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Application of New Transform " Kamal Transform'' to Ordinary Differential Equations with Non-Constant Coefficients

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ABSTRACT

In this paper we apply a new integral transform " Kamal transform" to solve ordinary differential equation with non-constant coefficients without resorting to a new frequency domain .

1.Introduction

A new integral transform, called Kamal transform defined for functions of exponential order, is proclaimed. We consider function in the set A, defined by

$$A=\left\{f(t):\exists M,k_1,k_2>0\,.\,|f(t)|<Me^{\frac{|t|}{k_j}},if\ \ t\in(-1)^j\times[0,\infty)\right\} \tag{1}$$

For a given function in the set A, the constant M must be finite, while k_1 and k_2 may be infinite, the variable v in Kamal transform is used to factor the variable t in the argument of the function $f(t)$. Specifically, for $f(t)$ in A . Kamal transform is defined by:

$$K[f(t)]=\int_0^\infty f(t)e^{\frac{-t}{v}}dt=G(v)\,,t\geq 0\ ,\quad k_1\leq v\leq k_2 \tag{2}$$

The next theorem very useful in study of differential equations having non-constant coefficients.

Theorem : Let $G(v)$ is Kamal transform of $f(t)$, then :

a) $K[tf(t)]=v^2\frac{d}{dv}(G(v))$

b) $K[tf'(t)]=v^2\frac{d}{dv}\left[\frac{1}{v}G(v)-f(0)\right]$

c) $K[tf''(t)]=v^2\frac{d}{dv}\left[\frac{1}{v^2}G(v)-\frac{1}{v}f(0)-f'(0)\right]$

d) $K[t^2f(t)]=v^4\frac{d^2}{dv^2}(G(v))+2v^3\frac{d}{dv}(G(v))$

e) $K[t^2f'(t)]=v^4\frac{d^2}{dv^2}\left(\frac{1}{v}G(v)-f(0)\right)+2v^3\frac{d}{dv}\left(\frac{1}{v}G(v)-f(0)\right)$

f) $K[t^2f''^{(t)}]=v^4\frac{d^2}{dv^2}\left(\frac{1}{v^2}G(v)-\frac{1}{v}f(0)-f'(0)\right)+2v^3\frac{d}{dv}\left(\frac{1}{v^2}G(v)-\frac{1}{v}f(0)-f'(0)\right)$

Proof : a) By the definition , we have:

$$G(v)=\int_0^\infty f(t)e^{\frac{-t}{v}}dt$$

Differentiate two sides with respect to v :

$$\frac{d}{dv}(G(v))=\frac{\partial}{\partial v}\int_0^\infty e^{\frac{-t}{v}}f(t)dt$$

And ,

$$G'(v)=\int_0^\infty\frac{\partial}{\partial v}\left(e^{\frac{-t}{v}}\right)f(t)dt=\frac{1}{v^2}\int_0^\infty e^{\frac{-t}{v}}tf(t)dt=\frac{1}{v^2}K[tf(t)] \tag{3}$$

Thus ,

$$K[tf(t)]=v^2\frac{d}{dv}(G(v)) \tag{4}$$

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To prove (b) and (c) , put $f(t) = f'(t)$ and $f(t) = f''(t)$ respectively in Eq.(4) .

d) By differentiating Eq.(3) with respect to v , we get:

$$\frac{d}{dv}(G'(v)) = \frac{\partial}{\partial v}\left\{\frac{1}{v^2}\left(\int_0^\infty e^{\frac{-t}{v}} t f(t) dt\right)\right\}$$

And ,

$$G''(v) = \frac{1}{v^2}\int_0^\infty \frac{t^2}{v^2} f(t) e^{\frac{-t}{v}} dt - \frac{2}{v^3}\int_0^\infty t f(t) e^{\frac{-t}{v}} dt$$

Thus

$$K[t^2 f(t)] = v^4 \frac{d^2}{dv^2}(G(v)) + 2v^3 \frac{d}{dv}(G(v)) \quad (5)$$

To prove (e) and (**f**) , put $f(t) = f'(t)$ and $f(t) = f''(t)$ respectively in Eq.(5) .

2. Application of Kamal Transform of Ordinary Differential Equations:

In this section we solve for some ordinary differential equations with variables coefficients .

Example 2.1

Let's consider the following ordinary differential equation :

$$ty'' + (t+1)y' + 2y = e^{-t} \quad , \quad y(0) = 0 \quad , y'(0) = 1 \quad (6)$$

Applying Kamal transform to both sides of Eq.(6)

$$K[ty'' + (t+1)y' + 2y] = K[e^{-t}] \quad (7)$$

Using the differential property of Kamal transform , Eq.(7) can be written as:

$$v^2 \frac{d}{dv}\left(\frac{1}{v^2}K(y) - \frac{1}{v}y(0) - y'(0)\right) + \left\{\frac{1}{v}K(y) - y(0) + v^2\frac{d}{dv}\left(\frac{1}{v}K(y) - y(0)\right)\right\} + 2K(y) = \frac{v}{1+v}$$

Where $K(y)$ is Kamal transform of the function $y(t)$

Now , applying the initial conditions , we get :

$$K'(y) - \frac{2}{v}K(y) + \frac{1}{v}K(y) + vK'(y) - K(y) + 2K(y) = \frac{v}{1+v}$$

