Available online at www.elixirpublishers.com (Elixir International Journal)

Statistics

Elixir Statistics 106 (2017) 46480-46486

ISSN: 2229-712X

Estimation of Population Mean in Calibration Ratio-Type Estimator under Systematic Sampling

Etebong P. Clement
Department of Mathematics and Statistics, University of Uyo, Uyo, Nigeria.

ARTICLE INFO

Article history:
Received: 14 July 2016;
Received in revised form: 25 April 2017;
Accepted: 6 May 2017;

Keywords
Calibration estimation,
Calibration asymptotic
Optimum estimator,
Efficiency,
Ratio-type estimator,
Optimality conditions,
Systematic sampling.

ABSTRACT
This paper introduces the theory of calibration estimator to ratio estimation in stratified systematic sampling scheme and proposes a class of calibration ratio-type estimators for estimating population mean $\bar{\mathbf{Y}}$ of the study variable $\mathbf{y}$ using auxiliary variable $\mathbf{x}$. The bias and variance of the proposed estimator have been derived under large sample approximation. Calibration Asymptotic optimum estimator ($\boldsymbol{CAOE}$) and its approximate variance estimator are derived. An empirical study to evaluate the relative performances of the proposed class of estimator against members of its class is carried out. Analytical and numerical results proved the dominance of the new proposal.

1. Introduction

In statistical surveys, when subpopulations within an overall population vary, it is advantageous to sample each subpopulation (stratum) independently. Stratification is the process of dividing members of the population into homogeneous subgroups before sampling. The strata should be mutually exclusive: every element in the population must be assigned to only one stratum. The strata should also be collectively exhaustive: no population element can be excluded. Then simple random sampling or systematic sampling is applied within each stratum. This often improves the representativeness of the sample by reducing sampling error.

The method of systematic sampling was first studied by Madow and Madow [1] and is widely used in survey of finite populations. Systematic sampling is a method of selecting sample members from a larger population according to a random starting point and a fixed, periodic interval. Typically, every "nth" member is selected from the total population for inclusion in the sample population. Systematic sampling is still thought of as being random, as long as the periodic interval is determined beforehand and the starting point is random.

Systematic sampling has got the nice feature of selecting the whole sample with just one random start. Apart from its simplicity, which is of considerable importance, this procedure in many situations provides estimators more efficient than simple random sampling and/or stratified random sampling for certain types of population [Cochran [2]; Gautschi [3]; Hajeck [4]].

The most challenging limitation of the ratio and product estimators is that of having efficiency not exceeding that of the regression estimator. Consequently, most authors have carried out researches towards the modification of the existing ratio and product estimators to provide better alternative estimators. Among these authors include; Singh and Vishwakarma [5, 6], Singh *et al.*[7], Sharma and Tailor [8], Onyeka [9], Tailor [10], Choudhury and Singh [11], Khare and Sinha [12] and Singh and Audu [13].

Clement and Enang [14] observed that most of these alternative estimators depend on some optimality conditions that are hardly satisfy in practice and suggested the use of calibration estimation to address these problems. Deville and Sarndal [15] first presented calibration estimators in survey sampling and calibration estimation has been studied by many survey statisticians. A few key references include [Arnab and Singh [16], Estavao and Sarndal [17], Kott [18], Singh [19, 20], Sarndal [21], Kim and Park [22], Clement *et al.* [23], Clement and Enang [24, 25] and Clement [26]].

In stratified random sampling, calibration approach is used to obtain optimum strata weights for improving the precision of survey estimates of population parameters. Kim, Sungur and Heo [27] , Koyuncu and Kadilar [28] defined some calibration estimators in stratified random sampling for population characteristics and Clement *et al* [29] defined calibration estimators for domain totals in stratified random sampling.

This paper introduces the theory of calibration estimator to ratio estimation in stratified systematic sampling scheme and proposes calibration ratio-type estimator for estimating population mean $\bar{Y}$ of the study variable y using auxiliary variable x.

2. Calibration Estimation in Stratified Systematic Sampling

Consider a finite population U of N elements

Tele:
E-mail address: epclement@yahoo.com

$$U = (U_1, U_2, \dots, U_N) \quad (1)$$

Suppose the finite population of equation (1) consists of H strata with N_h units in the hth stratum from which a simple random sample of size n_h is taken without replacement. The total population size be $N = \sum_{h=1}^{H} N_h$ and the sample size $n = \sum_{h=1}^{H} n_h$, respectively. Associated with the ith element of the hth stratum are y_{hi} and x_{hi} with $x_{hi} > 0$ being the covariate; where y_{hi} is the y value of the ith element in stratum h, and x_{hi} is the x value of the ith element in stratum h, $h = 1, 2, \dots, H$ and $i = 1, 2, \dots, N_h$ where y and x are the study variable and auxiliary variable respectively. For the hth stratum, let $W_h = N_h/N$ be the stratum weights and $f_h = n_h/N_h$, the sample fraction.

