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Fingero-Imbibition Phenomenon in Double Phase Flow through Homogeneous Porous Media with Magnetic Field Effect

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ABSTRACT

In this article, the phenomenon of fingero-imbibition in a particular displacement method concerning two immiscible fluids through a dipping homogeneous porous media with a magnetic field effect has been discussed analytically under certain conditions. This phenomenon provides a nonlinear partial differential equation as a governing equation, which can be solved by Homotopy Perturbation Sumudu transform method (HPSTM). The numerical and graphical results are discussed using MATLAB.

Introduction

It is a well-known physical phenomenon that whenever a porous medium full of some resident fluid is brought into contact with any other fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such phenomenon arising because of the difference in wetting abilities is called counter-current imbibition. Similarly, whenever a fluid in a porous medium is displaced by another fluid of lesser viscosity, then in place of the normal displacement of the whole front, protuberances (fingers [1]) may occur which shoot through the porous at relatively great speed. This phenomenon is called fingering or instabilities. And the phenomena of fingering and imbibition occurring simultaneously in displacement process is known as a fingero - imbibition phenomenon. This phenomenon has received much current importance, especially in petroleum engineering, hydrogeology, geophysics, reservoir engineering, etc. Many authors have investigated this problem from different viewpoints: e.g. Scheidegger [6, 7], Verma [9, 10], Mehta [4], Vyas [11], Parikh [5], etc.

Consider the basic assumptions as follows:

[1] The injected fluid is relatively less viscous than the native fluid.

[2] The injected fluid is conductive and the native fluid is non-conductive.

[3] A variable magnetic field, which is in the formation of native fluid, increases the velocity of the injected fluid by the gradient of the magnetic field effect term $\frac{\mu H^2}{8\pi}$, where μ is the permeability of magnetic field H [11].

Statement and Mathematical Formulation of the Problem:

Here we consider that

[1] There is a uniform injection of the fluid-water (w), into a finite cylindrical piece of a homogeneous porous medium of length L, that is completely saturated with a native fluid-oil (o).

[2] The cylinder is totally surrounded by an impermeable surface except at one end, that is labelled as an imbibition surface ($x = 0$) and this end is subjected to a nearby formation of the injected conductive fluid as shown in Figure 1. Also, Figure 2 shows the schematic demonstration of fingero-imbibition phenomenon.

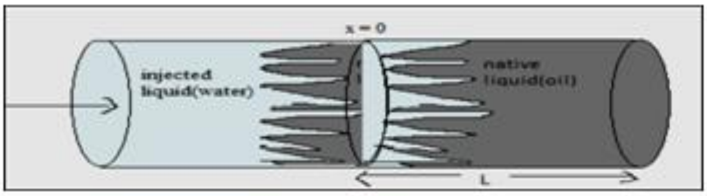

Figure 1. Demonstration of fingero-imbibition phenomenon in a finite cylindrical piece of a homogeneous porous medium.

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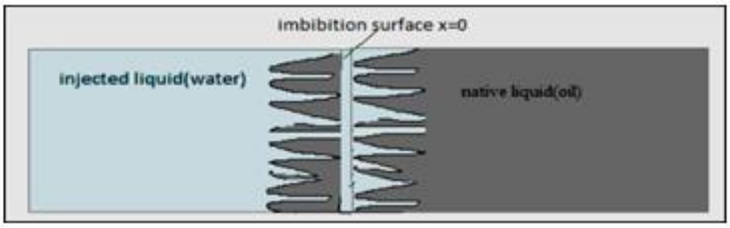

Figure 2. Schematic demonstration of fingero-imbibition phenomenon.

Nomenclature

H	Magnetic field intensity
μ	Permeability of magnetic field H
L	Length of a homogeneous porous medium
X	Linear coordinate
T	Time
V_w, V_o	Seepage velocity of water, oil
K	Permeability of a homogeneous porous medium
k_w, k_o	Relative permeability of water, oil
μ_w, μ_o	Constant kinematic viscosity of water, oil
P_w, P_o	Pressure of water, oil
$\emptyset$	Porosity of the medium
S_w, S_o	Saturation of water, oil
P_C	Capillary pressure
β	Capillary pressure coefficient
λ	Proportionality constant

Our main aim of the recent study is to find saturation of injected water (S_w), for that the seepage velocity of injected fluid-water ($\mathbf{V_w}$) and native fluid-oil (V_o) are given by Darcy's law (Scheidegger [7])

$$\boldsymbol{V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} + \frac{\mu H}{4\pi}\frac{\partial H}{\partial x}} \tag{1}$$

$$\boldsymbol{V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x}} \tag{2}$$

