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Viscous dissipation and heat source effects on free convective boundary layer slip flow in the presence of induced magnetic field

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ABSTRACT

In this analysis, the viscous dissipation and heat source effects on free convective boundary layer slip flow due to induced magnetic field is studied. The governing partial differential equations are converted into ordinary differential equations by using the similarity transformations. In which the coupled non linear differential equations are solved by using regular perturbation technique. The effects of various parameters on the velocity, temperature, induced magnetic field and also the local skin friction, the rate of heat transfer are presented through graphically and tabulated values in detailed.

Introduction

Free convection flow involving heat transfer occurs frequently in nature and in industrial processes. A few representative fields of interest in which the heat transfer plays an important role in designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution. Ostrach [1] is the initiator of the study of convection flow, made a technical note on the similarity solution of transient free convection flow past a semi infinite vertical plate by an integral method. Pop and Soundalgekar [2] have investigated the free convection flow past an accelerated infinite plate. Raptis et al. [3] have studied the unsteady free convective flow through a porous medium adjacent to a semi infinite vertical plate. Singh and Soundalgekar [4] have investigated the problem of transient free convection in cold water past an infinite vertical porous plate.

Magnetohydrodynamics is the science which deals with the motion of highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid [5]. Due to its importance the following researchers have investigated the magnetic field effect on the fluid flow problems [6–10]. Also, Jha [11] has analytically studied the natural convective flow along a vertical infinite plate under a constant magnetic field. Kim [12] considered effect of heat transfer on unsteady MHD convective flow past a semi-infinite vertical porous moving plate with variable suction. Sharma and Pareek [13] explained the behavior of steady free convective MHD flow past a vertical porous moving surface. Abd El-Naby et al. [14] presented a finite difference solution of radiation effects on MHD unsteady free-convection flow over a vertical porous plate. Alam et al. [15] have also studied the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field.

Viscous dissipation effects are usually ignored in macro scale systems, in laminar flow in particular, except for very viscous liquids at comparatively high velocities. However, even for common liquids at laminar Reynolds numbers, frictional effects in micro scale systems may change the energy equation [33]. Furthermore, viscous dissipation effects may be very significant for fluids with high viscosities and low specific heat capacities, even at relatively low Reynolds numbers. Accordingly, the viscous dissipation term should be considered in the micro scale systems. The viscous dissipative heat effects on the steady or unsteady free convection and on combined free and forced convection flows have been extensively studied by Ostrach [16-20]. Gebhart [21] analyze the effect of dissipation on natural convection. Soundalgekar [22] studied natural convection flow along vertical porous plate with suction and viscous dissipation. Soundalgekar [23] studied viscous dissipative effects on unsteady free convective flow past a vertical porous plate with constant suction. Tak and Lodha [24] analyzed the flow and heat transfer due to a stretching porous surface in presence of transverse magnetic field including heat due to viscous dissipation. Jaber [25] examined the effects of viscous dissipation and joule heating on magneto hydrodynamics flow of a fluid with variable properties past a stretching vertical plate.

The study of heat source/sink effects on heat transfer is very important in view of several physical problems. Afore-mentioned studies include only the effect of a uniform heat source/sink (i.e., temperature dependent heat source/sink) on heat transfer. Pal and Chatterjee [26, 27], Subhas Abel and Mahesha [28]), Rahman et al. [29], Bataller [30], have included the effect of a non-uniform heat source, but confined to the case of viscous fluids only. Chamkha and Khaled [31] investigated heat generation/absorption effects on hydromagnetic combined heat and mass transfer flow from an inclined plate.

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The effects of a heat source/sink on unsteady MHD convection through porous medium with combined heat and mass transfer was studied by Kamel [32]. Chamkha [33] solved the problem of unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Vajravelu [34] investigated the natural convection at a heated semi infinite vertical plate with internal heat generation.

In the recent years, in case of micro-electromechanical systems micro-scale fluid dynamics received much attention [35-37]. Due to the micro-scale dimensions, the fluid flow behaviour belongs to the slip flow regime and greatly differs from the traditional flow [35]. The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances [38, 39]. Cao and Baker [40] studied the mixed convective flow and heat transfer from a vertical plate by considering velocity slip and temperature jump boundary conditions and they obtained local non-similar solutions. Steady magneto-hydrodynamics (MHD) flow past a permeable surface with partial slip in the presence of the viscous dissipation effect under convective heating boundary conditions was studied by Mohammad [41]. MHD flow with slip effects and temperature-dependent heat source in a vertical wavy porous space was investigated by Srinivas and Muthuraj [42].

In almost all the previous papers, the authors neglected the induced magnetic field. However, in various physical states it is necessary to consider this effect in governing equations. This assumption is considered in order to simplify the mathematical analysis of the problem. Furthermore, the induced magnetic field produces its own magnetic field in the fluid; therefore, it can amend the original magnetic field. With this intention, the aim of the present investigation is to study the heat transfer characteristics on a viscous dissipative fluid over a vertical semi-infinite plate in the presence of an induced magnetic field. The convective type boundary conditions on temperature, partial slip and heat source are taking into account. The seek solutions for velocity, temperature and induced magnetic field are computed by regular perturbation technique. The effects of governing flow parameters are presented through graphs and tables.