Let the hth stratum means of the study variable y and auxiliary variable x $\left(\bar{y}_h = \sum_{i=1}^{n_h} y_{hi}/n_h\,;\ \bar{x}_h = \sum_{i=1}^{n_h} x_{hi}/n_h\right)$ be the unbiased estimator of the population mean $\left(\bar{Y}_h = \sum_{i=1}^{N_h} y_{hi}/N_h\,;\ \bar{X}_h = \sum_{i=1}^{N_h} x_{hi}/N_h\right)$ of y and x respectively, based on n_h observations.

Let $\bar{y}_{syh}$ be the mean of a systematic sample in stratum h, then the estimate of the population mean $\bar{Y}$ in stratified systematic sampling scheme is given by (Cochran [2]) as:

$$\bar{y}_{stsy} = \sum_{h=1}^{H} W_h \bar{y}_{syh} \quad (2)$$

2.1The proposed calibration ratio-type estimator

Solanki *et al.* [30] proposed a ratio-type estimator in simple random sampling without replacement (SRSWOR) as given by:

$$t(\alpha, \delta) = \bar{y}\left\{2 - \left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} exp\left[\frac{\delta(\bar{x} - \bar{X})}{(\bar{x} + \bar{X})}\right]\right\} \quad (3)$$

Following the Solanki *et al.* [30] ratio-type estimator, if $t(\alpha, \delta)$ is modified such that λ_h and $r_h\left(\frac{\bar{x}_{syh}}{\bar{X}_h}\right)^{\alpha_h} exp\left(\delta_h \frac{(\bar{x}_{syh} - \bar{X}_h)}{(\bar{x}_{syh} + \bar{X}_h)}\right)$ replaces the number 2 and $\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} exp\left[\frac{\delta(\bar{x} - \bar{X})}{(\bar{x} + \bar{X})}\right]$ respectively in equation (3)[see Etuk *et al.* [31]], then an alternative ratio estimator of mean $\bar{Y}$ in stratified systematic sampling is proposed as:

$$\bar{y}_{R,stsy} = \sum_{h=1}^{H} W_h \bar{y}_{syh}\left\{\lambda_h - r_h\left(\frac{\bar{x}_{syh}}{\bar{X}_h}\right)^{\alpha_h} exp\left(\delta_h \frac{(\bar{x}_{syh} - \bar{X}_h)}{(\bar{x}_{syh} + \bar{X}_h)}\right)\right\} \quad (4)$$

The class of ratio estimators as proposed in equation (4) is a modification of the Solanki *et al.* [30] ratio-type estimator of equation (3) for suitably chosen scalars $\lambda_h, r_h, \alpha_h, \delta_h$ such that λ_h and r_h satisfies the condition $\lambda_h = 1 + r_h\ ;\ -\infty \le r_h \le \infty$

Adapting the family of estimators of equation (4) to calibration estimation in stratified systematic sampling gives

$$\bar{y}^{*}_{R,stsy} = \sum_{h=1}^{H} \Omega^{*}_h \bar{y}_{syh}\left\{\lambda_h - r_h\left(\frac{\bar{x}_{syh}}{\bar{X}_h}\right)^{\alpha_h} exp\left(\delta_h \frac{(\bar{x}_{syh} - \bar{X}_h)}{(\bar{x}_{syh} + \bar{X}_h)}\right)\right\} \quad (5)$$

with the new weights Ω^{*}_h called the *calibration weights*. The calibration weights Ω^{*}_h are chosen such that a chi-square-type loss functions of the form:

$$\psi = \sum_{h=1}^{H} \frac{(\Omega^{*}_h - w_h)^2}{w_h Q_h} \quad (6)$$

is minimized subject to a calibration constraints of the form:

$$\sum_{h=1}^{H} \Omega^{*}_h \bar{x}_{syh} = \bar{X} \quad (7)$$

Minimizing the loss function (6) subject to the calibration constraints (7) leads to the calibration weights for stratified systematic sampling given by

$$\Omega^{*}_h = W_h + \left(\bar{X} - \sum_{h=1}^{H} W_h \bar{x}_{syh}\right)\frac{Q_h W_h \bar{x}_{syh}}{\sum_{h=1}^{H} Q_h W_h \bar{x}_{syh}^{2}} \quad (8)$$

Substituting (8) into (5) gives

$$\bar{y}^{*}_{R,stsy} = \sum_{h=1}^{H}\left[w_h + \left(\bar{X} - \sum_{h=1}^{H} w_h \bar{x}_{syh}\right)\frac{Q_h w_h \bar{x}_{syh}}{\sum_{h=1}^{H} Q_h W_h \bar{x}_{syh}^{2}}\right] \times$$

$$\bar{y}_{syh}\left\{\lambda_h - r_h\left(\frac{\bar{x}_{syh}}{\bar{X}_h}\right)^{\alpha_h} exp\left(\delta_h \frac{(\bar{x}_{syh} - \bar{X}_h)}{(\bar{x}_{syh} + \bar{X}_h)}\right)\right\} \quad (9)$$