And ,

$$K'(y) + \frac{v-1}{v(1+v)}K(y) = \frac{v}{(1+v)^2}$$

This is a linear differential equation for unknown function $K(y)$, have the Solution in the form :

$$K(y) = \frac{v^2}{(1+v)^2} + \frac{cv}{(1+v)^2} \quad (8)$$

Now , applying the inverse Kamal transform to Eq.(8) , then we find the solution of Eq.(6) is*:*

$$y(t) = K^{-1}\left[\frac{v^2}{(1+v)^2} + \frac{cv}{(1+v)^2}\right] = te^{-t}, \qquad where\ c = 0$$

Example 2.2:

Let's consider the ordinary differential equation :

$$ty'' + y' + ty = 0 \quad , \quad y(0) = 1 \quad , y'(0) = 0 \quad (9)$$

Applying Kamal transform to both sides of Eq.(9)

$$K[y'' + y' + ty] = 0 \quad (10)$$

Using the differential property of Kamal transform , Eq.(10) can be written as:

$$v^2\frac{d}{dv}\left(\frac{1}{v^2}K(y) - \frac{1}{v}y(0) - y'(0)\right) + \left(\frac{1}{v}K(y) - y(0)\right) + v^2\frac{d}{dv}(K(y)) = 0$$

Now , applying the initial conditions , we get

$$K'(y) - \frac{2}{v}K(y) + 1 + \frac{1}{v}K(y) - 1 + v^2K'(y) = 0$$

And ,

$$K'(y)(1+v^2) - \frac{1}{v}K(y) = 0$$

Or

$$\frac{K'(y)}{K(y)} = \frac{1}{v(1+v^2)} = \frac{1}{v} - \frac{v}{(1+v^2)}$$

Thus ,

$$K(y) = \frac{cv}{\sqrt{1+v^2}} \quad (11)$$

Now , applying the inverse Kamal transform to Eq.(11) ,then the solution of Eq.(9) is*:*

$$y(t) = cK^{-1}\left[\frac{v}{\sqrt{1+v^2}}\right] = J_0(t) \ , \ where \quad c = 1$$

Example 2.3:

Let's consider the ordinary differential equation :

$$t^2y' + 2ty = \sinh t \quad , \quad y(0) = \frac{1}{2} \quad (12)$$

Applying Kamal transform to both sides of Eq.(12)

$$K[t^2y' + 2ty] = K[\sinh t] \quad (13)$$

Using the differential property of Kamal transform , Eq.(13) can be written as:

$$v^4 \frac{d^2}{dv^2}\left(\frac{1}{v}K(y) - y(0)\right) + 2v^3 \frac{d}{dv}\left(\frac{1}{v}K(y) - y(0)\right) + 2v^2 K'(y) = \frac{v^2}{1 - v^2}$$

Now , applying the initial conditions , we get

$$v^3 K''(y) + 2v^2 K'(y) = \frac{v^2}{1 - v^2} = \sum_{j=1}^{\infty} v^{2j+2}$$

And ,

$$v^3 K''(y) + 2v^2 K'(y) = \sum_{j=1}^{\infty} v^{2j+2}$$

And ,

$$v^2 K''(y) + 2vK'(y) = \sum_{j=1}^{\infty} v^{2j+1}$$

Or

$$\frac{d}{dv}\left(v^2 K'(y)\right) = \sum_{j=1}^{\infty} v^{2j+1}$$

Thus ,

$$K(y) = \frac{\sum_{j=1}^{\infty} v^{2j+1}}{(2j+2)(2j+1)} + \frac{c_1}{v} + c_2 \quad (14)$$

Now , applying the inverse Kamal transform to Eq.(14) , then the solution of Eq.(12) is:

$$y(t) = K^{-1}\left[\frac{\sum_{j=1}^{\infty} v^{2j+1}}{(2j+2)(2j+1)} + \frac{c_1}{v} + c_2\right] = \frac{\sum_{j=1}^{\infty} t^{2j}}{(2j+2)!}, \quad where \quad c_1 = c_2 = 0$$

Example 2.4

Let's consider the following ordinary differential equation :

$$t^2 y'' + 4ty' + 2y = 12t^2 \quad , \ y(0) = y'(0) = 0 \quad (15)$$

Applying Kamal transform to both sides of Eq.(9)

$$K[t^2 y'' + 4ty' + 2y] = K[12t^2] \quad (16)$$

Using the differential property of Kamal transform , Eq.(16) can be written as:

$$\left\{v^4 \frac{d^2}{dv^2}\left(\frac{1}{v^2}K(y) - \frac{1}{v}y(0) - y'(0)\right) + 2v^3 \frac{d}{dv}\left(\frac{1}{v^2}K(y) - \frac{1}{v}y(0) - y'(0)\right)\right\} + \left\{4v^2 \frac{d}{dv}\left(\frac{1}{v}K(y) - y(0)\right)\right\} + 2K(y) = 24v^3$$

Now , applying the initial conditions , we get

$$\{v^2 K''(y) - 2vK'(y) + 2K(y)\} + 4vK'(y) - 4K(y) + 2K(y) = 24v^3$$

And ,

$$v^2 K''(y) + 2vK'(y) = 24v^3$$

or

$$\frac{d}{dv}\left(v^2 K'(y)\right) = 24v^3$$

And ,

$$K'(y) = \frac{1}{v^2}\left(\int 24v^3 \, dv + c_1\right)$$

Thus ,

$$K(y) = 2v^3 + \frac{c_1}{v} + c_2 \quad (17)$$

Now , applying the inverse Kamal transform to Eq.(17) , then the solution of Eq.(15) is:

$$y(t) = K^{-1}\left[2v^3 + \frac{c_1}{v} + c_2\right] = t^2 \ , where \quad c_1 = c_2 = 0$$

Conclusion

Application of Kamal transform to Solution of ordinary differential equation with variable Coefficients has been demonstrated.

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