Buckling Stress Values of Internally Pressurized Imperfect Thin Cylindrical Shell Under Uniform Axial Compression

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ABSTRACT

This research focused on determination of buckling stress values of internally pressurized unstiffened imperfect thin cylindrical shell under axial compression. The method of solution was carried out by the use of nonlinear large deflection theory and the effect of initial imperfections in the strain-displacement equations was considered. The Ritz method was used to determine the buckling stress parameter of the shell. Numerical examples were carried out; it was found that as imperfect ratio increases, the buckling stress values decreases at constant wavelength ratio, deflection parameters, radius of curvature, internal pressure and thickness of the shell. However, with the use of varying values of imperfect ratio, wavelength ratio, deflection parameters, and thickness of the shell at constant internal pressure and radius of curvature, the buckling stress value progressively to a maximum point known as the critical value and then depreciate progressively. This nonlinear analysis in the Ritz method and the imperfect ratio is responsible for the behaviour of the cylindrical shell.

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Introduction

Cylindrical shells have wide application as one of the important structural elements in many engineering fields. Such fields include civil, marine, mechanical, aeronautic and chemical engineering [1].

Cylindrical shell structures can fail either by yielding of buckling. The collapse of the structures precipitated by buckling is often a more serious problem than fracture or yielding. Buckling sometime occurs suddenly without warning, causing a catastrophic failure. Fracture or yielding, on the other hand, can also produce failure, but the elasticity of the material permits a redistribution of the stresses often allowing a progressive collapse rather than a sudden complete collapse characteristic of buckling. Once buckling is initiated within the structure, there is little or no chance of recovery unless the load is suddenly [2], [3]. In fact, buckling phenomenon in cylindrical shell occurs when most of the strain energy which is stored as membrane energy has been converted to bending energy requiring large deformation resulting to catastrophic failure [2]. Hence, the design of thin cylindrical shells should be based on buckling criteria [4]. Buckling behaviour of cylindrical shells (in particular, the critical buckling load) is not accurately predicted by linear elastic equations due to initial imperfections of the shell structure under the action of compressive loads. [5]

The imperfections include geometrical, structural and loading imperfections. These imperfections affect the load carrying capacity of the shell. The most dominant among these imperfections is geometrical imperfections [6], [7].

The geometrical imperfection is mostly due to deviation in circularity of the shell during its manufacturing.

The presence of this imperfection greatly reduces the buckling load predicted for a shell of perfect geometry. Thus, reliable prediction of buckling strength of these shell structures is important, because the buckling failure is catastrophic [8], [9], [10], [11].

The main objective of this research is to develop buckling stress of imperfect unstiffened thin cylindrical shell under uniform axial compression using the Ritz method. This was achieved by assuming the displacement function of the shell. Its stress function was obtained from the assumed displacement function from the compatibility equation which was carried out by nonlinear large deflection theory. The expression of the stored energy in the shell as well as work done by the external load was obtained using both the stress and displacement functions. The large deflection terms, effect of imperfection in the strain displacement and the external load were considered in the formulation of total strain energy of the imperfect shell. The resulted total strain energy was minimized using the Ritz method to determine the equation for obtaining the buckling stress values of the shell.

Let x and y be the axial and circumferential axis in the median surface of the undeformed cylindrical shell as shown in Fig. 1, w is the total radial deflection and w0 represents the initial radial deflection.

From the theory of elasticity, the strain – displacement relations of the cylindrical shell is as expressed in Eqns. (1a), (1b) and (1c) respectively

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2.0 Energy expression for the cylindrical shell
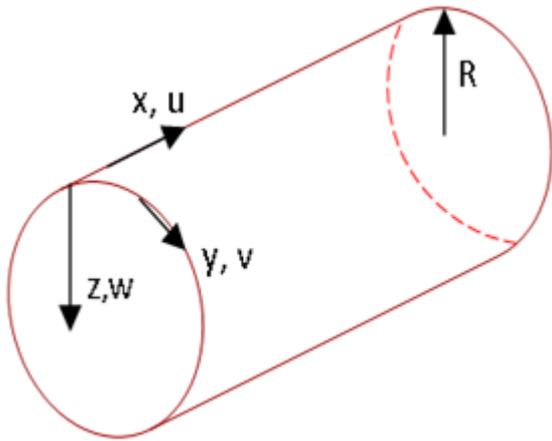
Fig 1. Coordinates and Displacement Components of a point on the Middle- surface of the shell.

