Efficient Product-cum-Dual to Product Estimators of Population Mean in Systematic Sampling

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ABSTRACT

This paper proposes a class of product-cum-dual to product estimators for estimating the population mean in systematic sampling using auxiliary information. The bias and variance of the proposed class of estimators have been derived under large sample approximation. Asymptotic optimum estimator (**AOE**) and its approximate variance estimator are derived and efficiency comparisons made with existing related estimators in theory. Analytical and numerical results show that at optimal conditions, the proposed class of estimators is always more efficient than all existing estimators under review.

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1. Introduction

The incorporation of auxiliary information is very important for the construction of efficient estimators for the estimation of population parameters and increasing the efficiency of the estimators in different sampling design. Using the knowledge of the auxiliary variables, several authors have proposed different estimation technique for the finite population mean of the study variable; [Cochran [1]; Kadilar and Cingi [2]; Singh et al. [3]; Khan and Arunachalam [4]; Lone and Tailor [5]; Khan [6]; Clement and Enang [7, 8]; Clement [9, 10]] have worked on the estimation of population parameters using auxiliary information.

In the present paper, the work is on the estimation of population mean using the knowledge of the auxiliary variables under systematic sampling. The method of systematic sampling was first studied by Madow and Madow [11] and is widely used in survey of finite populations. Systematic sampling is a method of selecting sample members from a larger population according to a random starting point and a fixed, periodic interval. Typically, every "nth" member is selected from the total population for inclusion in the sample population. Systematic sampling is still thought of as being random, as long as the periodic interval is determined beforehand and the starting point is random.

Systematic sampling has got the nice feature of selecting the whole sample with just one random start. Apart from its simplicity, which is of considerable importance, this procedure in many situations provides estimators more efficient than simple random sampling and/or stratified random sampling for certain types of population. Consequently, many Survey Statisticians have worked on the estimation of population mean in systematic sampling, these include :(Gautschi [12]; Hajeck [13]; Swain [14]; Shukla [15], Cochran [1], Singh and Solanki [16]; Singh and Jatwa [17]; Singh *et al.* [18]; Verma and Singh [19]; and Verma *et al*. [20]) among others.

Suppose a finite population consists of N units $U = (U_1, U_2, \dots, U_N)$ numbered from 1 to N in some order. A sample of size n units is taken at random from the first k units and every kth subsequent unit; then, $N = nk$ where n and k are positive integers; thus, there will be k samples (clusters) each of size n and observe the study variate y and auxiliary variate x for each and every unit selected in the sample. Let (y_{ij}, x_{ij}) for $i = 1, 2, \dots, k$ and $j = 1, 2, \dots, n$ denote the value of jth unit in the ith sample. Then, the systematic sample means are defined as follows: $\bar{y}_{sy} = 1/n \sum_{j=1}^{n} y_{ij}$ and $\bar{x}_{sy} = 1/n \sum_{j=1}^{n} x_{ij}$ are the unbiased estimators of the population means $\bar{Y} = 1/N \sum_{j=1}^{N} y_{ij}$ and $\bar{X} = 1/N \sum_{j=1}^{N} x_{ij}$ of y and x, respectively.

Let $S_y^2 = (1/(N-1)) \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{Y})^2$ and $S_x^2 = (1/(N-1)) \sum_{i=1}^{k} \sum_{j=1}^{n} (x_{ij} - \bar{X})^2$ be the population variances of the study variable and the auxiliary variable, respectively, with the corresponding population covariance $S_{xy} = (1/(N-1)) \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{Y})(x_{ij} - \bar{X})$. Also C_y^2 and C_x^2 are the known population coefficients of variation of the study variable and the auxiliary variable respectively.

This paper proposes a class of product-cum-dual to product type estimators for estimating the population mean in systematic sampling using auxiliary information. A comparative study is also carried out to compare the optimum estimators with respect to the classical sample mean per unit estimator in systematic sampling with the help of numerical data.

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2. Existing estimators in systematic sampling

This section gives a review of some existing estimators in survey sampling literature under the systematic sampling scheme with their variance expressions.