Setting the tuning parameter $Q_h = \bar{x}_{syh}^{-1}$ in equation (8) and substituting the results in equation (9) gives the proposed calibration ratio-type estimator under the stratified systematic sampling as:

$$\bar{y}^{*}_{R,stsy} = \frac{\sum_{h=1}^{H} W_h \bar{y}_{syh}}{\sum_{h=1}^{H} W_h \bar{x}_{syh}}\bar{X}\left\{\lambda_h - r_h\left(\frac{\bar{x}_{syh}}{\bar{X}_h}\right)^{\alpha_h} exp\left(\delta_h \frac{(\bar{x}_{syh} - \bar{X}_h)}{(\bar{x}_{syh} + \bar{X}_h)}\right)\right\} \quad (10)$$

2.2. Bias and variance estimator for the proposed estimator

Let $\bar{y}_{syh} = \bar{Y}_h(1 + e_0)$ *and* $\bar{x}_{syh} = \bar{X}_h(1 + e_1)$ (11)

$\lambda_h = 1 + r_h\ ;\ -1 \le r_h \le \infty$

$$\left.\begin{aligned} &E(e_1) = E(e_0) = 0 \\ &E(e_1^2) = \theta_h\{1+(n_h-1)\rho_x\}C_{hx}^2 \\ &E(e_0^2) = \theta_h\{1+(n_h-1)\rho_{hy}\}C_{hy}^2 \\ &E(e_1e_0) = \theta_h\{1+(n_h-1)\rho_{hx}\}^{\frac{1}{2}}\{1+(n-1)\rho_{hy}\}^{\frac{1}{2}}\rho_{xy}C_{hx}C_{hy} \end{aligned}\right\} \quad (12)$$

where

$$\theta_h = \left(\frac{N_h-1}{n_hN_h}\right);\ C_{hx}^2 = \frac{S_{hx}^2}{\bar{X}_h^2};\ C_{hy}^2 = \frac{S_{hy}^2}{\bar{Y}_h^2};\ K_h = \rho_{hxy}\frac{C_{hy}}{C_{hx}};\ \rho_{hxy} = \frac{S_{hxy}}{S_{hx}S_{hy}};$$

$S_{hx}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi}-\bar{X}_h)^2;\ S_{hy}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi}-\bar{Y}_h)^2$ and

$$S_{hxy} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi}-\bar{X}_h)\ (y_{hi}-\bar{Y}_h)$$

Expressing (5) in terms of the e's in (11) gives

$$\bar{y}_{R,stsy}^* = \sum_{h=1}^{H}\Omega_h^*\bar{Y}_h\,(1+e_0)\left\{\lambda_h - r_h(1+e_1)^{\alpha_h}exp\frac{\delta_he_1}{2}\left(1+\frac{1}{2}e_1\right)^{-1}\right\}$$

$$\bar{y}_{R,stsy}^* = \sum_{h=1}^{H}\Omega_h^*\bar{Y}_h\left\{\lambda_h - r_h(1+e_1)^{\alpha_h}\exp\frac{\delta_he_1}{2}\left(1+\frac{1}{2}e_1\right)^{-1}\right\} + \sum_{h=1}^{H}\Omega_h^*\bar{Y}_he_0\left\{\lambda_h - r_h(1+e_1)^{\alpha_h}\exp\frac{\delta_he_1}{2}\left(1+\frac{1}{2}e_1\right)^{-1}\right\} \quad (13)$$

Now, it is assumed that $|e_1| < 1$ so that expanding $(1+e_1)^{\alpha_h}$, $\left(1+\frac{1}{2}e_1\right)^{-1}$ and $exp\frac{\delta_he_1}{2}\left(1+\frac{1}{2}e_1\right)^{-1}$ as a series in power of e_1, multiplying out and retaining terms of the e's to the second degree, gives

$$\bar{y}_{R,stsy}^* - \bar{Y} = \sum_{h=1}^{H}w_h^*\bar{Y}_h\left[\left\{-\frac{r_h(2\alpha_h+\delta_h)}{2}\left(e_1 + \frac{(2\alpha_h+\delta_h-2)e_1^2}{4} + e_1e_0\right)\right\} + e_0\right] \quad (14)$$