1. Mathematical Formulation:

The two-dimensional steady magneto-hydrodynamic mixed convective heat transfer flow of a Newtonian, electrically conducting, viscous incompressible fluid over a porous vertical infinite plate with the viscous/magnetic dissipation of energy has been considered. The $\bar{x}$-axis is taken vertically upwards along the plate, $\bar{y}$-axis normal to the plate in the fluid region.It is assumed that the plate is electrically non-conducting and the applied magnetic field is of uniform strength (H_0) and perpendicular to the plate (see Fig.1).The magnetic Reynolds number of the flow is taken into consideration, so that the presence of induced magnetic field is also considered.

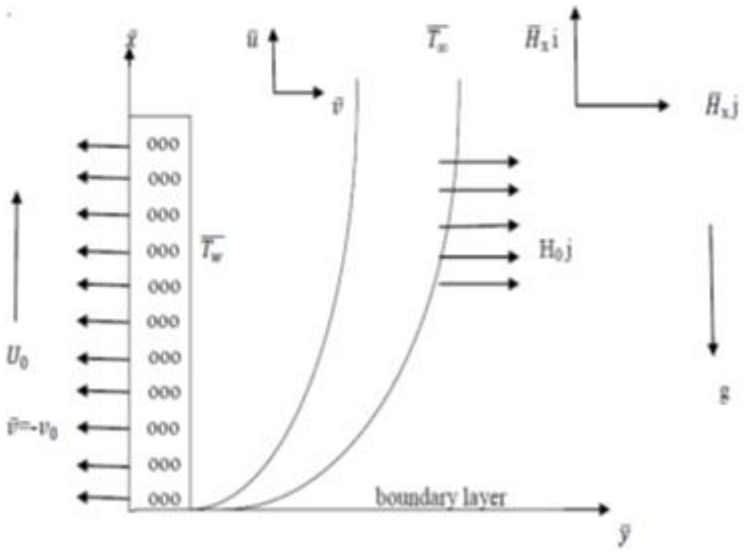

Fig 1. Physical configuration and coordinate system.

Let the plate be long enough in $\bar{x}$ -direction for the flow to be parallel. Let $(\bar{u}, \bar{v}, 0)$ be the be fluid velocity and ($\bar{H}_x$, $\bar{H}_y$, 0) be the magnetic induction vector at a point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid since the plate is infinite in length in $\bar{x}$ –direction, therefore all the physical quantities except possibly the pressure are assumed to be independent of the $\bar{x}$. The wall is maintained at a constant temperature $\overline{T_w}$ higher than ambient temperature $\overline{T_\infty}$. All the gas properties are considered constant except that the influence of density variation with temperature has been considered only in the body force term. The plate is subjected to a constant suction velocity. The equation of conservation of electric charge is $\nabla . J = 0.$ where $J = (J_x, J_y, J_z)$ .The direction of propagation is considered only along $\bar{y}$- axis and does not have any variation along the $\bar{y}$- axis and so $\dfrac{\partial J_y}{\partial y} = 0,$ which gives J_y =constant (see Ahmed [6, 9]).

Under above assumptions, the flow is governed by the following x -momentum equation.

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\mu_0 H_0}{\rho}\frac{\partial \bar{H}_x}{\partial \bar{y}} \quad (1)$$

The first in R.H.S of Eq. (1) shows the mixed convection term. It is assumed that the velocity gradient is very small and hence the viscous term in the above equation is vanished. Therefore at the absence induced magnetic field we may have -

$$\frac{\partial p}{\partial x} = -\rho_\infty g \quad (2)$$

Eliminating the pressure from Eqs. (1) and (2) and by using Boussinesq's approximation $\rho_\infty - \rho = \rho_\infty \beta(\bar{T} - \bar{T}_\infty)$, equation (1) takes the following form.

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\mu_0 H_0}{\rho}\frac{\partial \bar{H}_x}{\partial \bar{y}} \quad (3)$$

Similarity the equation of energy and magnetic induction are given below respectively.

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{v}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{1}{\sigma\rho C_p}\left(\frac{\partial \bar{H}_x}{\partial \bar{y}}\right)^2 + \bar{Q}\frac{\partial}{\partial \bar{y}}(\bar{T} - \bar{T}_\infty) \quad (4)$$

$$\bar{v}\frac{\partial \bar{H}_x}{\partial \bar{y}} = \frac{1}{\sigma\mu_0}\frac{\partial^2 \bar{H}_x}{\partial \bar{y}^2} + H_0\frac{\partial \bar{u}}{\partial \bar{y}} \quad (5)$$

The boundary conditions are

$$\bar{y}=0;\ \bar{u}=L\frac{\partial \bar{u}}{\partial u y},\quad \bar{v} = -v_0,\ -\kappa\frac{\partial \bar{T}}{\partial y} = -\left(T_w - T_\infty\right),\ \overline{H_x} = 0,$$

$$\bar{y} \to \infty : \bar{u} \to U_0,\ \bar{T} \to \bar{T}_\infty,\ \overline{H_x} \to 0 \quad (6)$$

The non-dimensional quantities are:

$$y = \frac{v_0 \bar{y}}{v},\ u = \frac{\bar{u}}{U_0},\ \theta = \frac{\bar{T} - \bar{T}_\infty}{\overline{T_w} - \bar{T}_\infty},\ \Pr = \frac{\rho v C_p}{k},\ Gr = \frac{vg\beta(\overline{T_w} - \bar{T}_\infty)}{U_0 v_0^{\,2}}, \quad (7)$$

$$H = \sqrt{\frac{\mu_0}{\rho}}\frac{\bar{H}_x}{U_0},\ E_c = \frac{U_0^2}{C_p(\overline{T_w} - \bar{T}_\infty)},\ \mathrm{P}_{rm} = \sigma v\mu_0,\ M = \sqrt{\frac{\mu_0}{\rho}}\frac{\bar{H}_0}{v_0},\ Q = \frac{\bar{Q}}{v_0},$$