2.1 The classical sample mean per unit estimator

The classical sample mean per unit estimator in systematic sampling is defined as given by:

$$\bar{y}_{sy}$$

with variance estimator as given by

$$V(\bar{y}_{sy}) = \left(\frac{N-1}{nN}\right)\bar{Y}^2 C_y^2\{1 + (n-1)\rho_y\}$$

2.2 The Swain classical ratio estimator

Swain [14] introduced the classical ratio estimator for population mean in systematic sampling as given by:

$$\bar{y}'_{sy} = \bar{y}'\left\{\frac{\bar{X}}{\bar{x}'}\right\}$$

with variance estimator as given by

$$V(\bar{y}'_R) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\{1 + (n-1)\rho_x\}[\rho^{*2}C_y^2 + (1 - 2K\rho^*)C_x^2]$$

2.3 The Shukla classical product estimator

Shukla [15] introduced the classical product estimator for population mean in systematic sampling as given by:

$$\bar{y}'_P = \bar{y}'\left\{\frac{\bar{x}'}{\bar{X}}\right\}$$

with variance estimator as given by

$$(\bar{y}'_P) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\{1 + (n-1)\rho_x\}[\rho^{*2}C_y^2 + (1 + 2K\rho^*)C_x^2]$$

2.4 The Srivenkataramana dual to ratio estimator

Srivenkataramana [21] introduced the dual to ratio estimator for population mean in systematic sampling as given by:

$$\bar{y}^*_R = \bar{y}^*\left\{\frac{\bar{x}^*}{\bar{X}}\right\}$$

with variance estimator as given by

$$V(\bar{y}^*_R) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\{1 + (n-1)\rho_x\}[\rho^{*2}C_y^2 + (g^2 - 2gK\rho^*)C_x^2]$$

2.5 The Bandyopadhyay dual to product estimator

Bandyopadhyay [22] introduced the dual to product estimator for population mean in systematic sampling as given by:

$$\bar{y}^*_P = \bar{y}'\left\{\frac{\bar{X}}{\bar{x}^*}\right\}$$

with variance estimator as given by

$$V(\bar{y}^*_P) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\{1 + (n-1)\rho_x\}[\rho^{*2}C_y^2 + (g^2 + 2gK\rho^*)C_x^2]$$

2.6 The regression estimator

Hansen *et al*. [23] suggested the combined regression estimator as:

$$\bar{y}_{REG} = \bar{y} + b(\bar{X} - \bar{x})$$

Under the systematic sampling, it is defined as:

$$\bar{y}_{sy,REG} = \bar{y}_{sy} + \hat{\beta}(\bar{X} - \bar{x}_{sy})$$

with variance estimator as given by

$$V(\bar{y}_{sy,REG}) = \left(\frac{N-1}{nN}\right)\bar{Y}^2 C_y^2\{1 + (n-1)\rho_y\}(1 - \rho^2)$$

3. The proposed class of estimators

The proposed 'product-cum-dual to product' estimators is given by

$$\bar{y}^*_{P'P^*} = \bar{y}'\left[\alpha\left(\frac{\bar{x}'}{\bar{X}}\right) + (1-\alpha)\left(\frac{\bar{X}}{\bar{x}^*}\right)\right] \tag{1}$$

where α is a suitably constant.

3.1 Bias and variance estimator for the proposed class of estimators

Let consider the following transformation and definitions:

Let $x_i^* = (1 + g)\bar{X} - g x_i$, $i = 1, 2, \ldots N$ and $g = \{n/(N - n)\}$

Then

$$\bar{x}^* = (1 + g)\bar{X} - g\bar{x}' \tag{2}$$

is also an unbiased estimator for $\bar{X}$ and the correlation of $(\bar{y}', \bar{x}^*)$ is negative.

Using the transformation in equation (2), then equation (1) becomes;

$$\bar{y}_{P'P^*}^* = \bar{y}'\left[\alpha\left(\frac{\bar{x}'}{\bar{X}}\right) + (1 - \alpha)\bar{X}[\{(1 + g)\bar{X} - g\bar{x}'\}^{-1}]\right] \tag{3}$$

Also, let

$$\left.\begin{array}{l}\bar{x}' = \bar{X}(1 + e_x)\\ \bar{y}' = \bar{Y}(1 + e_y)\end{array}\right\} \tag{4}$$

Where

$$\left.\begin{array}{l}E(e_x) = E(e_y) = 0\\[4pt] E(e_x^2) = \left(\frac{N-1}{nN}\right)\{1 + (n - 1)\rho_x\}C_x^2\\[4pt] E(e_y^2) = \left(\frac{N-1}{nN}\right)\{1 + (n - 1)\rho_y\}C_y^2\\[4pt] E(e_x e_y) = \left(\frac{N-1}{nN}\right)\{1 + (n - 1)\rho_x\}^{\frac{1}{2}}\{1 + (n - 1)\rho_y\}^{\frac{1}{2}}\rho C_x C_y\end{array}\right\} \tag{5}$$

