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Solving Linear and Nonlinear Schrodinger Equations Using Mixture of Kamal Transform and Homotopy Perturbation Method

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ABSTRACT

In this article, the Homotopy perturbation method (HPM) and Kamal transform are introduced for obtaining the approximate analytical solution of the Linear and Nonlinear Schrodinger Equations. The proposed method is an elegant combination of the new integral transform "Kamal Transform" and the Homotopy perturbation method.

1. Introduction

The linear and nonlinear Schrodinger equation is an example of a universal linear and nonlinear model that describes many physical systems. The equation can be applied to hydrodynamics, optics, nonlinear acoustics, quantum condensates, heat pulses in solids and various other nonlinear instability phenomena .Since analytic approaches to the Schrodinger equation have limited applicability in science and engineering problems, there is a growing interest in exploring new methods to solve the equation more accurately and efficiently . In recent years, many research workers have paid attention to study the solutions of nonlinear partial differential equations by using various methods. Among these the Adomian decomposition method Hashim, Noorani, Ahmed. Bakar, Ismail and Zakaria, (2006), the homotopy perturbation method Sweilam, Khader (2009), Jafari, Aminataei (2010), (2011), the differential transform method (2008), Homotopy Perturbation and Elzaki Transform [2-5], homotopy perturbation transform method and the variational iteration method. Homotopy perturbation method (HPM) was established in 1999 by He [6-9]. The method is a powerful and efficient technique to find the solutions of non-linear equations. It is worth mentioning that the HPM is applied without any discretization, restrictive assumption or transformation and is free from round off errors. Homotopy perturbation transform method and the variational iteration method .Various ways have been proposed recently to deal with these nonlinearities; one of these combinations is Kamal transform and homotopy perturbation method which is studies in this paper [1] , Kamal transform is a useful technique for solving linear differential equations . This paper uses homotopy perturbation method to decompose the nonlinear term, so that the solution can be obtained by iteration procedure. This means that we can use both Kamal transform and homotopy perturbation methods to solve many nonlinear problems. The mainaim of this paper is to consider the effectiveness of the Kamal transform homotopy perturbation method in solving Linear and Nonlinear Schrodinger Equations. This method KTHPM finds the solution without any discretization, linearization or restrictive assumptions and avoids the round-off errors, the results reveal that the KTHPM is very efficient, simple and can be applied to other nonlinear problems. The layout of the paper is as follows: In section 2, we introduce the basic idea, Application in 3 and conclusion in 4, respectively .

2. Basic Idea of Kamal Transform Homotopy Perturbation Method (KTHPM) :

The general form of nonlinear non-homogeneous partial differential equation can be considered as the follow :

$$\boldsymbol{Du(x,t)+Ru(x,t)+Nu(x,t)=f(x,t)} \qquad (1)$$

with the following initial conditions

$$\boldsymbol{u(x,0)=h(x),\qquad ut(x,0)=g(x)}$$

Where D is the second order linear differential operator $\boldsymbol{D=\frac{\partial^2}{\partial t^2}}$, is the linear differential operator of less order than D, N represents the general non-linear differential operator and $\boldsymbol{f(x,t)}$is the source term.

Taking Kamal transform(denoted throughout this paper by $\boldsymbol{K(.)}$) on both sides of Eq. (1), to get:

$$\boldsymbol{K[Du(x,t)]+K[Ru(x,t)]+K\,[Nu(x,t)]=K\,[f(x,t)]} \qquad (2)$$

Using the differentiation property of Kamal transform and above initial conditions, we have:

$$\boldsymbol{K[u(x,t)]=\ v^2K[f(x,t)]+vh(x)+v^2g(x)-v^2K[Ru(x,t)+Nu(x,t)]} \qquad (3)$$

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Operating with the Kamal inverse on both sides of Eq.(3) gives:

$$u(x,t) = G(x,t) - K^{-1}[v^2 K[Ru(x,t) + Nu(x,t)]] \tag{4}$$

Where $G(x,t)$ represents the term arising from the source term and the prescribed initial condition.
Now, we apply the homotopy perturbation method

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t). \tag{5}$$

And the nonlinear term can be decomposed as:

$$Nu(x,t) = \sum_{n=0}^{\infty} p^n H_n(u) \tag{6}$$

Where $H_n(u)$ are He's polynomial and given by:

$$H_n(u_0, u_1, u_2 \dots u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{n=0}^{\infty} p^i u_i\right)\right]_{p=0}, \; n = 0,1,2,\dots \tag{7}$$