Using the transformations (7), the non–dimensional governing equations in sets of Ordinary are as follows:

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + M\frac{dH}{dy} = -Gr\theta \quad (8)$$

$$\frac{d^2\theta}{dy^2} + \Pr\frac{d\theta}{dy} = -\left(Ec\Pr\left(\frac{du}{dy}\right)^2 + \frac{Ec\Pr}{\Pr m}\left(\frac{dH}{dy}\right)^2\right) + Q\frac{d\theta}{dy} \quad (9)$$

$$\frac{d^2H}{dy^2} + M\Pr m\frac{du}{dy} + \Pr m\frac{dH}{dy} = 0 \quad (10)$$

The corresponding boundary conditions are

$$y = 0;\ u = h\frac{du}{dy},\ \theta = -\gamma\left(1-\theta\right);\ H = 0;$$

$$y \to \infty;\ u \to 1,\ \theta \to 0;\ H \to 0; \quad (11)$$

3. Method of Solutions

In order to solve the Eqs. (8)-(10) under the boundary conditions (11), we note that Ec for all incompressible fluids and it is assumed the solutions of the form

$$u(y) = u_0(y) + E_c u_1(y) + O(E_c^{\,2}) \quad (12)$$

$$\theta(y) = \theta_0(y) + E_c\theta_1(y) + O(E_c^{\,2}) \quad (13)$$

$$H(y) = H_0(y) + E_c H_1(y) + O(E_c^{\,2}) \quad (14)$$

We now substitute Eqs. (12)-(14) into Eqs. (8)-(11) and equating the coefficient of the same degree terms and neglecting term $O(E_c^{\,2})$, the following Ordinary differential equations are obtained.

$$u_0^{\,11} + u_0^{\,1} = -MH_0^1 - Gr\theta_0 \quad (15)$$

$$u_1^{11} + u_1^1 = -MH_1^1 - Gr\theta_1 \tag{16}$$

$$H_0^{11} + \mathrm{P}_{rm} H_0^1 = -M\,\mathrm{P}_{rm} u_0^1 \tag{17}$$

$$H_1^{11} + \mathrm{P}_{rm} H_1^1 = -M\,\mathrm{P}_{rm} u_1^1 \tag{18}$$

$$\theta_0^{11} + \mathrm{P}_r(1+Q)\theta_0^1 = 0 \tag{19}$$

$$\theta_1^{11} + \mathrm{P}_r(1+Q)\theta_1^1 = -P_r\left(u_0^1\right)^2 - \frac{\Pr}{\Pr m}\left(H_0^1\right)^2 \tag{20}$$

The boundary conditions reduce to

$$y = 0; u_0 = h\left(\frac{\partial u_0}{\partial y}\right), u_1 = h\left(\frac{\partial u_1}{\partial y}\right), \theta_0^1 = -\gamma(1-\theta_0), \theta_1^1 = \gamma\theta_1, H_0 = 0, H_1 = 0;$$

$$y \to \infty; u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, H_0 \to 0, H_1 \to 0; \tag{21}$$

3.1. Skin friction

The boundary layer produces a drag force on the plate due to the viscous stress which developed at wall and is defined by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0^1(0) + E_c u_1^1(y) \tag{22}$$

3.2. Rate of Heat transfer

The coefficient heat transfer can be calculated in non-dimensional form at the plate, which is generally known as Nusselt number as follows:

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \theta_0^1(0) + E_c\theta_1^1(0) \tag{23}$$

3.3 **Complete solutions applying boundary conditions.**

$$u_0(y) = (P_3 e^{-A_3 y} + P_2 e^{-A_1 y})$$

$$u_1(y) = E_c\begin{pmatrix} P_{23}e^{-P_{15}y} + P_{16}e^{-A_1 y} + P_{17}e^{-2A_3 y} + P_{18}e^{-2A_1 y} + P_{19}e^{-(A_1+A_3)y} + P_{20}e^{-2P_{rm}y} \\ +P_{21}e^{-(P_{rm}+A_3)y} + P_{22}e^{-(p_{rm}+A_1)y} \end{pmatrix}$$

$$u(y) = (P_3 e^{-A_3 y} + P_2 e^{-A_1 y}) + E_c\begin{pmatrix} P_{23}e^{-P_{15}y} + P_{16}e^{-A_1 y} + P_{17}e^{-2A_3 y} + P_{18}e^{-2A_1 y} + \\ P_{19}e^{-(A_1+A_3)y} + P_{20}e^{-2P_{rm}y} + P_{21}e^{-(P_{rm}+A_3)y} + P_{22}e^{-(p_{rm}+A_1)y} \end{pmatrix}$$

$$\theta_1 = E_c\begin{pmatrix} P_{13}e^{-A_1 y} + P_7 e^{-2A_3 y} + P_8 e^{-2A_1 y} + P_9 e^{-(A_1+A_3)y} + P_{10}e^{-2P_{rm}y} \\ +P_{11}e^{-(P_{rm}+A_3)y} + P_{12}e^{-(P_{rm}+A_1)y} \end{pmatrix}$$

$$\theta_0 = P_1 e^{-A_1 y}$$

$$\theta(y) = P_1 e^{-A_1 y} + E_c\begin{pmatrix} P_{13}e^{-A_1 y} + P_7 e^{-2A_3 y} + P_8 e^{-2A_1 y} + P_9 e^{-(A_1+A_3)y} + \\ P_{10}e^{-2P_{rm}y} + P_{11}e^{-(P_{rm}+A_3)y} + P_{12}e^{-(P_{rm}+A_1)y} \end{pmatrix}$$