Expressing equation (3) in terms of the e's in equation (4) gives

$$\bar{y}_{P'P^*}^* = \bar{Y}(1 + e_y)[\alpha(1 + e_x) + (1 - \alpha)(1 - ge_x)^{-1}]$$
$$\bar{y}_{P'P^*}^* - \bar{Y} = \bar{Y}[e_y + (1 - \alpha)g^2 e_x^2 + \{g + \alpha(1 - g)\}(e_x e_y + e_x)] \tag{6}$$

Taking expectation of both sides of equation (6) and using the results in equation (5), gives the bias of $\bar{y}_{P'P^*}^*$ to the first order of approximation (i.e. to terms of order $o(n^{-1})$) as:

$$B(\bar{y}_{P'P^*}^*) = \left\{\left(\frac{N-1}{nN}\right)1 + (n - 1)\rho_x\right\}[(1 - \alpha)g^2 + \{g + \alpha(1 - g)K\rho^*\}]C_x^2 \tag{7}$$

where

$$\rho^* = \frac{\{1 + (n - 1)\rho_y\}^{\frac{1}{2}}}{\{1 + (n - 1)\rho_x\}^{\frac{1}{2}}} \,, K = \rho\frac{C_y}{C_x}$$

Squaring both sides of equation (6) and retaining terms to the second degree, gives

$$(\bar{y}_{P'P^*}^* - \bar{Y})^2 = \bar{Y}^2[e_y + (1 - \alpha)g^2 e_x^2 + \{g + \alpha(1 - g)\}(e_x e_y + e_x)]^2$$
$$(\bar{y}_{P'P^*}^* - \bar{Y})^2 = \bar{Y}^2\left[e_y^2 + \{g + \alpha(1 - g)\}[2e_x e_y + \{g + \alpha(1 - g)\}e_x]\right] \tag{8}$$

Taking expectation of equation (8) and using the results in (5), gives the variance of $\bar{y}_{P'P^*}^*$ to the first order of approximation as:

$$V(\bar{y}_{P'P^*}^*) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\{1 + (n - 1)\rho_x\}[\rho^{*2}C_y^2 + \{g + \alpha(1 - g)\}[2K\rho^* + \{g + \alpha(1 - g)\}]C_x^2] \tag{9}$$

3.2   Optimality conditions for the proposed class of estimators

To investigate the optimal conditions for the proposed class of estimators let,

$$\frac{\partial V(\bar{y}_{P'P^*}^*)}{\partial\alpha} = 0$$

So that

$$\alpha = -\left(\frac{K\rho^* + g}{1 - g}\right)$$
$$= \alpha_{opt} \qquad (say) \tag{10}$$

Substituting the value of α_{opt} in (10) for α in (1) gives the asymptotically optimum estimator (**AOE**) for population mean ($\bar{Y}$) in systematic sampling as:

$$\bar{y}_{P'P^*}^* = \bar{y}'\left[\frac{(1 + K\rho^*)}{(1 - g)}\bar{X}\{(1 + g)\bar{X} - g\bar{x}'\}^{-1} - \frac{(K\rho^* + g)}{(1 - g)}\left(\frac{\bar{x}'}{\bar{X}}\right)\right] \tag{11}$$

Similarly, substituting the value of α_{opt} in (10) for α in (9) gives the variance of asymptotically optimum estimator (**AOE**) $\bar{y}_{P'P^*,opt}^*$ (or minimum variance of $\bar{y}_{P'P^*}^*$) as:

$$V\left(\bar{y}^*_{P'P^*}\right) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\rho^{*2}C_y^2\{1 + (n-1)\rho_x\}\left(1 - \rho^2\right) \tag{12}$$

Following from the above, the following theorem is established:

Theorem

Given

$$\bar{y}^*_{P'P^*} = \bar{y}'\left[\alpha\left(\frac{\bar{x}'}{\bar{X}}\right) + (1 - \alpha)\left(\frac{\bar{X}}{\bar{x}^*}\right)\right]$$