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{dw_0}{\partial x}\right)^2 \tag{1a}$$

$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \frac{w-w_0}{R} \tag{1b}$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\cdot\frac{\partial w_0}{\partial y} \tag{1c}$$

The stresses and strains in the middle surface of the shell in the case of plane stress are related to each other by the following equations.

$$\sigma_x = \frac{E}{1-\mu^2}(\epsilon_x + \mu \epsilon_y) \tag{2a}$$

$$\sigma_y = \frac{E}{1-\mu^2}(\epsilon_y + \mu \epsilon_x) \tag{2b}$$

$$\sigma_{xy} = \frac{E}{2(1+\mu)} \epsilon_{xy} \tag{2c}$$

Substituting Eqns. (1a), (1b) and (1c) into their related equations in Eqns. (2a), (2b) and (2c), the followings were obtained;

$$\sigma_x = \frac{E}{1-\mu^2}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \mu\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right)\right]\right\} \tag{3a}$$

$$\sigma_y = \frac{E}{1-\mu^2}\left\{\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right) + \mu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]\right\} \tag{3b}$$

$$\sigma_{xy} = \frac{E}{2(1-\mu^2)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\cdot\frac{\partial w_0}{\partial y}\right] \tag{3c}$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}; \ \sigma_y = \frac{\partial^2 F}{\partial x^2}; \ \sigma_{xy} = \frac{-\partial^2 F}{\partial x \partial y} \tag{4}$$

Eliminating variables u and v in Eqns. (3) and (4), the relation between stress function F and radial component displacement, w was expressed as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial x^2}\cdot\frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R}\frac{\partial^2 w_0}{\partial x^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right] \tag{5a}$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is called Laplace operator.

$$(\nabla^2)^2 F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial x^2}\cdot\frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R}\frac{\partial^2 w_0}{\partial x^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right] \tag{5b}$$

For simplicity, w was assumed to be proportional to w_0. Thus,

$$Л = \frac{w_0}{w} \tag{6}$$

here Л is called the imperfection ratio. It is independent of x and y. With the expression from Eqns (5b) and (6), the compatibility equation was expressed as;

$$\left(\frac{1}{1-Л}\right)\nabla^4 F = E(1+Л)\left[\left(\frac{\partial^2 w}{\partial x \partial y^2}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}\right] - \frac{E}{R}\frac{\partial^2 w}{\partial x^2} \tag{7}$$

Where ∇^4 is called Bilharmonic operator.

Equation (7) is the compatibility equation of perfect thin cylindrical shell.

The strain energy of isotropic medium referred to arbitrary orthogonal coordinates was expressed as:

$$U = \frac{1}{2}\iiint_{vol}\sigma_{ij}\epsilon_{ij}dvol = \frac{1}{2}\iiint_{vol}[\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_{xy}2\epsilon_{xy} + \sigma_{xz}2\epsilon_{xz} + \sigma_{yz}2\epsilon_{yz}]dxdydz \tag{8}$$

Substituting Eqns. 1(a-c), 2(a-c), 3(a-c) and 4 into Eqn. (8a), we have expressions stated in Eqns. (8) and (9) respectively:

i.The extensional strain energy in the shell was expressed as;

$$U_e = \frac{h}{2E}\int_0^L\int_0^{2\pi R}\left\{\left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2}\right)^2 + 2(1+\mu)\left[\left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 - \frac{\partial^2 F}{\partial x^2}\cdot\frac{\partial^2 F}{\partial y^2}\right]\right\}dxdy \tag{8a}$$

ii.The work of the external force applied at the ends of the shell.