$$H_0(y) = P_6 e^{-P_{rm}y} + P_4 e^{-A_3 y} + P_5 e^{-A_1 y}$$

$$H_1(y) = E_c(P_{32}e^{-P_{rm}y} + P_{24}e^{-P_{15}y} + P_{25}e^{-A_1 y} + P_{26}e^{-2A_3 y} + P_{27}e^{-2A_1 y} +$$
$$P_{28}e^{-(A_1+A_3)y} + P_{29}e^{-2P_{rm}y} + P_{30}e^{-(P_{rm}+A_3)y} + P_{31}e^{-(P_{rm}+A_1)y})$$

$$H(y) = (P_6 e^{-P_{rm}y} + P_4 e^{-A_3 y} + P_5 e^{-A_1 y}) + E_c(P_{32}e^{-P_{rm}y} + P_{24}e^{-P_{15}y} + P_{25}e^{-A_1 y} +$$
$$P_{26}e^{-2A_3 y} + P_{27}e^{-2A_1 y} + P_{28}e^{-(A_1+A_3)y} + P_{29}e^{-2P_{rm}y} + P_{30}e^{-(P_{rm}+A_3)y} + P_{31}e^{-(P_{rm}+A_1)y})$$

4. Results and discussion

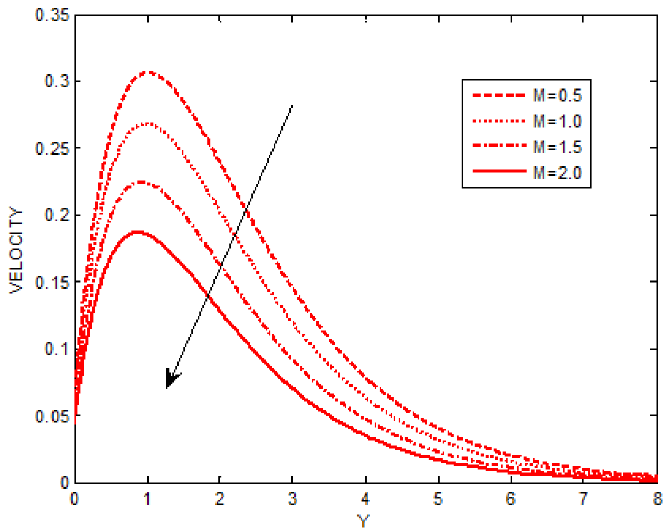

Fig 2. Velocity profiles for different values of M.

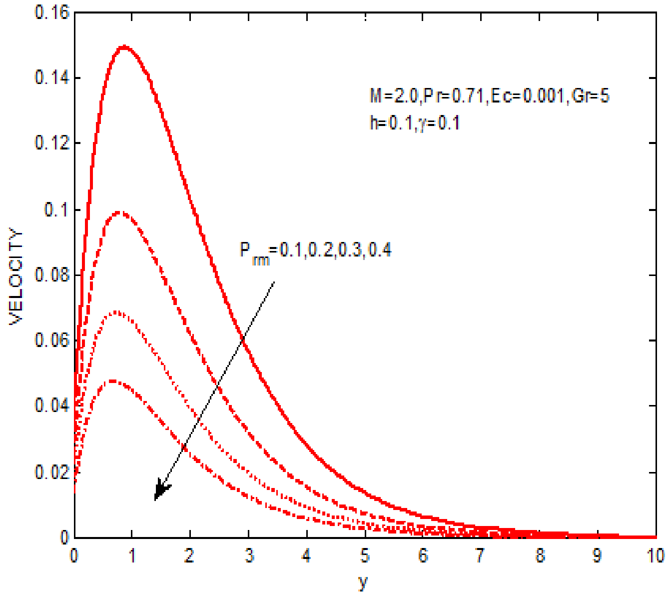

Fig 3. Velocity profiles for different values of Prm.

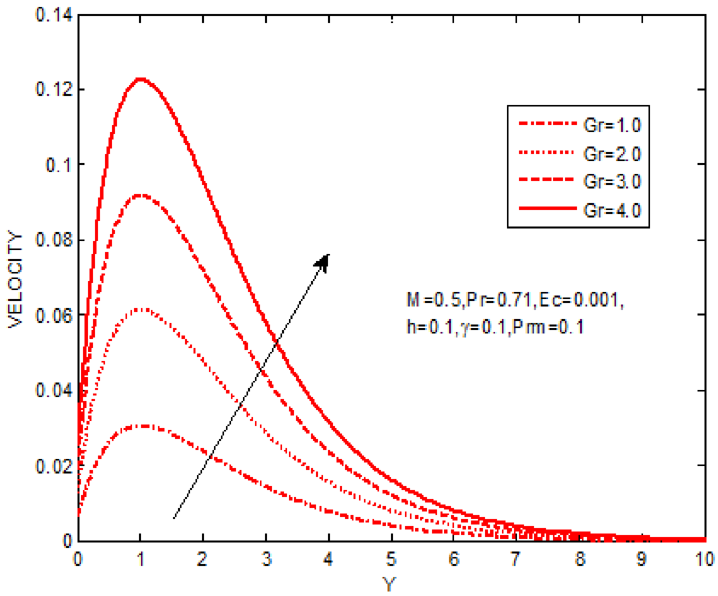

Fig 4. Velocity profiles for different values of Gr.