Substituting Eqs. (6) and (5) in Eq. (4) we get:

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = G(x,t) - pK^{-1}\left[v^2 K\left[R\sum_{n=0}^{\infty} p^n u_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(u)\right]\right] \tag{8}$$

Which is the coupling of Kamal transform and the homotopy perturbation method using He's polynomials. Comparing the coefficient of like powers of $\mathbf{p}$, the following approximations are obtained:

$$p^0: \; u_0(x,t) = G(x,t),$$
$$p^1: \; u_1(x,t) = -K^{-1}[v^2 K[Ru_0(x,t) + H_0(u)]],$$
$$p^2: \; u_2(x,t) = -K^{-1}[v^2 K[Ru_1(x,t) + H_1(u)]],$$
$$p^3: \; u_3(x,t) = -K^{-1}[v^2 K[Ru_2(x,t) + H_2(u)]],$$

Then the solution is;

$$u(x,t) = \lim_{p\to 1} u_n(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots \tag{9}$$

To show the capability of the method, KTHPM applied to some examples in the next section

3. Applications

Example 3.1

Let's consider the following linear homogeneous Schrodinger Equation:

$$u_t + iu_{xx} = 0 \tag{10}$$

With the initial condition;

$$u(x,0) = 1 + \cosh 2x \tag{11}$$

Applying Kamal transform of both sides of Eq. (10),

$$K[u_t] = -K[iu_{xx}] \tag{12}$$

Using the differential property of Kamal transform Eq.(12) can be written as:

$$\frac{1}{v} K[u(x,t)] - u(x,0) = -K[iu_{xx}] \tag{13}$$

Using initial condition (11), Eq. (13) can be written as:

$$K[u(x,t)] = v(1 + \cosh 2x) - vK[iu_{xx}] \tag{14}$$

The inverse Kamal transform implies that:

$$u(x,t) = (1 + \cosh 2x) - K^{-1}[vK[iu_{xx}]] \tag{15}$$

Now, we apply the homotopy perturbation method, we get:

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = (1 + \cosh 2x) - pK^{-1}\left[ivK\left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right)_{xx}\right] \tag{16}$$

Comparing the coefficient of like powers of p, the following approximations are obtained ;

$$p^0: \; u_0(x,t) = 1 + \cosh 2x$$
$$p^1: \; u_1(x,t) = -K^{-1}[ivK[u_0(x,t)_{xx}]] = -K^{-1}[ivK[4\cosh 2x]]$$
$$= -K^{-1}[4iv^2 \cosh 2x] = -(4it)\cosh 2x$$
$$p^2: \; u_2(x,t) = -K^{-1}[ivK[u_1(x,t)_{xx}]] = -K^{-1}[ivK[-16it \; \cosh 2x]]$$
$$= K^{-1}[16i^2 v^3 \cosh 2x] = \frac{(4it)^2}{2!}\cosh 2x$$
$$p^3: \; u_3(x,t) = -K^{-1}\left[ivK\left[\frac{(4it)^2}{2!} 4\cosh 2x\right]\right] = -\frac{(4it)^3}{3!}\cosh 2x$$

Therefore , the solution $u(x,t)$ is given by:

$$u(x,t) = 1 + cosh\, 2x\left(1 - (4it) + \frac{(4it)^2}{2!} - \frac{4(it)^3}{3!} + \cdots\right) \tag{17}$$

In series form, and

$$u(x,t) = 1 + \cosh 2x\, e^{-4it} \tag{18}$$

Example 3.2

Let's consider the following nonlinear homogeneous Schrodinger Equation :

$$iu_t + u_{xx} + 2|u|^2 u = 0 \tag{19}$$

With the initial condition

$$u(x,0) = e^{ix} \tag{20}$$

Applying the Kamal transform of both sides of Eq. (19),

$$K[u_t] = K[i(u_{xx} + 2|u|^2 u)] \tag{21}$$

Using the differential property of Kamal transform Eq. (21) can be written as:

$$\frac{1}{v}K[u(x,t)] - u(x,0) = K[i(u_{xx} + 2u^2\bar{u})\ ] \tag{22}$$

Where $|u|^2u = u^2\bar{u}$ and $\bar{u}$ is the conjugate of u. Using initial condition (20), Eq. (22) can be written as:

$$K[u(x,t)] = ve^{ix} + vK[i(u_{xx} + 2u^2\bar{u})\ ] \tag{23}$$

The inverse Kamal transform implies that:

$$u(x,t) = e^{ix} + K^{-1}[iv\ K[(u_{xx} + 2u^2\bar{u})\ ]] \tag{24}$$

Now, we apply the homotopy perturbation method, we get:

