

ON Ω_{gb}^+ -closed sets in simple extension topological spaces

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ABSTRACT

This paper serves as a platform to discuss and bring out the concept of kernel, separation axiom and continuity of Ω_{gb}^+ and $\bar{\Omega}_{gb}^+$ -closed sets, under the light of simple extension topological spaces.

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Keywords

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1. Introduction

A new class of generalized open sets called b-open sets in topological spaces was defined by Andrijevic [2]. The class of all b open sets generates the same topology as the class of all pre-open sets. In 1986, Maki [11] introduced the concept of generalized Λ sets and defined the associated closure operators by using the work of Levine [8] and Dunham [5]. Caldas and Dontchev [3] introduced Λ_s -sets, V_s -sets, $g\Lambda_s$ -sets and gV_s -sets. Ganster and et al. [6] introduced the notion of pre Λ -sets and pre V -sets and obtained new topologies via these sets. M.E. Abd El-Monsef et al. [1] defined $b\Lambda$ -sets and bV -sets on a topological space and proved that it forms a topology. In 1963 Levine [9] introduced the concept of a simple extension of a topology τ as $\tau(B) = \{(B \cap O) \cup O' / O, O' \in \tau \text{ and } B \notin \tau\}$. Sr. I. Arockiarani and F. Nirmala Irudayam [12] introduced the concept of b^+ -open sets in extended topological spaces. Caldas and Jafari[4] introduced the notions of $\Lambda_\delta - T_0$, $\Lambda_\delta - T_1$ and $\Lambda_\delta - T_2$ topological spaces. S. Reena and F. Nirmala Irudayam [14] devised a new form of continuity and T. Noiri, Sr. I. Arockiarani and F. Nirmala Irudayam [13] coined the idea of Ω_{gb}^{+*} , $\bar{\Omega}_{gb}^{+*}$ sets in simple extended topological spaces. T. Madhumathi and F. Nirmala Irudayam [10] proposed the idea of $\Omega_{gb}^+(S)$ and $\bar{\Omega}_{gb}^+(S)$ sets in simple extension ideal topological spaces.

2. Preliminaries

All through the paper the space X is a SETS in which no separation axioms are assumed unless and otherwise stated.

Definition 2.1

A subset A of a topological space (X, τ) is said to be,

- (i) b-open set[2], if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$ and b-closed set $\text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \subseteq A$.
- (ii) a generalized closed (briefly g-closed) [7] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (iii) a generalized b-closed (briefly bg-closed) [6] if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (iv) πgb -closed[15] if $\text{bcl}(A) \subseteq A$ whenever $A \subseteq U$ and U is π -open in (X, τ) . By $\pi GB^+C(X, \tau)$ we mean the family of all πgb -closed subsets of the space (X, τ)

Definition 2.2[12]: A subset A of a topological space (X, τ) is said to be,

- (i) b^+ -open set if $A \subseteq \text{cl}^+(\text{int}(A)) \cup \text{int}(\text{cl}^+(A))$ and b-closed set $\text{cl}^+(\text{int}(A)) \cup \text{int}(\text{cl}^+(A)) \subseteq A$.
- (ii) a generalized⁺ closed (briefly g^+ -closed) if $\text{cl}^+(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (iii) a generalized b^+ -closed (briefly bg^+ -closed) if $\text{bcl}^+(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
- (iv) πgb^+ -closed[14] if $\text{bcl}^+(A) \subseteq A$ whenever $A \subseteq U$ and U is π^+ -open in (X, τ^+) . By $\pi GB^+C(X, \tau^+)$ we mean the family of all πgb^+ -closed subsets of the space (X, τ^+) .

Definition 2.3[10]: Let S be a subset of a topological space (X, τ^+) we define the sets $\Omega_{gb}^+(S)$ and $\bar{\Omega}_{gb}^+(S)$ as follows, $\Omega_{gb}^+(S) = \bigcap \{G | G \in \pi GB^+O(X, \tau^+) \text{ and } S \subseteq G\}$, $\bar{\Omega}_{gb}^+(S) = \bigcup \{F | F \in \pi GB^+C(X, \tau^+) \text{ and } S \supseteq F\}$.

Definition 2.4[14]: A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called

- (i) π^+ -irresolute if $f^{-1}(V)$ is π^+ -closed in (X, τ^+) for every π^+ -closed set V of (Y, σ^+) .
- (ii) b^+ -irresolute if for each b^+ -open set V in (Y, σ^+) , $f^{-1}(V)$ is b^+ -open in (X, τ^+) .
- (iii) b^+ -continuous if for each open set V in (Y, σ^+) , $f^{-1}(V)$ is b^+ -open in (X, τ^+) .