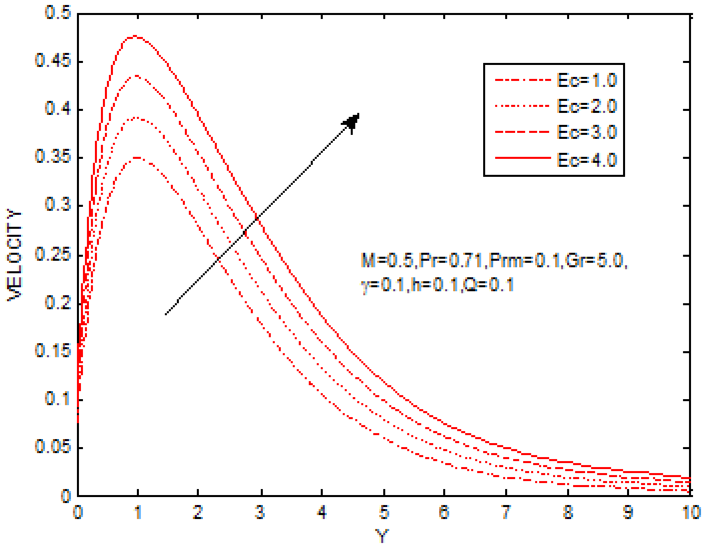

Fig 5. Velocity profiles for different values of Ec.

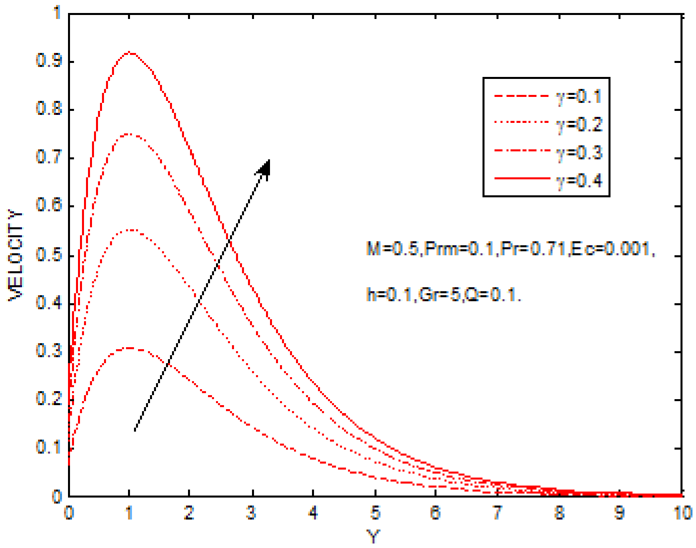

Fig 6. Velocity profiles for different values of γ .

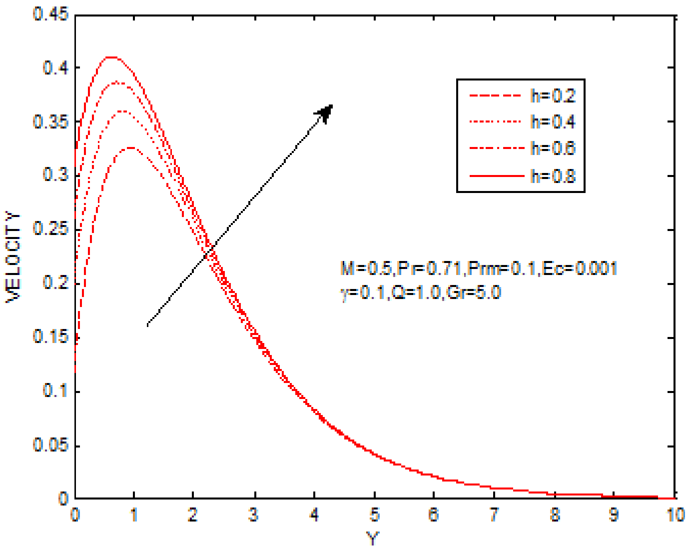

Fig 7. Velocity profiles for different values of h.

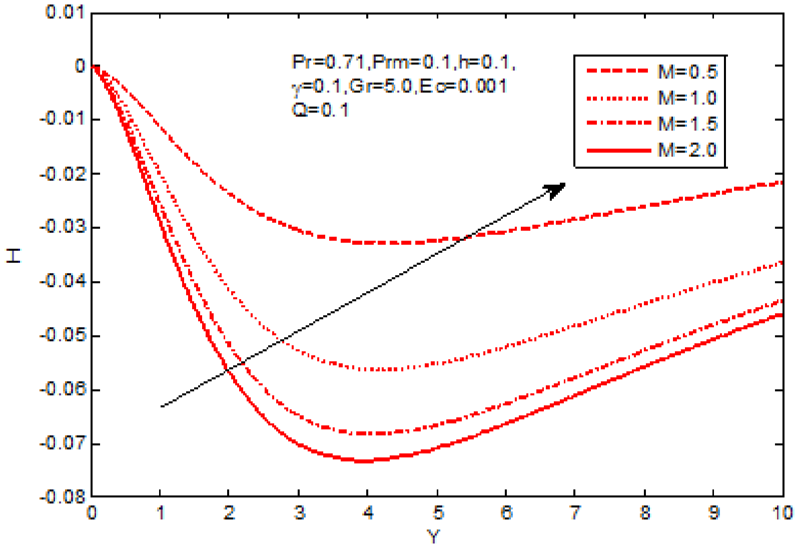

Fig 8. Induced magnetic field profiles for different values of M.