$$U_c = \sigma_c h\int_0^L\int_0^{2\pi R}\left[\frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2} - \mu\frac{\partial^2 F}{\partial x^2}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]dxdy \tag{9}$$

Where σ_c = constant stress over the shell thickness.

iiiThe potential due to the internal pressure, p

$$= \int_0^L\int_0^{2\pi R}p(w-w_0)\,dxdy \tag{10}$$

$$w = f_1 + f_2\cos\frac{mx}{R}\cos\frac{ny}{R} + f_3\cos\frac{2mx}{R} + f_4\cos\frac{2ny}{R} \tag{11}$$

Where m and n are the numbers of waves in axial and circumferential directions respectively. The corresponding stress function for cylindrical shell subjected to compressive force acting concentrically:

$$F = -\sigma_c\frac{y^2}{2} + \frac{PR}{h}\frac{x^2}{2} + a_{11}\cos\frac{mx}{R}\cos\frac{ny}{R} + a_{22}\cos\frac{2mx}{R}\cos\frac{2ny}{R} + a_{02}\cos\frac{2ny}{R} + a_{20}\cos\frac{2mx}{R} + a_{13}\cos\frac{mx}{R}\cos\frac{3ny}{R} + a_{31}\cos\frac{3mx}{R}\cos\frac{ny}{R} \tag{12}$$

The coefficients $a_{11}, a_{22}, a_{02}, a_{20}, a_{31}, a_{13}$ in Eqn. (12) were determined in terms of $f_2, f_3,$ and f_4 from the compatibility equation as expressed in Eqns. 13(a-f):

$$\forall_{20} = \frac{a_{20}}{Eh^2} = (1-Л)\frac{\beta}{4}\varphi_4 - (1-Л^2)\left(\frac{\bar{\mu}^2}{32}\varphi_2^2\right) \tag{13a}$$

$$\forall_{02} = \frac{a_{02}}{Eh^2} = -(1-Л^2)\frac{\varphi_2^2}{32\bar{\mu}^2} \tag{13b}$$

$$\forall_{11} = \frac{a_{11}}{Eh^2} = (1-Л)\frac{\varphi_2^2\beta}{(1+\bar{\mu}^2)^2} - (1-Л^2)\frac{2\bar{\mu}^2}{(1+\bar{\mu}^2)^2}\varphi_2(\varphi_3+\varphi_4) \tag{13c}$$

$$\forall_{22} = \frac{a_{22}}{Eh^2} = -(1-Л^2)\frac{\bar{\mu}^2}{(1+\bar{\mu}^2)^2}(\varphi_3\varphi_4) \tag{13d}$$

$$\forall_{31} = \frac{a_{31}}{Eh^2} = -(1-Л^2)\frac{2\bar{\mu}^2}{(9+\bar{\mu}^2)^2}(\varphi_2\varphi_4) \tag{13e}$$

$$\forall_{13} = \frac{a_{13}}{Eh^2} = -(1-Л^2)\frac{2\bar{\mu}^2}{(1+9\bar{\mu}^2)^2}(\varphi_2\varphi_3) \tag{13f}$$

Where

$$\bar{\mu} = \frac{n}{m}, \beta = \frac{R}{m^2 h}, \qquad \varphi_i = \frac{f_i}{h}, \text{and } i = 2, 3, 4$$

$\bar{\mu}$ *is* called wavelength ratio in axial and circumferential direction

3.0 Expression of Total Potential for unstiffened Cylindrical Shell Subjected to Internal Pressure and Axial Compressive Force