3. Ω_{gb}^+ -KERNEL

Definition 3.1: Let (X, τ^+) be a topological space, $A \subset X$. Then Ω_{gb}^+ -kernel of A is defined by $\Omega_{gb}^+-\text{Ker}(A) = \bigcap \{G / G \in \Omega_{gb}^+O(X, \tau^+) \text{ and } A \subset G\}$

Definition 3.2: A point $x \in X$ is called Ω_{gb}^+ -cluster point of A if for every Ω_{gb}^+ -open set U containing x , $A \cap U \neq \emptyset$.

Let (X, τ^+) be a topological space and A, B be subsets of X , Let $x, y \in X$ then we have the following lemmas.

Lemma 3.3: $A \subset \Omega_{gb}^+-\text{Ker}(A)$

Proof: Let $x \notin \Omega_{gb}^+ - \text{Ker}(A)$ then there exists $V \in \Omega_{gb}^+ - O(X, \tau^+)$ such that $A \subset V$ and $x \notin V$. Hence $x \notin A$.

Lemma 3.4: If $A \subset B$, then $\Omega_{gb}^+ - \text{Ker}(A) \subset \Omega_{gb}^+ - \text{Ker}(B)$.

Proof: Let $x \notin \Omega_{gb}^+ - \text{Ker}(B)$. Then there exists $G \in \Omega_{gb}^+ - O(X, \tau^+)$ such that $B \subset G$ and $x \notin G$. Since $A \subset B$, $A \subset G$ and hence $x \notin \Omega_{gb}^+ - \text{Ker}(A)$.

Lemma 3.5: $\Omega_{gb}^+ - \text{Ker}(A) = \Omega_{gb}^+ - \text{Ker}(\Omega_{gb}^+ - \text{Ker}(A))$.

Proof: Let $x \in \Omega_{gb}^+ - \text{Ker}(\Omega_{gb}^+ - \text{Ker}(A))$ then for every Ω_{gb}^+ -open set, $G \supset \Omega_{gb}^+ - \text{Ker}(A)$, $x \in G$. Since $A \subset \Omega_{gb}^+ - \text{Ker}(A)$, for every Ω_{gb}^+ -open set $G \supset A$, $x \in G$. Hence $x \in \Omega_{gb}^+ - \text{Ker}(A)$. Therefore $\Omega_{gb}^+ - \text{Ker}(\Omega_{gb}^+ - \text{Ker}(A)) \subset \Omega_{gb}^+ - \text{Ker}(A)$. Also $\Omega_{gb}^+ - \text{Ker}(A) \subset \Omega_{gb}^+ - \text{Ker}(\Omega_{gb}^+ - \text{Ker}(A))$. Hence $\Omega_{gb}^+ - \text{Ker}(A) = \Omega_{gb}^+ - \text{Ker}(\Omega_{gb}^+ - \text{Ker}(A))$.

Lemma 3.6: $y \in \Omega_{gb}^+ - \text{Ker}(\{x\})$ if $x \in \Omega_{gb}^+ - \text{cl}(\{y\})$

Proof: Let $y \notin \Omega_{gb}^+ - \text{Ker}(\{x\}) \Leftrightarrow$ there exists a Ω_{gb}^+ -open set $V \supset \{x\}$ such that $y \notin V \Leftrightarrow$ there exists a Ω_{gb}^+ -open set $V \supset \{x\}$ such that $\{y\} \cap V = \emptyset \Leftrightarrow x$ is not a Ω_{gb}^+ -cluster point of $\{y\} \Leftrightarrow x \notin \Omega_{gb}^+ - \text{cl}(\{y\})$

4. $\Omega_{gb}^+ - T_k$ SPACES

Definition 4.1: (X, τ^+) is $\Omega_{gb}^+ - T_0$ if for each pair of distinct points x, y of X , there exists a Ω_{gb}^+ -open set containing one of points but not the other.

Theorem 4.2: (X, τ^+) is $\Omega_{gb}^+ - T_0$ iff for each pair of distinct points x, y of X , $\Omega_{gb}^+ - \text{cl}(\{x\}) \neq \Omega_{gb}^+ - \text{cl}(\{y\})$.