Taking expectation of both sides of (14) and using the results in (12), gives the bias of $\bar{y}_{R,stsy}^*$ to the first order of approximation (i.e. to terms of order $o(n_h^{-1})$) as:

$$E(\bar{y}_{R,stsy}^* - \bar{Y}) = \sum_{h=1}^{H}\Omega_h^*\bar{Y}_h\,\gamma_h\left[-\frac{r_h(2\alpha_h+\delta_h)}{2}C_{hx}^2\left(\frac{(2\alpha_h+\delta_h-2)}{4} + K_h\right)\right]$$

$$\mathrm{Bias}(\bar{y}_{R,stsy}^*) = \sum_{h=1}^{H}\Omega_h^*\bar{Y}_h\,\gamma_hC_{hx}^2\left[-\frac{r_h(2\alpha_h+\delta_h)}{2}\left(\frac{(2\alpha_h+\delta_h-2)+4K_h}{4}\right)\right] \quad (15)$$

If $\alpha_h = -\frac{1}{2}\delta_h$, then the $Bias(\bar{y}_{R,stsy}^*)$ is equal to zero. Therefore, the estimator $\bar{y}_{R,stsy}^*$ with $\alpha_h = -\frac{1}{2}\delta_h$ is almost unbiased.

Squaring both sides of (14) and retaining terms to the second degree, gives

$$\left(\bar{y}_{R,stsy}^* - \bar{Y}\right)^2 = \sum_{h=1}^{H}\Omega_h^{*2}\bar{Y}_h^2\left[\frac{r_h^2(2\alpha_h+\delta_h)^2e_1^2}{4} - r_h(2\alpha_h+\delta_h)e_1e_0 + e_0^2\right] \quad (16)$$

Taking expectation of both sides of (16) and using the results in (12), gives the variance of $\bar{y}_{R,stsy}^*$ to the first order of approximation as:

$$V\left(\bar{y}_{R,stsy}^*\right) = \sum_{h=1}^{H}\Omega_h^{*2}\bar{Y}_h^2\theta_h\varphi_h\left\{\rho^{*2}C_{hy}^2 + \frac{r_h(2\alpha_h+\delta_h)}{4}[r_h(2\alpha_h+\delta_h) - 4K_h\rho^*]C_{hx}^2\right\} \quad (17)$$

Setting the tuning parameter $Q_h = \bar{x}_{syh}^{-1}$ in equation (8) and substituting for Ω_h^* in equation (17) gives

$$V\left(\bar{y}_{R,stsy}^*\right) = \left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right)^2\sum_{h=1}^{H}W_h^2\bar{Y}_h^2\theta_h\varphi_h\left\{\rho^{*2}C_{hy}^2 + \frac{1}{4}[\eta^2 - 4K_h\eta\rho^*]C_{hx}^2\right\} \quad (18)$$

Where $\eta = r_h(2\alpha_h+\delta_h)$, $\varphi_h = 1+(n_h-1)\rho_{hx}$, $\theta_h = \left(\frac{N_h-1}{n_hN_h}\right)$, $\hat{\bar{X}}_{st} = \sum_{h=1}^{H}W_h\bar{X}_h$, $\rho^* = \left\{\frac{1+(n_h-1)\rho_{hy}}{1+(n_h-1)\rho_{hx}}\right\}^{\frac{1}{2}}$

2.3 Optimal conditions for the proposed calibration ratio estimator

To investigate the optimal condition for the proposed calibration ratio-type estimator let,

$$\frac{\partial V\left(\bar{y}_{R,stsy}^*\right)}{\partial\eta} = 0$$

So that

$$\begin{aligned} \alpha_h &= \left(\frac{2\rho^*K_h - \delta_hr_h}{2r_h}\right) \\ &= \alpha_{h,opt} \qquad (say) \end{aligned} \quad (19)$$

Substituting the value of $\alpha_{h,opt}$ in (19) for α_h in (5) gives the calibration asymptotically optimum estimator ($CAOE$) for population mean ($\bar{Y}$) in stratified systematic sampling as:

$$\bar{y}^*_{R,stsy} = \sum_{h=1}^{H}\Omega^*_h\bar{y}_{syh}\left\{\lambda_h - r_h\left(\frac{\bar{x}_{syh}}{\bar{X}_h}\right)^{\left(\frac{2\rho^*K_h-\delta_h r_h}{2r_h}\right)}\exp\left(\delta_h\frac{(\bar{x}_{syh}-\bar{X}_h)}{(\bar{x}_{syh}+\bar{X}_h)}\right)\right\} \quad (20)$$

Similarly, substituting the value of $\alpha_{h,opt}$ in (19) for α_h in (17) gives the variance of calibration asymptotically optimum estimator ($CAOE$) $V_{opt}(\bar{y}^*_{R,stsy})$ (or minimum variance of $\bar{y}^*_{R,stsy}$) as:

$$V_{opt}(\bar{y}^*_{R,stsy}) = \sum_{h=1}^{H}\Omega^{*2}_h\bar{Y}^2_h\theta_h\varphi_h\{\rho^{*2}C^2_{hy} - K^2_h\rho^{*2}C^2_{hx}\} \quad (21)$$