The total potential of the system, $\prod$ is the sum of the strain energies and it is expressed as follows,

$$\prod = U_e + U_c + U_P \tag{14}$$

The non-dimensional form of the strain energies were as shown in Eqns (15), (16) and (17)

$$\bar{U}_e = \frac{U_e R}{\pi E h^3 L} = \bar{P}^2 + \bar{\sigma}_c^2 + 2\mu\bar{P}\bar{\sigma}_c + \forall_{11}^2 \frac{(1+\bar{\mu}^2)^2}{4\beta^2} +$$

$$\forall_{22}^2 \frac{4(1+\bar{\mu}^2)^2}{\beta^2} + \frac{8}{\beta^2}(\bar{\mu}^4 \forall_{02}^2 + \forall_{20}^2) + \frac{1}{4\beta^2}[(9+\bar{\mu}^2)\forall_{31}^2 +$$

$$(1+9\bar{\mu}^2)^2 \forall_{13}^2] \tag{15}$$

$$\bar{U}_c = \frac{U_c R}{\pi E h^3 L} = -2\left[\bar{\sigma}_c^2 + \mu\bar{P}\bar{\sigma}_c + \left(\frac{\bar{\sigma}_c}{8\beta}\varphi_2^2 + \frac{\bar{\sigma}_c}{\beta}\varphi_4^2\right)(1-Л^2)\right] \tag{16}$$

$$\bar{U}_P = \frac{U_P R}{\pi E h^3 L} = 2\bar{P}(1-Л^2)\frac{\bar{\mu}^2}{\beta}\left(\frac{\varphi_2^2}{8} + \varphi_3^2\right) - (\bar{P} + \mu\sigma_c) \tag{17}$$

Where $\bar{P} = \frac{PR^2}{Eh^2}$ and $\bar{\sigma}_c = \frac{\sigma_c R}{Eh}$ respectively   The non-dimensional form of the total potential of the system, $\bar{\prod}$, is expressed as shown in Eqn (17b)

$$\bar{\prod} = \bar{U}_e + \bar{U}_c + \bar{U}_P \tag{17b}$$

3.1 Minimization of Total Potential Energy, $\bar{\prod}$, of Internally Pressurized Unstiffened Thin Cylindrical Shell Subjected to Axial Compression

The total potential energy of internally pressurized thin cylindrical subjected to axial compression must be minimum when the structure is in equilibrium.

The variation of potential with respect to each of the arbitrary parameters vanished for equilibrium, this gave rise to Eqn.(18):

$$\frac{\partial\bar{\prod}}{\partial\varphi_2} = 0, \frac{\partial\bar{\prod}}{\partial\varphi_3} = 0, \frac{\partial\bar{\prod}}{\partial\varphi_4} = 0 \tag{18}$$

Evaluation of $\frac{\partial\bar{\prod}}{\partial\varphi_2} = 0,\ \frac{\partial\bar{\prod}}{\partial\varphi_3} = 0,\ \frac{\partial\bar{\prod}}{\partial\varphi_4} = 0$ yielded Eqns (19) – (21):

$$\phi_1 \frac{\beta}{1-Л} = T_1^* + \beta^2(T_2^* + T_3^*\lambda + T_4^*\lambda^2) + \varphi_2^2 T_5^* \tag{19}$$

$$\phi_2 \frac{\beta}{1-Л} = \bar{\bar{B}}_1 + \beta^2\bar{\bar{B}}_2\lambda^2 + \varphi_2^2\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda}\right) \tag{20}$$

$$\phi_3 \frac{\beta}{1-Л} = H_1 + \beta^2(H_2 + H_3\lambda^2) + \varphi_2^2\left(H_4 + \frac{H_5}{\lambda}\right) \tag{21}$$

he notations used in Eqns. (19),(20) and (21) were defined as follows;

$$\phi_1 = \bar{\sigma}_c - \bar{\mu}^2\bar{P} \tag{22a}$$

$$T_1^* = \frac{(1+\bar{\mu}^2)^2}{12(1-\mu^2)} \tag{22b}$$

$$T_2^* = \Lambda_2 = \frac{1}{(1+\bar{\mu}^2)^2} \tag{22c}$$

$$T_3^* = \Lambda_3 = -\left[\frac{2(2+Л)(1+\lambda_1)}{(1+\bar{\mu}^2)^2} + \frac{\lambda_1}{2}\right]\bar{\mu}^2 \tag{22d}$$