Then to first degree of approximation

$$V\left(\bar{y}^*_{P'P^*,opt}\right) \leq V(\bar{y}^*_P)$$

with equality holding if

$$\alpha = -\left(\frac{K\rho^* + g}{1 - g}\right)$$

where

$$\bar{x}^* = (1 + g)\bar{X} - g\bar{x}' \ ,$$

$$\rho^* = \frac{\{1 + (n-1)\rho_y\}^{\frac{1}{2}}}{\{1 + (n-1)\rho_x\}^{\frac{1}{2}}} \ , K = \rho\frac{C_y}{C_x}$$

$$V\left(\bar{y}^*_{P'P^*,opt}\right) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\rho^{*2}C_y^2\{1 + (n-1)\rho_x\}\left(1 - \rho^2\right) \ ^{and}$$

$$V(\bar{y}^*_P) = \left(\frac{N-1}{nN}\right)\bar{Y}^2\{1 + (n-1)\rho_x\}[\rho^{*2}C_y^2 + \{g + \alpha(1 - g)\}[2K\rho^* + \{g + \alpha(1 - g)\}]C_x^2]$$

4. Membership of the proposed class of estimators

This section shows that by altering the values of α in equation (1), some of the existing estimators listed in section 2 become special cases of the proposed class of estimators.

Property 1

If $\alpha = 1$, the equation (1) reduces to the Shukla [15] classical product estimator in systematic sampling as given by:

$$\bar{y}'_P = \bar{y}'\left\{\frac{\bar{x}'}{\bar{X}}\right\}$$

Property 2

If $\alpha = 0$, the equation (1) reduces to the Bandyopadhyay [22] dual to product estimator in systematic sampling as given by:

$$\bar{y}^*_P = \bar{y}'\left\{\frac{\bar{X}}{\bar{x}^*}\right\}$$

5. Analytical Study

5.1. Efficiency comparisons

The asymptotically optimum estimator *(AOE)* is compared with the existing estimators in systematic sampling listed in section 2 with respect to their variance expressions.

5.1.1 Comparison with the classical sample mean per unit estimator

$$V\left(\bar{y}^*_{P'P^*,opt}\right) < V(\bar{y}_{sy})$$

$$\left(\frac{N-1}{nN}\right)\bar{Y}^2\rho^{*2}C_y^2\{1 + (n-1)\rho_x\}\left(1 - \rho^2\right) < \left(\frac{N-1}{nN}\right)\bar{Y}^2C_y^2\{1 + (n-1)\rho_y\}$$

$$\rho^{*2}\{1 + (n-1)\rho_x\}\left(1 - \rho^2\right) < \{1 + (n-1)\rho_y\}$$

The optimality condition is:

$$\rho^* < \left[\frac{\{1 + (n-1)\rho_y\}}{\{1 + (n-1)\rho_x\}\left(1 - \rho^2\right)}\right]^{\frac{1}{2}}$$

When this optimality condition is satisfied, the proposed class of estimators is more efficient than the classical sample mean per unit estimator.

5.1.2   Comparison with the Swain classical ratio estimator

$$V\left(\bar{y}^*_{P'P^*,opt}\right) < V(\bar{y}'_R)$$

$$\left(\frac{N-1}{nN}\right)\bar{Y}^2\rho^{*2}C_y^2\{1 + (n-1)\rho_x\}\left(1 - \rho^2\right) < \left(\frac{N-1}{nN}\right)\bar{Y}^2\{1 + (n-1)\rho_x\}[\rho^{*2}C_y^2 + (1 - 2K\rho^*)C_x^2]$$

$$\rho^{*2}C_y^2(1-\rho^2) < \rho^{*2}C_y^2 + (1-2K\rho^*)C_x^2$$
$$K^2\rho^{*2} - 2K\rho^* + 1 > 0$$

The optimality conditions are:

(i) $\qquad \rho^* > 1/K$

(ii) $\qquad K > 1/\rho^*$

When either of these optimality conditions is satisfied, the proposed class of estimators is more efficient than the Swain classical ratio estimator.

5.1.3 Comparison with the Shukla classical product estimator

$$V\left(\overline{y}_{P'P^*,opt}^*\right) < V(\overline{y}_P')$$
$$\left(\frac{N-1}{nN}\right)\overline{Y}^2\rho^{*2}C_y^2\{1+(n-1)\rho_x\}(1-\rho^2) < \left(\frac{N-1}{nN}\right)\overline{Y}^2\{1+(n-1)\rho_x\}[\rho^{*2}C_y^2 + (1+2K\rho^*)C_x^2]$$
$$\rho^{*2}C_y^2(1-\rho^2) < \rho^{*2}C_y^2 + (1+2K\rho^*)C_x^2$$
$$K^2\rho^{*2} + 2K\rho^* + 1 > 0$$

The optimality conditions are:

(i) $\qquad \rho^* > -1/K$

(ii) $\qquad K > -1/\rho^*$

When either of these optimality conditions is satisfied, the proposed class of estimators is more efficient than the Shukla classical product estimator.