Setting the tuning parameter $Q_h = \bar{x}^{-1}_{syh}$ in equation (8) and substituting for Ω^*_h in equation (21) gives

$$V_{opt}(\bar{y}^*_{R,stsy}) = \left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right)^2\sum_{h=1}^{H}W^2_h\theta_h\varphi_h\rho^{*2}S^2_{hy}\{1-\rho^2_{hxy}\} \quad (22)$$

Following from the above, the following theorem is established:

Theorem

Given

$$\bar{y}^*_{R,stsy} = \sum_{h=1}^{H}\Omega^*_h\bar{y}_{syh}\left\{\lambda_h - r_h\left(\frac{\bar{x}_{syh}}{\bar{X}_h}\right)^{\alpha_h}exp\left(\delta_h\frac{(\bar{x}_{syh}-\bar{X}_h)}{(\bar{x}_{syh}+\bar{X}_h)}\right)\right\}$$

Then to first degree of approximation

$$V_{opt}(\bar{y}^*_{R,stsy}) \le V(\bar{y}^*_{R,stsy})$$

with equality holding if

$$\alpha_h = \left(\frac{2\rho^*K_h-\delta_h r_h}{2r_h}\right)$$

where

$$V(\bar{y}^*_{R,stsy}) = \left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right)^2\sum_{h=1}^{H}W^2_h\bar{Y}^2_h\theta_h\varphi_h\left\{\rho^{*2}C^2_{hy} + \frac{1}{4}[\eta^2 - 4K_h\eta\rho^*]C^2_{hx}\right\},$$

$$V_{opt}(\bar{y}^*_{R,stsy}) = \left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right)^2\sum_{h=1}^{H}W^2_h\theta_h\varphi_h\rho^{*2}S^2_{hy}\{1-\rho^2_{hxy}\},\ \eta = r_h(2\alpha_h+\delta_h),\ \varphi_h = 1+(n_h-1)\rho_{hx},$$

$$\theta_h = \left(\frac{N_h-1}{n_hN_h}\right),\ \hat{\bar{X}}_{st} = \sum_{h=1}^{H}W_h\bar{X}_h,\ \rho^* = \left\{\frac{1+(n_h-1)\rho_{hy}}{1+(n_h-1)\rho_{hx}}\right\}^{\frac{1}{2}}$$

3. Adaptation of existing estimators to calibration estimation

This section adapts some existing estimators relevant to the study to calibration estimation under the stratified systematic sampling.

3.1 Calibration Stratified Random Sampling Estimator

The classical stratified random sampling estimator is given by:

$$\bar{y}_{st} = \sum_{h=1}^{H}W_h\bar{y}_h$$

Adapting this estimator to Calibration estimation gives

$$\bar{y}^*_{st} = \sum_{h=1}^{H}\Omega^*_h\bar{y}_h \quad (23)$$

where Ω^*_h are the calibration weights as earlier defined

Expressing (23) in terms of the **e's** in (11) gives

$$\bar{y}^*_{st} - \bar{Y} = \sum_{h=1}^{H}\Omega^*_h\bar{Y}_h\, e_0 \quad (24)$$

Let

$$E(e^2_0) = \left(\frac{1}{n_h}-\frac{1}{N_h}\right)C^2_{hy} \quad (25)$$

Squaring and taking expectation of both sides of (24) and using the results of (25) gives the variance of $\bar{y}^*_{st}$ to the first order of approximation as

$$V(\bar{y}^*_{st}) = \sum_{h=1}^{H}\Omega^{*2}_h\gamma_h\,\bar{Y}^2_hC^2_{hy} \quad (26)$$

where $\gamma_h = \left(\frac{1}{n_h}-\frac{1}{N_h}\right)$

Setting the tuning parameter $Q_h = \bar{x}^{-1}_{syh}$ in equation (8) and substituting for Ω^*_h in equation (26) gives

$$V(\bar{y}^*_{st}) = \left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right)^2\sum_{h=1}^{H}W^2_h\gamma_hS^2_{hy} \quad (27)$$

3.2 Calibration Stratified Systematic Sampling Estimator

If $\bar{y}_{syh}$ is the mean of a systematic sample in stratum h, then the estimate of the population mean $\bar{Y}$ in stratified systematic sampling scheme is given by Cochran [2] as:

$$\bar{y}_{stsy} = \sum_{h=1}^{H} W_h \bar{y}_{syh} \tag{28}$$

Adapting this estimator to Calibration estimation gives

$$\bar{y}_{stsy}^{*} = \sum_{h=1}^{H} \Omega_h^{*} \bar{y}_{syh} \tag{29}$$

where Ω_h^{*} are the calibration weights as earlier defined

Expressing (29) in terms of the e's in (11) gives

$$\bar{y}_{stsy}^{*} - \bar{Y} = \sum_{h=1}^{H} \Omega_h^{*} \bar{Y}_h e_0 \tag{30}$$