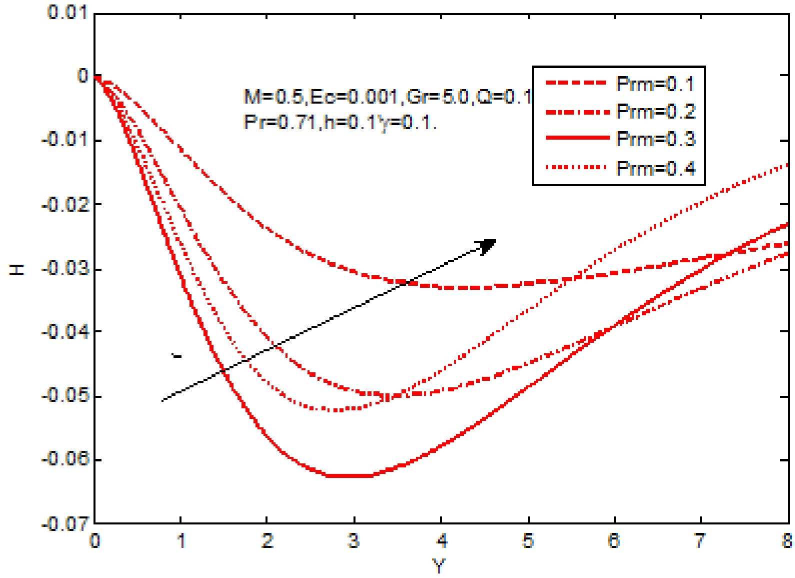

Fig 9. Induced magnetic field profiles for different values of Prm.

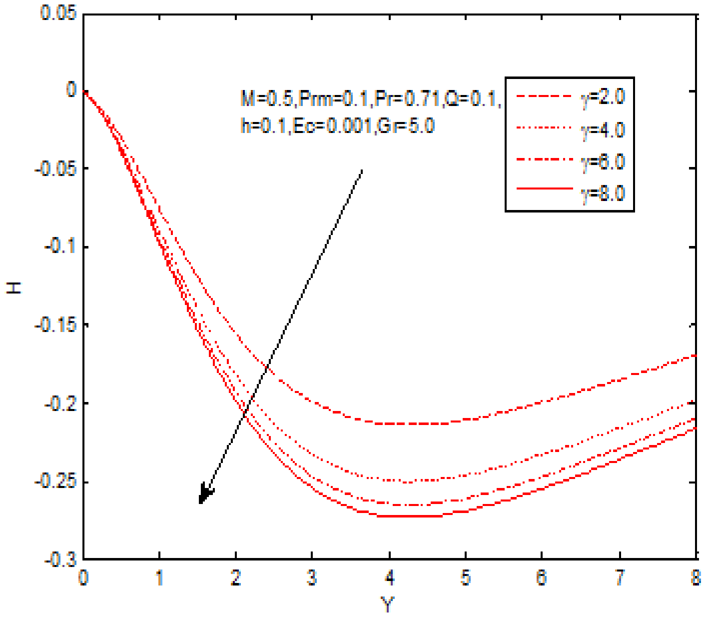

Fig 10. Induced magnetic field profiles for different values of γ.

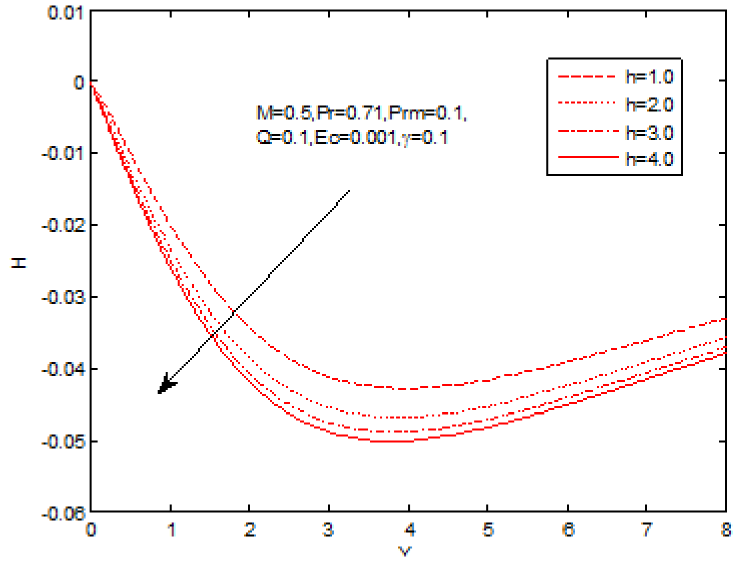

Fig 11. Induced magnetic field profiles for different values of h.

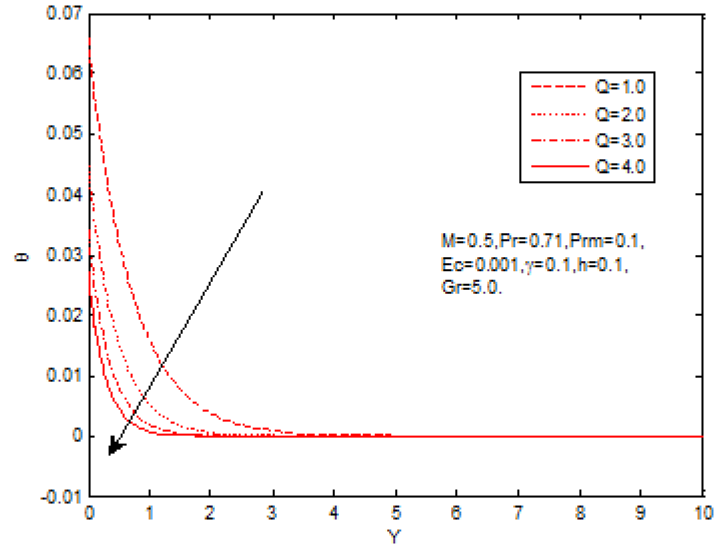

Fig 12. Temperature profiles for different values of M.