Proof: Necessity: Let (X, τ^+) be a $\Omega_{gb}^+ - T_0$ space. Let $x, y \in X$ such that $x \neq y$. Then there exists a Ω_{gb}^+ -open set V containing one of the points but not the other, say $x \in V$ and $y \notin V$. Then V^c is Ω_{gb}^+ -closed set containing y but not x . But $\Omega_{gb}^+ - \text{cl}(\{y\})$ is the smallest Ω_{gb}^+ -closed set containing y . Therefore $\Omega_{gb}^+ - \text{cl}(\{y\}) \subset V^c$. Hence $x \notin \Omega_{gb}^+ - \text{cl}(\{y\})$. Thus $\Omega_{gb}^+ - \text{cl}(\{x\}) \neq \Omega_{gb}^+ - \text{cl}(\{y\})$.

Sufficiency: Suppose $x, y \in X$, $x \neq y$ and $\Omega_{gb}^+ - \text{cl}(\{x\}) \neq \Omega_{gb}^+ - \text{cl}(\{y\})$. Let $z \in X$ such that $z \in \Omega_{gb}^+ - \text{cl}(\{x\})$ but $z \notin \Omega_{gb}^+ - \text{cl}(\{y\})$. If $x \in \Omega_{gb}^+ - \text{cl}(\{y\})$, then $\Omega_{gb}^+ - \text{cl}(\{x\}) \subset \Omega_{gb}^+ - \text{cl}(\{y\})$ and hence $z \in \Omega_{gb}^+ - \text{cl}(\{y\})$. This is a contradiction. Therefore $x \notin \Omega_{gb}^+ - \text{cl}(\{y\})$. That implies $x \in (\Omega_{gb}^+ - \text{cl}(\{y\}))^c$. Therefore $(\Omega_{gb}^+ - \text{cl}(\{y\}))^c$ is a Ω_{gb}^+ -open set containing x but not y . Hence (X, τ^+) is $\Omega_{gb}^+ - T_0$.

Definition 4.3: (X, τ^+) is $\Omega_{gb}^+ - T_1$ if for any pair of distinct points x, y of X , there is a Ω_{gb}^+ -open set U in X such that $x \in U$ and $y \notin U$ and there is a Ω_{gb}^+ -open set V in X such that $y \in V$ and $x \notin V$.

Remark 4.4: Every $\Omega_{gb}^+ - T_1$ space is $\Omega_{gb}^+ - T_0$ space. But the converse need not be true. For example, let $X = \{a, b, c\}$, $\tau = \{X, \Phi, \{a\}, \{a, b\}\}$ and $B = \{b\}$, $\tau^+ = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$. Then (X, τ^+) is $\Omega_{gb}^+ - T_0$ space but not $\Omega_{gb}^+ - T_1$ space.

Theorem 4.5: In a space (X, τ^+) , the following are equivalent

- (1) (X, τ^+) is $\Omega_{gb}^+ - T_1$
- (2) For every $x \in X$, $\{x\} = \Omega_{gb}^+ - \text{cl}(\{x\})$.
- (3) The intersection of all Ω_{gb}^+ -open sets containing the point x in X is $\{x\}$.

Proof: (1) \Rightarrow (2): Suppose $y \neq x$ in X . Then there exists a Ω_{gb}^+ -open set V such that $x \in V$ and $y \notin V$. If $x \in \Omega_{gb}^+ - \text{cl}(\{y\})$, then x is a cluster point of $\{y\}$. That implies for every Ω_{gb}^+ -open set U containing x , $\{y\} \cap U \neq \emptyset$. Here V is a Ω_{gb}^+ -open set containing x . Therefore $\{y\} \cap V \neq \emptyset$ implies $y \in V$. This is a contradiction. Thus $x \notin \Omega_{gb}^+ - \text{cl}(\{y\})$. Hence for a point x , $y \notin \Omega_{gb}^+ - \text{cl}(\{x\})$. Thus $\{x\} = \Omega_{gb}^+ - \text{cl}(\{x\})$. (2) \Rightarrow (3): $x \in \Omega_{gb}^+ - \text{cl}(\{y\}) \Leftrightarrow x$ is a Ω_{gb}^+ -cluster point of $\{x\} \Leftrightarrow$ for every Ω_{gb}^+ -open set U containing x , $\{x\} \cap U \neq \emptyset$ if and only if $x \in \bigcap \{G/G \in \Omega_{gb}^+ - O(X, \tau^+) \text{ and } \{x\} \subset G\}$. Therefore $\Omega_{gb}^+ - \text{cl}(\{x\}) = \bigcap \{G/G \in \Omega_{gb}^+ - O(X, \tau^+) \text{ and } \{x\} \subset G\}$. (3) \Rightarrow (1): Let $x \neq y$ in X . By (3), and $\{x\} \subset G$. Hence there exists one Ω_{gb}^+ -open set V containing x but not y . Similarly, there exists one Ω_{gb}^+ -open set U containing y but not x . Hence (X, τ) is $\Omega_{gb}^+ - T_1$.