$$T_4^* = \Lambda_4 = 4(1+Л)\bar{\mu}^4\left[\frac{(1+\lambda_1)^2}{(1+\bar{\mu}^2)} + \frac{\lambda_1}{(9+\bar{\mu}^2)^2} + \frac{1}{(1+9\bar{\mu}^2)^2}\right] \tag{22e}$$

$$T_5^* = \Lambda_5 = (1+Л)\left(\frac{1+\bar{\mu}^4}{16}\right) \tag{22f}$$

$$\phi_2 = -\bar{\mu}^2\bar{P} \tag{23a}$$

$$\bar{\bar{B}}_1 = \frac{\bar{\mu}^4}{3(1-\mu^2)} \tag{23b}$$

$$\bar{\bar{B}}_2 = B_2 = (1+Л)\frac{2\bar{\mu}^4}{(1+\bar{\mu}^2)^2}\lambda_1^2 \tag{23c}$$

$$\bar{\bar{B}}_3 = B_3 = (1+Л)\left[\frac{1}{(1+9\bar{\mu}^2)^2} + \frac{(1+\lambda_1)}{(1+\bar{\mu}^2)^2}\right]\frac{\bar{\mu}^4}{2} \tag{23d}$$

$$\bar{\bar{B}}_4 = B_4 = -\frac{\bar{\mu}^2}{4(1+\bar{\mu}^2)^2} \tag{23e}$$

$$\phi_3 = \bar{\sigma}_c \tag{24a}$$

$$H_1 = \frac{1}{3(1-\mu^2)} \tag{24b}$$

$$H_2 = \aleph_2 = \frac{1}{4} \tag{24c}$$

$$H_3 = \aleph_3 = (1+Л)\frac{2\bar{\mu}^4}{(1+\bar{\mu}^2)^2} \tag{24d}$$

$$H_4 = \aleph_4 = (1+Л)\frac{\bar{\mu}^4}{2}\left[\frac{1}{(9+\bar{\mu}^2)^2} + \left(1+\frac{1}{\lambda_1}\right)\frac{1}{(1+\bar{\mu}^2)^2}\right] \tag{24e}$$

$$H_5 = \aleph_5 = -\left[\frac{1}{(1+\bar{\mu}^2)^2} + \frac{1+Л}{8}\right]\frac{\bar{\mu}^2}{4\lambda_1} \tag{24f}$$

Where $\lambda = \frac{\varphi_3}{\beta}\ and\ \lambda_1 = \frac{\varphi_4}{\varphi_3}$

Eliminating φ_2 and β from Eqns. (19), (20) and (21), the following equation was obtained:

$$\bar{\bar{M}}_1\bar{\sigma}_c^2 + \bar{\bar{M}}_2\bar{\sigma}_c + \bar{\bar{M}}_3 = 0 \tag{25}$$

where

$$\bar{\bar{M}}_1 = \frac{1}{(1-Л^2)}\left[\frac{\vec{\eta}_2\vec{\eta}_3}{\bar{\bar{\omega}}_3^2}\left(H_4 + \frac{H_5}{\lambda} - T_5^*\right)^2 + \frac{\bar{\bar{\omega}}_2}{\bar{\bar{\omega}}_3}\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda}\right)^2 - \left(\frac{\bar{\bar{\omega}}_2\vec{\eta}_3}{\bar{\bar{\omega}}_3^2} + \frac{\vec{\eta}_2}{\bar{\bar{\omega}}_3}\right)\left(H_4 + \frac{H_5}{\lambda} - T_5^*\right)\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda}\right)\right] \tag{26a}$$