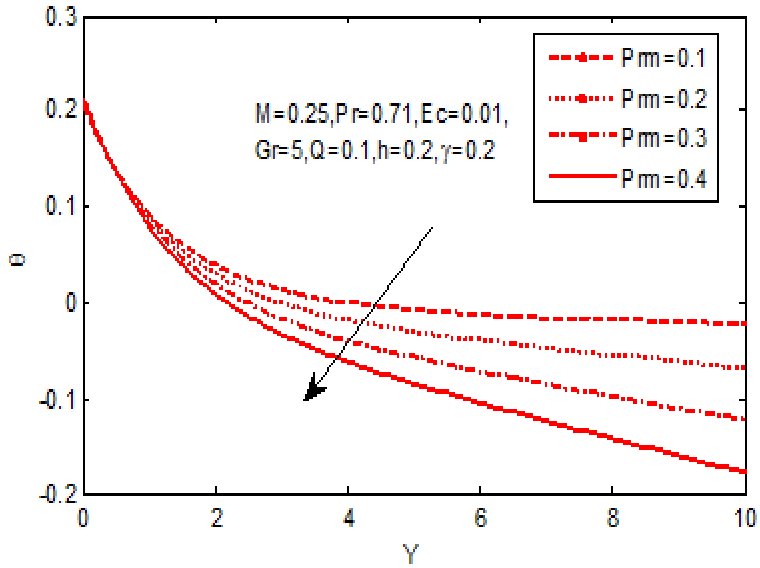

Fig 13. Temperature profiles for different values of Prm.

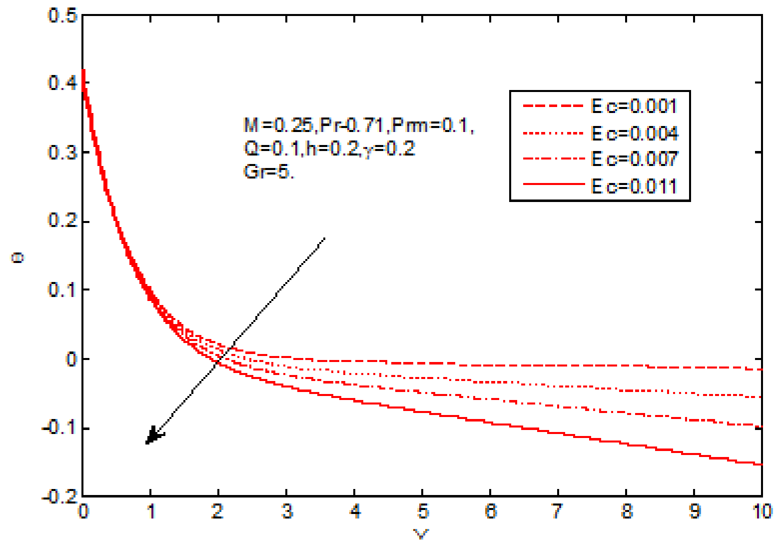

Fig 14. Temperature profiles for different values of Ec.

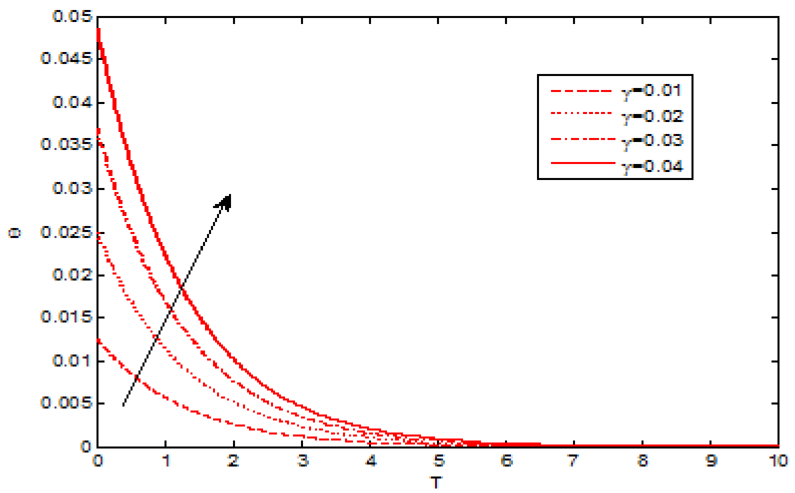

Fig 15. Temperature profiles for different values of γ .

Table 1. Skin-friction for cooling of the plate (Gr>0).

h	Prm	Ec	M	Gr	γ	Skin-friction
0.5	1.0	0.001	0.25	10	0.1	-4.9099
0.5	1.0	0.001	0.25	10	0.2	-6.8080
0.5	1.0	0.001	0.25	10	0.3	-7.0394
0.5	2.0	0.001	0.25	10	0.1	8.3577
0.5	2.0	0.001	0.25	10	0.2	15.4024

Table 2. Skin-friction for cooling of the plate (Gr<0).