Theorem 4.6: A space (X, τ^+) is $\Omega_{gb}^+ - T_1$ if the singletons are $\Omega_{gb}^+ -$ closed sets

Proof: Suppose (X, τ^+) is $\Omega_{gb}^+ - T_1$. Let $x \in X$ and $y \in \{x\}^c$. Then $x \neq y$ and so there exists a Ω_{gb}^+ -open set U_y such that $y \in U_y$ but $x \notin U_y$. Therefore $y \in U_y \subset \{x\}^c$. That is, $\{x\}^c = \bigcup \{U_y/y \in \{x\}^c\}$ is Ω_{gb}^+ -open. Hence $\{x\}$ is Ω_{gb}^+ -closed. Conversely, let $x, y \in X$ with $x \neq y$. Then $y \in \{x\}^c$ and $\{x\}^c$ is a Ω_{gb}^+ -open set containing y but not x . Similarly $\{y\}^c$ is a Ω_{gb}^+ -open set containing x but not y . Hence (X, τ^+) is a $\Omega_{gb}^+ - T_1$.

Definition 4.7: (X, τ^+) is $\Omega_{gb}^+ - T_2$ if for each pair of distinct points x and y in X there exists a Ω_{gb}^+ -open set U and a Ω_{gb}^+ -open set V in X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Remark 4.8: Every $\Omega_{gb}^+ - T_2$ space is $\Omega_{gb}^+ - T_1$.

Theorem 4.9: For a topological space (X, τ^+) , the following are equivalent:

- (1) (X, τ^+) is $\Omega_{gb}^+ - T_2$.
- (2) If $x \in X$, then for each $y \neq x$, there is a Ω_{gb}^+ -open set U containing x such that $y \notin \Omega_{gb}^+ - \text{cl}(U)$.
- (3) For each $x \in X$, $\{x\} = \bigcap \{\Omega_{gb}^+ - \text{cl}(U) \mid U \text{ is a } \Omega_{gb}^+ - \text{open set containing } x\}$.

Proof: (1) \rightarrow (2): Let $x \in X$. Then for each $y \neq x$, there exists Ω_{gb}^+ -open sets A and B such that $x \in A$, $y \in B$ and $A \cap B = \emptyset$. Then $x \in A \subset X - B = F$. Then F is Ω_{gb}^+ -closed. $A \subset F$ and $y \notin F$. That implies $y \notin \bigcap \{F/F \text{ is } \Omega_{gb}^+ - \text{closed and } A \subset F\} = \Omega_{gb}^+ - \text{cl}(A)$.

(2) \rightarrow (1): Let $x, y \in X$ and $x \neq y$. By (2), there exists a Ω_{gb}^+ -open set U containing x such that $y \notin \Omega_{gb}^+ - \text{cl}(U)$. Therefore $y \in X - (\Omega_{gb}^+ - \text{cl}(U))$, $X - (\Omega_{gb}^+ - \text{cl}(U))$ is Ω_{gb}^+ -open and $\notin X - (\Omega_{gb}^+ - \text{cl}(U))$. Also $U \cap X - (\Omega_{gb}^+ - \text{cl}(U)) = \emptyset$. Hence (X, τ^+) is $\Omega_{gb}^+ - T_2$.

(3) \leftrightarrow (1): Obvious.

5. $\Omega_{gb}^+ -$ CONTINUOUS AND $\Omega_{gb}^+ -$ IRRESOLUTE FUNCTIONS

Definition 5.1: A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called Ω_{gb}^+ -continuous if every $f^{-1}(V)$ is Ω_{gb}^+ -closed in (X, τ^+) for every closed set V of (Y, σ^+) .

Definition 5.2: A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is called Ω_{gb}^+ -irresolute if $f^{-1}(V)$ is Ω_{gb}^+ -closed in (X, τ^+) for every Ω_{gb}^+ -closed set V in (Y, σ^+) .

Definition 5.3: A function $f: X \rightarrow Y$ is said to be pre b^+ -closed if $f(U)$ is b^+ -closed in Y for each b^+ -closed set in X .

Remark 5.4: Composition of two Ω_{gb}^+ -continuous functions need not be Ω_{gb}^+ -continuous.