$$\bar{\bar{M}}_2 = -\frac{\bar{P}\bar{\mu}^2}{(1-Л^2)}\left\{\frac{2\vec{\eta}_2\vec{\eta}_3}{\bar{\bar{\omega}}_3^2}\left(H_4 + \frac{H_5}{\lambda}\right)\left(H_4 + \frac{H_5}{\lambda} - T_5^*\right) + \frac{2\bar{\bar{\omega}}_2}{\bar{\bar{\omega}}_3}\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda} - T_5^*\right) - \left(\frac{\bar{\bar{\omega}}_2\vec{\eta}_3}{\bar{\bar{\omega}}_3^2} + \frac{\vec{\eta}_2}{\omega_3}\right)\left[2\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda}\right)\left(H_4 + \frac{H_5}{\lambda}\right) + T_5^{*2} - T_5^*\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda} + H_4 + \frac{H_5}{\lambda}\right)\right]\right\} \tag{26b}$$

$$\bar{\bar{M}}_3 = \frac{\bar{P}\bar{\mu}^4}{(1-Л^2)}\left[\frac{\vec{\eta}_2\vec{\eta}_3}{\bar{\bar{\omega}}_3^2}\left(H_4 + \frac{H_5}{\lambda}\right)^2 + \frac{\bar{\bar{\omega}}_2}{\bar{\bar{\omega}}_3}\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda} - T_5^*\right)^2 - \left(\frac{\bar{\bar{\omega}}_2\vec{\eta}_3}{\bar{\bar{\omega}}_3^2} + \frac{\vec{\eta}_2}{\bar{\bar{\omega}}_3}\right)\left(H_4 + \frac{H_5}{\lambda}\right) + \left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda} - T_5^*\right)\right] + \frac{\vec{\eta}_3^2\bar{\bar{\omega}}_2^2}{\bar{\bar{\omega}}_3^2} - \frac{2\vec{\eta}_2\vec{\eta}_3\bar{\bar{\omega}}_2}{\bar{\bar{\omega}}_3} + \vec{\eta}_2^2 \tag{26c}$$

Where $\vec{\eta}_2$, and $\vec{\eta}_3$ were defined as follows as derived:

$$\vec{\eta}_2 = T_1^*\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda}\right) - B_1 T_5^* \tag{27a}$$

$$\vec{\eta}_3 = (T_2^* + T_3^*\lambda + T_4^*\lambda^2)\left(\bar{\bar{B}}_3 + \frac{\bar{\bar{B}}_4}{\lambda}\right) - \bar{\bar{B}}_2 T_5^*\lambda^2 \tag{27b}$$

Also, $\bar{\bar{\omega}}_2$ and $\bar{\bar{\omega}}_3$ were defined as follows:

$$\bar{\bar{\omega}}_2 = T_1^*\left(H_4 + \frac{H_5}{\lambda}\right) - H_1 T_5^* \tag{28a}$$

$$\bar{\bar{\omega}}_3 = (T_2^* + T_3^*\lambda + T_4^*\lambda^2)\left(H_4 + \frac{H_5}{\lambda}\right) - (H_2 + H_3\lambda^2)T_5^* \tag{28b}$$

Equation (25) is the governing equation for determining the critical buckling stress of an internally Pressurized Unstiffened Thin Cylindrical Shell Under Axial Compressive Force.

4.0 Results and Discussions

4.1 Results

Numerical Examples For the purpose of this work, the following numerical examples were done: Buckling stress parameters of the cylindrical shells for $0.1 \leq Л \leq 0.9$ were determined With $\lambda = \lambda_1 = 1,\ m = 5,\ \bar{\mu} = 1,\ h = 0.05\ metre,\ \bar{P} = 2\ and\ R = 2\ metres$. The Results were as shown in Table 1, while its graphical representation is as shown in Fig.2

Table 1. Buckling Stress Vs Imperfect ratio with other parameters constant of Imperfect unstiffened cylindrical shell.