Squaring and taking expectation of both sides of (30) and using the results of (12) gives the variance of $\bar{y}_{stsy}^{*}$ to the first order of approximation as

$$V\left(\bar{y}_{stsy}^{*}\right) = \sum_{h=1}^{H} \Omega_h^{*2} \theta_h \rho_h^{*2} S_{hy}^2 \tag{31}$$

Setting the tuning parameter $Q_h = \bar{x}_{syh}^{-1}$ in equation (8) and substituting for Ω_h^{*} in equation (31) gives

$$V\left(\bar{y}_{stsy}^{*}\right) = \left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right)^2 \sum_{h=1}^{H} W_h^2 \theta_h \rho_h^{*2} S_{hy}^2 \tag{32}$$

3.3 Calibration Swain Ratio Estimator

Swain [32] introduced the classical ratio estimator in systematic sampling as given by:

$$\bar{y}_{S,sy} = \bar{y}_{sy} \left(\frac{\bar{X}}{\bar{x}_{sy}}\right) \tag{33}$$

We modify this estimator under the stratified systematic sampling as:

$$\bar{y}_{S,stsy} = \sum_{h=1}^{H} W_h \; \bar{y}_{syh} \left(\frac{\bar{X}}{\bar{x}_{syh}}\right) \tag{34}$$

Adapting this estimator to Calibration estimation gives

$$\bar{y}_{S,stsy}^{*} = \sum_{h=1}^{H} \Omega_h^{*} \bar{y}_{syh} \left(\frac{\bar{X}}{\bar{x}_{syh}}\right) \tag{35}$$

where Ω_h^{*} are the calibration weights as earlier defined

Expressing (35) in terms of the e's in (11) gives

$$\bar{y}_{S,stsy}^{*} - \bar{Y} = \sum_{h=1}^{H} \Omega_h^{*} \bar{Y}_h \left(e_0 - e_1 - e_1 e_0 + \left(3e_1^2/2\right)\right) \tag{36}$$

Squaring and taking expectation of both sides of (36) and using the results of (12) gives the variance of $\bar{y}_{S,stsy}^{*}$ to the first order of approximation as

$$V\left(\bar{y}_{S,stsy}^{*}\right) = \sum_{h=1}^{H} \Omega_h^{*2} \theta_h \varphi_h \left\{\rho_h^{*2} S_{hy}^2 + R_h^2 S_{hx}^2 - 2R_h \rho_h^{*} S_{hxy}\right\} \tag{37}$$

Setting the tuning parameter $Q_h = \bar{x}_{syh}^{-1}$ in equation (8) and substituting for Ω_h^{*} in equation (37) gives

$$V\left(\bar{y}_{S,stsy}^{*}\right) = \left(\frac{\bar{X}}{\hat{\bar{X}}_{st}}\right)^2 \sum_{h=1}^{H} W_h^2 \theta_h \varphi_h \left\{\rho_h^{*2} S_{hy}^2 + R_h^2 S_{hx}^2 - 2R_h \rho_h^{*} S_{hxy}\right\} \tag{38}$$

4. Empirical Study

To judge the relative performances of the proposed calibration ratio estimator over members of its class, data statistics given in table 1 was considered.

Table 1: Data Statistics

Parameter	Stratum 1	Stratum 2	Stratum 3	Total
N_h	6	8	11	$N = 25$
n_h	3	3	4	$n = 10$
$\bar{X}_h$	6.813	10.12	7.967	$\bar{X} = 8.3792$
$\bar{Y}_h$	417.33	503.375	340.00	$\bar{Y} = 410.84$
S_{hx}^2	15.9712	132.66	38.438	$S_x^2 = 59.7368$
S_{hy}^2	74775.467	259113.70	65885.60	$S_y^2 = 1237702$
S_{hxy}	1007.0547	5709.1629	1404.71	$S_{xy}^2 = 2524.79$
ρ_{hxy}	0.9215	0.9738	0.8827	$\rho = 0.9285$
γ_h	0.1667	0.2083	0.1591	$R = 49.0309$
w_h^2	0.0576	0.1024	0.1936	$\rho^{*} = 0.9409$
ρ_{hx}	0.8378	0.9042	0.7875	
ρ_{hy}	0.7036	0.7634	0.7875	

Two measuring criteria; variance and percent relative efficiency ($\boldsymbol{PRE}$) were used to compare the performance of each estimator.

