Model with a Viscous Fluid in a Kaluza- Klein Metric”, Astrophys. Space Sci. 301,47 (2006)
[41]. Mukhopadhyay, U., Ghosh,P.P, Choudhury, S.B.D.:” Λ - CDM Universe: A Phenomenological Approach with Many Possiblities”, Int. J. Mod. Phys. D17, 301 (2008)
[42]. Setare, M.R. : “ The Holographic Dark Energy in Non- Flat Brans- Dicke Cosmology”, Phys. Lett.B 644, 99(2007a)
[43]. Setare, M. R. :” Interacting Holographic Phantom”, Eur. Phys. J. C 50, 991(2007b)
[44]. Setare, M.R. : “ Interacting Holographic Generalized Chaplygin Gas Model”, Phys.Lett. B654,1 (2007c)
[45] Setare, M. R. , Saridakis, E. N.:” Holographic Dark Energy With Varying Gravitational Constant”, Int. J.Mod. Phys. D 18,549 (2009)
[46]. Nojiri,S., Odintsov, S.D.:” Introduction to Modified Gravity and Gravitational Alternative for Dark Energy”, Int. J. Geo. Methods Mod. Phys. 4, 115 , arXiv:hep-th/0601213v5(2007)
[47]. Nojiri,S., Odintsov, S.D.: ” Unified Cosmic History in Modified Gravity from F® Theory to Lorentz Non- Invarient Models “,Phys. Rep. 505, 59 ,arXiv:1011.0544v4[gr-qc].
[48].Weaver, M.: “Dynamics of Magnets Bianchi VIo Cosmologies “, Class. Quantum Gravity 17, 421 (2000)
[49]. Ibanez, J., Vander Hoogen, R. J. Coley, A. A. :Isotropization of Scalar Field Bianchi Models with an Exponential Potential”, Phys. Rev. D 51, 928
[50]. Socorro, J., Medina, E. R. : “ Supersymmetric Quantum Mechanics for Bianchi Class A Models”, Phys. Rev. D61, 087702 (2000)
[51]. Radinschi, I.: Energy Associated with the Bianchi Type VIo Universe “,Chin. J. Phys. 39, 393(2001).
[52]. Pradhan , A. Bali, R. : “ Magnetized Bianchi Type \$S VI-{0}\$ Barotropic Massive String Universe with Decaying Vacuum Energy Density \$^\$”, Electron. J. Theor. Phys. 5, 91 (2008)
[53]. Bali, R. ,Pradhan A., Amirhashchi, H.:”Bianchi Type VIo Magnetized Barotropic Bulk Viscous Fluid Massive string Universe in General Relativity”, Int. J. Theor. Phys. 47, 2594(2008)
[54]. Amirhashchi, H. ,Pradhan, A. Saha, B. : “ Variable Equation of State fot Bianchi Type VIo Dark Energy Models”, Astrophys. Sapce Sci. 333, 295(2011c)
[55]. Berman, M. S. :” Special Law of Variation for Hubble Parameters “, Nuovo Cimento B 74, 182(1983)
[56].Berman, M. S. ,Gomide, F. M. : “ Cosmological Models with Constant Deceleration Parameter “, Gen . Relativ. Gravit, 20, 191(1988)
[57]. Vishwakarma, R.G.: “ A Study of Angular Size Redshift Relation for Models in which Lambda Decays as the Energy Density “, Class. Quant. Gravity 18, 1159-1172(2000)
[58]. Thorne, K. S. : “ Primardial Element Formation, Primordial Magnetic Fields and the Isotropy of the Universe “, Astrophys. J. 148,51(1967)
[59]. Kantowski,R., Sachs, R.K.:” Some Spatially Homogeneous Anisotropic Relativistic Cosmological Models”, J. Math. Phys. 7, 433(1966).
[60]. Kristian, J. , Sachs, R. K.:” Observations in Cosmology” , Astrophys. J. 143, 379(1966)
[61].Collins, C. B., Glass, E. N., Wilkinson, D. A.: Exact Spatially Homogeneous Cosmologies “,Gen. Relativ. Gravit. 12, 805(1980)
[62]. Huterer, D., Turner, M. S.:” Probing Dark Energy: Methods and Stradegies.”, Phys., Rev. D64, 123527(2001)
[63]. Weller, J., Albrecht, A,: ” Future Supernovae Observation as a Probe of Dark Energy”, Phys. Rev. D 65, 103512(2002)
[64].Chevallier, M., Polarski, D: Accelerating Universe with Scaling Dark Matter “,Int. J. Mod. Phys. D10, 213(2001)
[65]. Linder, E. V.: “ Exploring the Expansion History of the Universe”, Phys. Rev. Lett. 90, 91301(2003)
[66]. Linder, E. V.: “ The Dyanamics of Quintessence, the Quintessence of Dynamics”, Gen. Relaiv. Gravit. 40329(2008)
[67]. MacCallum, M. A. H.: “ A Class of Homogeneous Cosmological Model III: Asymptotic Behavior Communications “, Math. Phys. 20, 57(1997)
[68].Perlmutter, S.,et al.(Supernova Cosmology Project Collaboration),”Measurements of Ω and Λ from 42 High- Redshift Supernovae” Astrophys. J. 517, 5 (1999).
[69]. Tonry,J.L.,et al. (SDSScollaboration): “Cosmological Results From High-z Supernovae”, Astrophys.J.594,1(2003)
[70]. Riess, A. G., et al. (Supernova Cosmology Project Collaboration):Type Ia Supernovae Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for past Deceleration and Constraint on Dark Energy Evalution”, Astrophys.J.607,665 (2004).