5.1.4 Comparison with the Srivenkataramana dual to ratio estimator

$$V\left(\overline{y}_{P'P^*,opt}^*\right) < V(\overline{y}_R^*)$$
$$\left(\frac{N-1}{nN}\right)\overline{Y}^2\rho^{*2}C_y^2\{1+(n-1)\rho_x\}(1-\rho^2) < \left(\frac{N-1}{nN}\right)\overline{Y}^2\{1+(n-1)\rho_x\}[\rho^{*2}C_y^2 + (g^2 - 2gK\rho^*)C_x^2]$$
$$\rho^{*2}C_y^2(1-\rho^2) < \rho^{*2}C_y^2 + (g^2 - 2gK\rho^*)C_x^2$$
$$K^2\rho^{*2} - 2gK\rho^* + g^2 > 0$$

The optimality conditions are:

(i) $\qquad \rho^* > g/K$

(ii) $\qquad K > g/\rho^*$

When either of these optimality conditions is satisfied, the proposed class of estimators is more efficient than the Srivenkataramana dual to ratio estimator.

5.1.5 Comparison with the Bandyopadhyay dual to product estimator

$$V\left(\overline{y}_{P'P^*,opt}^*\right) < V(\overline{y}_P^*)$$
$$\left(\frac{N-1}{nN}\right)\overline{Y}^2\rho^{*2}C_y^2\{1+(n-1)\rho_x\}(1-\rho^2) < \left(\frac{N-1}{nN}\right)\overline{Y}^2\{1+(n-1)\rho_x\}[\rho^{*2}C_y^2 + (g^2 + 2gK\rho^*)C_x^2]$$
$$\rho^{*2}C_y^2(1-\rho^2) < \rho^{*2}C_y^2 + (g^2 + 2gK\rho^*)C_x^2$$
$$K^2\rho^{*2} + 2gK\rho^* + g^2 > 0$$

The optimality conditions are:

(i) $\qquad \rho^* > -g/K$

(ii) $\qquad K > -g/\rho^*$

When either of these optimality conditions is satisfied, the proposed class of estimators is more efficient than the Bandyopadhyay dual to product estimator.

5.1.6 Comparison with the regression estimator

$$V\left(\overline{y}_{P'P^*,opt}^*\right) < V(\overline{y}_{sy,REG})$$
$$\left(\frac{N-1}{nN}\right)\overline{Y}^2\rho^{*2}C_y^2\{1+(n-1)\rho_x\}(1-\rho^2) < \left(\frac{N-1}{nN}\right)\overline{Y}^2C_y^2\{1+(n-1)\rho_y\}(1-\rho^2)$$
$$\rho^{*2}\{1+(n-1)\rho_x\} < \{1+(n-1)\rho_y\}$$

The optimality condition is:

$$\rho^* < \left[\frac{\{1+(n-1)\rho_y\}}{\{1+(n-1)\rho_x\}}\right]^{\frac{1}{2}}$$

When this optimality condition is satisfied, the proposed class of estimators is more efficient than the regression estimator.

5.2 The percent relative efficiency (PRE)

The percent relative efficiency (PRE) of an estimator θ with respect to the classical sample mean per unit estimator in systematic sampling ($\bar{y}_{sy}$) is defined by:

$$E(\theta, \bar{y}_{sy}) = \frac{V(\bar{y}_{sy})}{V(\theta)} \times 100$$

6. Empirical Study

This section investigates the theoretical results, and tests the optimality and efficiency performances of the proposed class of estimators over other existing ones considered in this study, using live data of Population I and Population II respectively.

Population I

The details of population parameters are:

$$N = 15, n = 3, \bar{Y} = 80, \bar{X} = 44.47, S_x^2 = 149.55, S_y^2 = 2000, \rho_y = 06652, \rho_x = 0.7070,$$

$$\rho = 0.9848, g = 0.25, C_y = 0.56, C_x = 0.20, S_{xy} = 538.57.$$

[Tailor *et al.* [24]]

Population II

The details of population parameters are:

$$N = 176, n = 16, \bar{Y} = 282.6136, \bar{X} = 6.9943, C_y^2 = 0.3019, C_x^2 = 0.1791, \rho_y = 0.6342, \rho_x =$$

$$0.6986, \rho = 0.6741, K = 0.8752.$$

[Murthy [25]]

Two measuring criteria; variance and percent relative efficiency (PRE) are used to compare the performance of each estimator. The variance and PREs for the estimators are given in Table (1).