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = e^{ix} + pK^{-1}[ivK[(\sum_{n=0}^{\infty} p^n u_n(x,t))_{xx} + \sum_{n=0}^{\infty} p^n H_n(u)\ ]] \tag{25}$$

Where $H_n(u)$are He's polynomial [8, 9] that represents the nonlinear terms. The first few components of He's polynomials, are given by

$$H_0(u) = 2u_0{}^2\bar{u}_0$$
$$H_1(u) = 2(u_0{}^2\bar{u}_0 + 2u_1u_0\bar{u}_0)$$

Comparing the coefficients of like powers of p, we have

$$p^0:\ u_0(x,t) = e^{ix}$$
$$p^1:\ u_1(x,t) = K^{-1}\big[ivK[u_0(x,t)_{xx} +\ H_0(u)]\big] = (it)e^{ix}$$
$$p^2:\ u_2(x,t) = K^{-1}\big[ivK[u_1(x,t)_{xx} +\ H_1(u)]\big] = \frac{(it)^2}{2!}e^{ix}$$

Proceeding in a similar manner, we have:

$$p^3:\ u_3(x,t) = \frac{(it)^3}{3!}e^{ix}$$

Therefore , the solution $u(x,t)$ is given by:

$$u(x,t) = e^{ix}\left(1 + (it) + \frac{(it)^2}{2!} + \frac{(it)^3}{3!} + \cdots\right) \tag{26}$$

In series form, and $u(x,t) = e^{i(x+t)}$ (27)

Example 3.3

Let's consider the following nonlinear inhomogeneous Schrodinger Equation :

$$iu_t = -\frac{1}{2}u_{xx} + u\cos^2 x + |u|^2u\ \ ,t \geq 0 \tag{28}$$

With the initial condition :

$$u(x,0) = \sin x \tag{29}$$

Applying the Kamal transform of both sides of Eq. (28), subject to the initial condition (29), we have

$$K[u(x,t)] = v\sin x - ivK[-\frac{1}{2}u_{xx} + u\cos^2 x + u^2\bar{u})\ ] \tag{30}$$

Where $|u|^2u = u^2\bar{u}$ and $\bar{u}$ is the conjugate of , and the inverse of Kamal transform implies that:

$$u(x,t) = \sin x - K^{-1}\left[ivK[-\frac{1}{2}u_{xx} + u\cos^2 x + u^2\bar{u})\ ]\right] \tag{31}$$

Now, we apply the homotopy perturbation method, we get:

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = \sin x - pK^{-1}\left[iv\ K[-\frac{1}{2}(\sum_{n=0}^{\infty} p^n u_n(x,t))_{xx} + \cos^2 x\sum_{n=0}^{\infty} p^n u_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(u)\ ]\right] \tag{32}$$

Comparing the coefficients of like powers of p, we have

$$p^0:\ u_0(x,t) = \sin x\ ,$$
$$p^1:\ u_1(x,t) = \left(\frac{-3it}{2}\right)\sin x\ ,$$
$$p^2:\ u_2(x,t) = \frac{1}{2!}\left(\frac{-3it}{2}\right)^2\sin x\ ,$$
$$p^3:\ u_3(x,t) = \frac{1}{3!}\left(\frac{-3it}{2}\right)^3\sin x$$

Therefore the solution $u(x,t)$ is given by:

$$u(x,t) = \sin x\left(1 + \left(\frac{-3it}{2}\right) + \frac{1}{2!}\left(\frac{-3it}{2}\right)^2 + \frac{1}{3!}\left(\frac{-3it}{2}\right)^3 + \cdots\right) \tag{33}$$

In series form, and

$$u(x,t) = \sin x\, e^{\frac{-3it}{2}} \tag{34}$$

4. Conclusions

In this article, a new modification of HPM, called KTHPM, has been applied for solving linear and non-linear Schrödinger equations with initial conditions. The results show that KTHPM is a powerful tool for obtaining exact and approximate solution of linear and nonlinear equations. By using this method we obtain a new, efficient recurrent relation, to solve linear and nonlinear Schrodinger equations.

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