Imperfect ratio, Л	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buckling Stress Parameter	3.0330	2.8148	2.5499	2.2660	1.9783	1.6951	1.4213	1.1590	0.9097

Table 2. Buckling Stress of Imperfect unstiffened cylindrical shell from varying properties of the

Thickness, h (metre)	Imperfect ratio, Л	Wavelength Ratio, $\bar{\mu}$	Deflection Parameter, λ	Deflection Parameter 1, λ_1	Buckling Stress
0.05	0.1	1.0	1.0	0.5	0.2599
0.10	0.2	1.2	2.0	1.0	2.1697
0.15	0.3	1.4	3.0	1.5	0.4098
0.15	0.4	1.6	4.0	2.0	0.08383
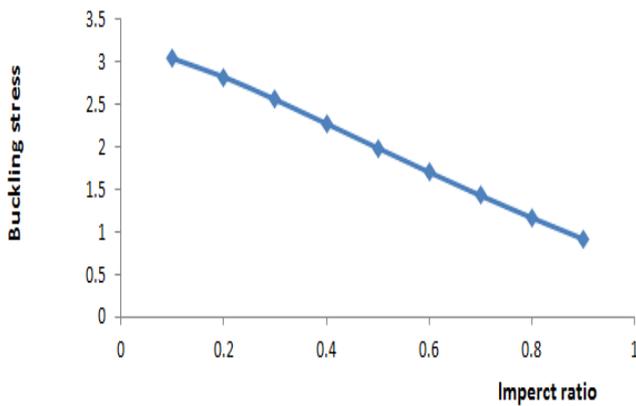
Fig 2. Graph of buckling stress Vs Imperfect ratio with other parameters constant.
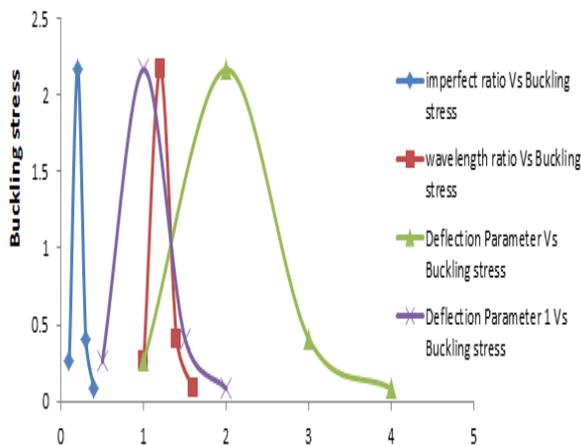
Fig 3. Graph of Buckling stress Vs Varying Imperfect ratio, Wavelength ratio, and Deflection Parameters.

Buckling stress parameter of the cylindrical shells with $\mathbf{m = 5}$, $\mathbf{\bar{P} = 2 \text{ and } R = 2 \text{ metres}}$ and varying thickness, h, imperfect ratio, Л , wavelength ratio, $\bar{\mu}$, and deflection parameters $\boldsymbol{\lambda} \text{ and } \boldsymbol{\lambda_1}$. The results were as shown in Table 2, while its graphical representation is as shown in Fig.3

4.2 Discussion of Results

As shown in Fig.2, with constant values of $\boldsymbol{\lambda}$, $\boldsymbol{\lambda_1}$ $\mathbf{m}$, $\boldsymbol{\bar{\mu}}$, $\mathbf{h}$, $\mathbf{\bar{P}}$ and $\mathbf{R}$, as the imperfect ratio of the imperfect unstiffened cylindrical shell increases, the buckling stress of the shell decreases. While in Fig. 3, with increase in $\mathbf{h}$, Л, $\boldsymbol{\lambda}$, $\boldsymbol{\lambda_1}$, $\boldsymbol{\bar{\mu}}$ at constant $\mathbf{m}$, $\mathbf{\bar{P}}$ and $\mathbf{R}$, buckling stress increases progressively to a certain point called the critical buckling stress and then depreciate progressively.

5.0 Conclusion

The use of nonlinear large deflection theory in the Ritz method for determination of buckling stress of imperfect unstiffened thin cylindrical shell is very convenient. It made the derivation of buckling stress parameter equation easy.

The buckling stress values obtained from the derived equation would be useful in stability design of imperfect unstiffened thin cylindrical shell.

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