$$\boldsymbol{Var(\bar{y}_{st}) = \sum_{h=1}^{H} w_h^2 \gamma_h S_{hy}^2 = 8274.8790}$$

$$\boldsymbol{V(\bar{y}_{st}^*) = \left(\frac{\bar{X}}{\hat{\hat{X}}_{st}}\right)^2 \sum_{h=1}^{H} W_h^2 \gamma_h S_{hy}^2 = 7,889.2696}$$

$$\boldsymbol{V(\bar{y}_{stsy}^*) = \left(\frac{\bar{X}}{\hat{\hat{X}}_{st}}\right)^2 \sum_{h=1}^{H} W_h^2 \theta_h \,\rho_h^{*2} S_{hy}^2 = 27,266.3554}$$

$$\boldsymbol{V(\bar{y}_{S,stsy}^*) = \left(\frac{\bar{X}}{\hat{\hat{X}}_{st}}\right)^2 \sum_{h=1}^{H} W_h^2 \theta_h \,\varphi_h \left\{\rho_h^{*2} S_{hy}^2 + R_h^2 S_{hx}^2 - 2R_h \rho_h^* S_{hxy}\right\} = 3,768.7418}$$

$$\boldsymbol{V_{opt}(\bar{y}_{R,stsy}^*) = \left(\frac{\bar{X}}{\hat{\hat{X}}_{st}}\right)^2 \sum_{h=1}^{H} W_h^2 \theta_h \varphi_h \rho^{*2} S_{hy}^2 \left\{1 - \rho_{hxy}^2\right\} = 2,675.3892}$$

The percent relative efficiency ($\boldsymbol{PRE}$) of an estimator $\boldsymbol{\theta}$ with respect to the usual unbiased estimator in stratified random sampling ($\bar{y}_{st}$) is defined by

$$\boldsymbol{PRE(\theta, \bar{y}_{st}) = \frac{Var(\bar{y}_{st})}{Var(\theta)} \times 100} \tag{39}$$

The percent relative efficiency of the usual unbiased estimator in stratified sampling ($\bar{y}_{st}$), Calibration stratified random sampling estimator ($\bar{y}_{st}^*$), calibration stratified systematic sampling $(\bar{y}_{stsy}^*)$, calibration Swain ratio estimator $(\bar{y}_{S,stsy}^*)$, and the proposed calibration ratio-type estimator in stratified systematic sampling $(\bar{y}_{R,stsy}^*)$ with respect to $\bar{y}_{st}$were computed and presented in table 2.

Table 2: Performance of estimators from analytical study

Estimator	Variance	PRE(θ, y_{st})
$\bar{y}_{st}$	8274.8790	100
$\bar{y}_{st}^*$	7,889.2696	104.8878
$\bar{y}_{stsy}^*$	27,266.3554	30.3483
$\bar{y}_{S,stsy}^*$	3,768.7418	219.5661
proposed	2,675.3892	309.2963

4.2 Discussion of Results

Numerical results from Table (2) show that the proposed estimator $(\bar{y}_{R,stsy}^*)$ has 209 percent gains in efficiency while the Calibration stratified random sampling estimator $(\bar{y}_{st}^*)$ has 5 percent gains in efficiency; this shows that the proposed estimator $(\bar{y}_{R,stsy}^*)$ is 204 percent more efficient than the Calibration stratified random sampling estimator ($\bar{y}_{st}^*$). Similarly, the proposed estimator $(\bar{y}_{R,stsy}^*)$ is 89 percent more efficient than the calibration Swain ratio estimator in stratified systematic sampling $(\bar{y}_{S,stsy}^*)$. Again, in using the proposed estimator $(\bar{y}_{R,stsy}^*)$, one will have 279 percent efficiency gains over the calibration stratified systematic sampling $(\bar{y}_{stsy}^*)$.

5. Conclusion

This paper introduces the theory of calibration estimator to ratio estimation, proposes calibration ratio-type estimator in stratified systematic sampling. It derives the estimator of variance for the proposed estimator and analyses its properties. Analysis showed that the estimator of variance of the proposed calibration ratio-type estimator in stratified systematic sampling is more efficient than the estimators of variance of the unbiased estimator in stratified sampling ($\bar{y}_{st}$), Calibration stratified random sampling estimator ($\bar{y}_{st}^*$), calibration stratified systematic sampling $(\bar{y}_{stsy}^*)$, and calibration Swain ratio estimator in stratified systematic sampling $(\bar{y}_{S,stsy}^*)$. It is observed that the new calibration ratio estimator is very attractive and should be preferred in practice as it provides consistent and more precise parameter estimates.

References

[1] Madow, W.G. and Madow, L.H., (1944): On the theory of systematic sampling, *.Ann. Math. Statist*, 15, 1-24.
[2] Cochran, W.G. (1977). *Sampling Techniques*, 3rd edition, New York: Wiley.
[3] Gautschi, W. (1957). Some remarks on systematic sampling. *Ann. Math. Statist*, **28**, 385–394.
[4] Hajeck, J. (1959): Optimum strategy and other problems in probability sampling *Casopis pro Pestovani Matematiky*, **84**, 387–423.
[5] Singh H.P. and Vishwakarma, G. K. (2006): Combined ratio-product estimator of finite population mean in stratified sampling. Metodologia de Encuestas. **7** (1), 35-44