Example 5.5: Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $B = \{c\}$, $\tau^+ = \{X, \Phi, \{a\}, \{c\}, \{a, b\}\}$. $\sigma = \{X, \phi, X, \{a\}, \{a, b\}\}$ and $B = \{b\}$, $\sigma^+ = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$. $\eta = \{X, \phi, \{c\}, \{a, c\}\}$ and $B = \{b\}$, $\eta^+ = \{X, \Phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. Define $f: (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$. Define $g: (X, \sigma) \rightarrow (X, \eta)$ by $g(a) = a$, $g(b) = b$, $g(c) = c$. Then f and g are Ω_{gb}^+ -continuous but $g \circ f$ is not Ω_{gb}^+ -continuous.

Proposition 5.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be π^+ -irresolute and pre b^+ -closed. Then $f(A)$ is Ω_{gb}^+ -closed in Y for every Ω_{gb}^+ -closed set A of X .

Proof: Let A be Ω_{gb}^+ -closed in X . Let $f(A) \subset V$ is π -open in Y . Then $A \subset f^{-1}(V)$ and A is Ω_{gb}^+ -closed in X implies $b^+cl(A) \subset f^{-1}(V)$. Hence $f(b^+cl(A)) \subset V$. Since f is pre b^+ -closed, $b^+cl(f(A)) \subset b^+cl(f(b^+cl(A))) = f(b^+cl(A)) \subset V$. Hence $f(A)$ is Ω_{gb}^+ -closed in Y .

Definition 5.7: A topological space X is a Ω_{gb}^+ -space if every Ω_{gb}^+ -closed set is closed.

Proposition 5.8: Every Ω_{gb}^+ -space is Ω_{gb}^+ - $T_{1/2}$ space.

Theorem 5.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function.

(1) If f is Ω_{gb}^+ -irresolute and X is Ω_{gb}^+ - $T_{1/2}$ space, then f is b^+ -irresolute.

(2) If f is Ω_{gb}^+ -continuous and X is Ω_{gb}^+ - $T_{1/2}$ space, then f is b^+ -continuous.

Proof: (1) Let V be b^+ -closed in Y . Since f is Ω_{gb}^+ -irresolute, $f^{-1}(V)$ is Ω_{gb}^+ -closed in X . Since X is Ω_{gb}^+ - $T_{1/2}$ space, $f^{-1}(V)$ is b^+ -closed in X . Hence f is b^+ -irresolute. (2) Let V be closed in Y . Since f is Ω_{gb}^+ -continuous, $f^{-1}(V)$ is Ω_{gb}^+ -closed in X . By assumption, it is b^+ -closed. Hence f is b^+ -continuous.

Definition 5.10: A function $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is π^+ -open map if $f(U)$ is π^+ -open in Y for every π^+ -open in X .

Theorem 5.11: If the bijective $f: (X, \tau^+) \rightarrow (Y, \sigma^+)$ is b^+ -irresolute and π^+ -open map, then f is Ω_{gb}^+ -irresolute.

Proof: Let V be Ω_{gb}^+ -closed in Y . Let $f^{-1}(V) \subset U$ where U is π^+ -open in X . Hence $V \subset f(U)$ and $f(U)$ is π^+ -open implies $b^+cl(V) \subset f(U)$. Since f is b^+ -irresolute, $(f^{-1}(b^+cl(V)))$ is b^+ -closed. Hence $b^+cl(f^{-1}(V)) \subset b^+cl(f^{-1}(b^+cl(V))) = f^{-1}(b^+cl(V)) \subset U$. Therefore, f is Ω_{gb}^+ -irresolute.

Theorem 5.12: If $f: X \rightarrow Y$ is π^+ -open, b^+ -irresolute, pre b^+ -closed surjective function. If X is Ω_{gb}^+ - $T_{1/2}$ space, then Y is Ω_{gb}^+ - $T_{1/2}$ space.

Proof: Let F be a Ω_{gb}^+ -closed set in Y . Let $f^{-1}(F) \subset U$ where U is π^+ -open in X . Then $F \subset f(U)$ and F is a Ω_{gb}^+ -closed set in Y implies $b^+cl(F) \subset f(U)$. Since f is b^+ -irresolute, $b^+cl(f^{-1}(F)) \subset b^+cl(f^{-1}(b^+cl(F))) = f^{-1}(b^+cl(F)) \subset U$. Therefore $f^{-1}(F)$ is Ω_{gb}^+ -closed in X . Since X is Ω_{gb}^+ - $T_{1/2}$ space, $f^{-1}(F)$ is b^+ -closed in X . Since f is pre b^+ -closed, $f(f^{-1}(F)) = F$ is b^+ -closed in Y . Hence Y is Ω_{gb}^+ - $T_{1/2}$ space.

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