γ	h	Prm	Ec	M	Gr	Skin-friction
0.5	0.1	1.0	0.001	0.25	-10	-2.7696
0.5	0.2	1.0	0.001	0.25	-10	-3.9943
0.5	0.3	1.0	0.001	0.25	-10	-4.6422
0.5	0.1	2.0	0.005	0.25	-10	-26.1952
0.5	0.2	3.0	0.005	0.5	-10	-22.2002
0.5	0.1	3.0	0.001	0.25	-20	47.6996

Table 3. Nusselt number.

Pr	Prm	Ec	M	Gr	h	γ	Nu
0.71	1.0	0.001	0.25	10	0.5	0.1	0.2054
1	1.0	0.001	0.25	10	0.5	0.1	-0.1672
3	1.0	0.001	0.25	10	0.5	0.1	-0.1053
0.71	1.0	0.001	0.25	10	0.5	0.2	0.7992
0.71	1.0	0.001	0.25	10	0.5	0.3	1.5596
0.71	1.0	0.001	0.25	10	0.5	0.4	2.3819

For different values of Magnetic parameter M, the velocity profiles are plotted in Fig.2.From this figure, we notice that as the value of M increases the velocity decreases. This is due to fact that the application of Transverse magnetic field, in which turns act as a Lorentz force. For different values of Magnetic Pandtl number (P_{rm}), the velocity profiles plotted Fig.3.From this figure we notice, that as the values of P_{rm} increases the velocity decreases in particular that a short distance from the wall. For different values of Thermal Grashof number Gr, the velocity profile plotted Fig.4. From Fig.5, we notice that as that the Gr values increases the velocity increases rapidly near the wall. From the graph, we can observe that Eckert number (Ec) has a significant effect on velocity. The Ec is increased, hence increases the velocity of the fluid flow. For different values of convective parameter γ the velocity profile plotted Fig.6. From this figure, we notice that as that Gamma (convective parameter) values increases the velocity increases rapidly near the wall. For different values of h (Slip parameter), the velocity profile plotted Fig.7. From this figure, we notice that as that h (Slip parameter) values increases the velocity increases rapidly near the wall. For different values of M on Induced magnetic profiles plotted Fig.8. From this graph, we notice that as the value of M increases as the induced magnetic field negatively increases, due to induced magnetic flux reversal arises for all distances in to the boundary layer, transverse to the plate in all combinations of M values of H are at peak short distance from the wall and tends to zero in the free stream. For different values of M on Induced magnetic profiles plotted Fig.9. the values of Prm are set to be less than a unity, which implies that the magnetic diffusion rate exceeds the viscous diffusion rate. The Prm increases the momentum diffusivity increases. The magnetic field diffuses in the medium causing a corresponding increase in the induced magnetic field magnitudes. Fig.10 shows that the behavior of induced magnetic field with various values on convective parameter γ, it was distinguished that induced magnetic profiles decreases with in boundary layer with increase in convective parameter. Thus effects of increasing values of the convective parameter gamma are to decreasing the induced magnetic field. Fig.11 indicated that the induced magnetic component H, with distinct values of slip parameter h, Induced magnetic component H decreases as the slip parameter h are increased indicating the fact that slips at the surface de-accelerates the fluid motion. For different values of heat source parameter Q, The Temperature profiles are presented in Fig.12.Here we find that as the values of Q increases, the temperature profiles decreases with decreasing in the thermal boundary layer thickness. The influences of Magnetic prandtl number Prm on the temperature profiles are plotted in Fig.13. We examine that temperature distribution across the boundary layer decrease with decreases of Prm. Fig.14 depicts temperature profiles against 'y' for distinct values of Eckert number (Ec).The magnetic of the temperature is minimum at the plate and then for away from plate increases the boundary layer. Fig.15. Depicts temperature profiles for various values of convective parameter γ it is perceived that temperature will distribute increase with an increasing in γ for it's observed that at near the plate rapidly increases for away the plate it converges boundary point.

5. Conclusions

This study presented the flow, heat transfer and induced magnetic field effects of a free convective fluid over a vertical semi-infinite plate with the effects of heat source, viscous dissipation and velocity slip. Analytical results are carried out in this study and the influence of governing flow parameters on the flow, heat transfer and Nusselt number are field effects as well as the Skin friction are analyzed and discussed through the graphs and tables.

The main remarks for the present study as follows:

1. The rising values of velocity slip parameter improve the wall and friction co-efficient.
2. The convective parameter enhances the fluid temperature.
3. The heat source parameter depreciates the fluid temperature.
4. The influence of magnetic Prandtl number on velocity distribution is very significant
5. The improvement of velocity slip parameter and convective parameter boost up the friction at the wall.
6. The rising values of Prandtl number reduce the heat transfer rate.

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Nomenclature

C_p	Specific heat at constant pressure		*Greek symbols*
E_c	Eckert number	β	Coefficient of volume expansion due to temperature
g	acceleration due to gravity	μ_0	magnetic diffusivity
G_m	mass Grashof number	ϑ	Kinematic viscosity
Gr	thermal Grashof number	k	Thermal conductivity
H	induced magnetic field	ρ	Density
H_0	uniform magnetic field	σ	electrical conductivity
H_x	induced magnetic field along x-axis	θ	dimensionless fluid temperature
J	current density		
M	Hartmann number		
P_r	Prandtl number		
P_{rm}	Magnetic Parndtl number		
$\bar{T}$	Temperature		
$\bar{T}_W$	Fluid temperature at the surface		
$\bar{T}_\infty$	Fluid temperature in the free stream		
u	Velocity component in x-direction		
U_0	dimensionless free stream velocity		
ϑ_0	suction velocity		