[6] Singh, H.P. and Vishwakarma, G. K. (2007). Modified exponential ratio and product estimators for finite population mean in double sampling. Austrian Journal of Statistics **36** (3), 217-225.
[7] Singh R, Chauhan, P, Sawan, N and Smarandache, F. (2009). Improvement in estimating the population mean using exponential estimator in simple random sampling. Bulletin of Statistics and Economics **3** (9), 13-18.
[8] Sharma and Tailor (2010). A new ratio-cum-dual to ratio estimator of finite population mean in simple random sampling. Global Journal of Science **10** (1): 27-31.
[9] Onyeka, A, C. (2012). Estimation of population mean in post-stratified sampling using known value of some population parameter (s). Statistics in Transition-New Series, **13** (1), 65-78.
[10] Tailor, R (2012), An almost unbiased ratio-cum-product estimator of population mean using known coefficients of variation of auxiliary variables. International Journal of Statistics and Economics, **8** (12), 70-85.
[11] Choudhury, S. and Singh, B.K. (2012). A class of chain ratio-cum-dual to ratio type estimator with two auxiliary characters under double sampling in sample surveys. Statistics in Transition-New Series, **13** (3), 519-536.
[12] Khare, B. B. and Sinha, R.R. (2012): Combined class of estimators for ratio and product of two population means in presence of non-response. International Journal of Statistics and Economics, **8 (**12), 12-20.
[13] Singh, R, and Audu, A. (2013). Efficiency of ratio estimator in stratified random sampling using information on auxiliary attribute. International Journal of Engineering and Innovative Technology **2** (1)116-172.
[14] Clement, E. P. and Enang, E. I. (2015): Calibration approach alternative ratio estimator for population mean in stratified sampling. *International Journal of Statistics and Economics,* **16** (1), 83-93.
[15] Deville, J.C. and Sarndal, C. E. (1992): Calibration estimators in survey sampling Journal of the American Statistical Association, **87**, 376-382.
[16] Arnab, R. and Singh, S. (2005). A note on variance estimation for the generalized regression predictor Australia and New Zealand Journal of Statistics.**47** (2), 231-234.
[17] Estavao, V. M. and Sarndal, C.E. (2006). Survey estimates by calibration on complex auxiliary information. International Statistical Review **74**, 127-147.
[18] Kott, P.S. (2006): Using calibration weighting to adjust for non-response and coverage errors Survey Methodology **32**, 133-142.
[19] Singh, S. (2006). Survey Statistician celebrate golden jubilee year- 2003 of the linear regression estimator. *Metrika* 1-18.
[20] Singh, S. (2006): Calibrated empirical likelihood estimation using a displacement function: Sir R. A. Fisher's Honest Balance. Presented at INTERFACE 2006.Pasadena, CA, USA.
[21] Sarndal, C-E. (2007): The calibration approach in survey theory and practice. Survey Methodology, **33**, 99-119.
[22] Kim, J. K. and Park, M. (2010): Calibration estimation in survey sampling, International Statistical Review **78** (1), 21-29.
[23] Clement, E. P., Udofia, G.A. and Enang, E. I. (2015). Estimation for domains in stratified random sampling in the presence of non-response. *Aligarch Journal of Statistics, 35*, 117-132.
[24] Clement, E. P. and Enang, E. I. (2015): Multivariate calibration estimation for domain in stratified random sampling. *International Journal of Modern Mathematical Sciences, 13* (2), 187-197.
[25] Clement, E. P. and Enang, E. I. (2017): On the efficiency of ratio estimator over the regression estimator, Communications in Statistics-Theory and Methods 46 (11), 5357-5367, DOI: 10.1080/03610926.2015.1100741
[26] Clement, E. P. (2015). Calibration Approach Separate Ratio Estimator for Population Mean in Stratified Sampling. International Journal of Modern Mathematical Sciences, 13(4), 377-384
[27] Kim, J.M. Sungur, E.A. and Heo T.Y. (2007): Calibration approach estimators in stratified sampling. Statistics and Probability Letters, **77** (1), 99-103.
[28] Koyuncu, N. and Kadilar, C. (2013). Calibration estimators using different measures in stratified random sampling. International Journal of Modern Engineering Research, **3** (1), 415-419.
[29] Clement, E. P., Udofia, G.A. and Enang, E. I. (2014). Sample design for domain calibration estimators. International Journal of Probability and Statistics, **3** (1), 8-14.
[30] Solanki, R. S., Singh, H. P. & Rathour, A. (2012): An alternative estimator for estimating the finite population mean using auxiliary information in sample surveys. *ISRN Probability and Statistics* Article ID 657682, doi:10.5402/2012/657682
[31] Etuk, S. I., Enang, E. I & Ekpenyong, E. J. (2016): A modified class of ratio estimators for population mean, Communications in Statistics -Theory and Methods, 45 (4), 837-849, DOI: 10.1080/03610926.2013.851238
[32] Swain, A.K.P.C**.** (1964). The use of systematic sampling in ratio estimate, *Jour. Ind. Stat. Assoc.*, **2 (213)**, 160–164