For the efficiency comparisons, it is observed as follows:

(1)
$$\rho^* = 0.9825 < \left[\frac{\{1+(n-1)\rho_y\}}{\{1+(n-1)\rho_x\}(1-\rho^2)}\right]^{\frac{1}{2}} = 5.6445$$

(2) (i) $\quad \rho^* = 0.9825 > 1/K = 0.3627$

(ii) $\quad K = 2.7574 > 1/\rho^* = 1.0178$

(3) (i) $\quad \rho^* = 0.9825 > -1/K = -0.3627$

(ii) $\quad K = 2.7574 > -1/\rho^* = -1.0178$

(4) (i) $\quad \rho^* = 0.9825 > g/K = 0.0907$

(ii) $\quad K = 2.7574 > g/\rho^* = 0.2545$

(5) (i) $\quad \rho^* = 0.9825 > -g/K = -0.0907$

(ii) $\quad K = 2.7574 > -g/\rho^* = -0.2545$

(6)
$$\rho^* = 0.9825 \leq \left[\frac{\{1+(n-1)\rho_y\}}{\{1+(n-1)\rho_x\}}\right]^{\frac{1}{2}} = 0.9825$$

Therefore since all of the optimality conditions are satisfied in accordance with section 5.1, it is concluded that the proposed estimator is more efficient than the other existing ones considered in this study with reference to population I.

Similarly, investigation revealed that all the optimality conditions are also satisfied in accordance with section 5.1 for population II as is evident in Table (1).

Table 1: Variance and PREs for the estimators

S/No.	Estimator	Population I		Population II	
		Variance	PREs	Variance	PREs
1	$\bar{y}_{sy}$	1455.0788	100	15742.2749	100
2	$\bar{y}_R'$	605.6027	240.2695	8859.1359	177.6954
3	$\bar{y}_P'$	2688.6837	54.1186	43030.3283	36.5841
4	$\bar{y}_R^*$	1206.3990	120.6134	14137.0401	111.3548
5	$\bar{y}_P^*$	1727.4096	84.2347	17553.1593	89.6834
6	$\bar{y}_{sy,REG}$	43.9434	3311.2568	8588.9852	183.2845
7	Proposed	43.9448	3311.1513	8588.5507	183.2937

7. Discussion of results

The results of efficiency comparisons using Table (1) showed that all of the optimality conditions are satisfied in accordance with section 5.1 for the two populations considered in the study, it is concluded that the proposed class of estimators is more efficient than the other existing estimators considered in this study.

Numerical results for the percent relative efficiency ($PREs$) in table (1) reveals that the proposed estimator ($\bar{y}_{P'P^*,opt}^*$) has 3211 percent and 83 percent gains in efficiency for population I and population II respectively. This shows that the proposed

estimator $(\overline{y}^*_{P'P^*,opt})$ is 3071 percent and 6 percent more efficient than the Swain classical ratio estimator $(\overline{y}'_R)$ in systematic sampling for population I and population II respectively. Similarly, the proposed estimator $(\overline{y}^*_{P'P^*,opt})$ is 3257 percent and 147 percent more efficient than the Shukla classical product estimator $(\overline{y}'_P)$ in systematic sampling for population I and population II respectively.

In using the proposed estimator $(\overline{y}^*_{P'P^*,opt})$ one will have 3191 percent and 72 percent efficiency gains over the Srivenkataramana dual to ratio estimator $(\overline{y}^*_R)$ in systematic sampling under the two populations respectively. Similarly, one would have 3227 percent and 94 percent efficiency gains over the Bandyopadhyay dual to product estimator $(\overline{y}^*_P)$ in systematic sampling under the two populations respectively if the proposed estimator $(\overline{y}^*_{P'P^*,opt})$ is used.

The numerical results also show that the proposed estimator fares better as the classical regression estimator $(\overline{y}_{sy,REG})$ in systematic sampling. Generally, it can also be deduced that the proposed class of estimators always fare better than all the existing estimators under each of the population considered in the study.

8. Conclusion

Sequel to the discussion of results above, it is concluded that the proposed class of estimators at optimal condition fares better as the classical regression estimator and is more efficient than all the other existing estimators under each of the population considered in the study; thus providing a better and efficient alternative estimator in practical situations.

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