The equations of continuity are as follows:

$$\emptyset \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = \mathbf{0} \tag{3}$$

$$\emptyset \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = \mathbf{0} \tag{4}$$

For the imbibition phenomenon, the capillary pressure P_c [2] can be written as

$$\boldsymbol{P_c = P_o - P_w} \tag{5}$$

$$\therefore \frac{\partial P_c}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x} \tag{6}$$

and $\boldsymbol{V_o + V_w = 0}$ (7)

By equations (1), (2) and (7), we get

$$\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} + \frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} - \frac{\mu H}{4\pi}\frac{\partial H}{\partial x} = 0 \tag{8}$$

Solving equations (6) and (8) for $\frac{\partial P_w}{\partial x}$, we get

$$\frac{\partial P_w}{\partial x} = \frac{-k\left(\frac{k_o}{\mu_o}\right)\frac{\partial P_c}{\partial x} + \frac{\mu H \partial H}{4\pi\,\partial x}}{k\left(\frac{k_o}{\mu_o} + \frac{k_w}{\mu_w}\right)} \tag{9}$$

By equation (1) and (9),

$$\boldsymbol{V_w = k\left(\frac{k_w k_o}{k_w\mu_o + k_o\mu_w}\right)\frac{\partial P_c}{\partial x} + \left(\frac{k_o\mu_w}{k_w\mu_o + k_o\mu_w}\right)\frac{\mu H}{4\pi}\frac{\partial H}{\partial x}} \tag{10}$$

Substituting the value of V_w into the continuity equation (3), we get

$$\phi\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}[k(\frac{k_w k_o}{k_w\mu_o + k_o\mu_w})\frac{\partial P_c}{\partial x} + (\frac{k_o\mu_w}{k_w\mu_o + k_o\mu_w})\frac{\mu H}{4\pi}\frac{\partial H}{\partial x}] = 0 \tag{11}$$

According to Scheidegger [7] approximation, we have

$$\left(\frac{k_w k_o}{k_w\mu_o + k_o\mu_w}\right) \approx \frac{k_o}{\mu_o} \tag{12}$$

By equation (11), we get

$$\phi\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}[k(\frac{k_o}{\mu_o})\frac{\partial P_c}{\partial x} + (\frac{k_o}{\mu_o})\frac{\mu H}{4\pi}\frac{\partial H}{\partial x}] = 0 \tag{13}$$

Choosing two phases such that their permeability and viscosity are different (Scheidegger [6]), but their ratios are equal and constant, i.e. $\left(\frac{k_w}{\mu_w}\right) = \left(\frac{k_o}{\mu_o}\right)$

Thus by equation (13), we get

$$\emptyset\frac{\partial S_w}{\partial t}+\frac{\partial}{\partial x}\left[k\left(\frac{k_o}{\mu_o}\right)\frac{\partial P_c}{\partial x}+\frac{\mu H}{4\pi}\frac{\partial H}{\partial x}\right]=0 \quad (14)$$

Again k_w and k_o are assumed to be functions of water saturation S_w and oil saturation S_o respectively and assume linear relative permeability for water and oil as [7]

$$k_w=S_w \text{ and } k_o=S_o=1-\alpha S_w;\ \alpha=1.11 \quad (15)$$

Also, capillary pressure can be represented by a function of saturation of water [2, 4] as

$$P_c=-\beta S_w \quad (16)$$

By equations (14), (15) and (16), we get

$$\phi\frac{\partial S_w}{\partial t}+\frac{k}{\mu_o}\frac{\partial}{\partial x}[(1-\alpha S_w)(-\beta)(\frac{\partial S_w}{\partial x})+\frac{\partial}{\partial x}(\frac{\mu H}{4\pi}\frac{\partial H}{\partial x})]=0 \quad (17)$$

Here we have considered that the magnetic field effect is in *x* direction only [10]. Also, because of one-dimensional fingero-imbibition phenomenon, Verma [9] gave the value of magnetic field effect as $H=\frac{\lambda}{x^n}$, where *n* is an integer. And for the definiteness Verma [9] has taken *n = -1*

$$\therefore H=\lambda x$$

By equation (17), we get

$$\emptyset\frac{\partial S_w}{\partial t}-\frac{k\beta}{\mu_o}\frac{\partial}{\partial x}\left[(1-\alpha S_w)\left(\frac{\partial S_w}{\partial x}\right)\right]+\frac{\mu}{4\pi}\lambda^2=0 \quad (18)$$

Using dimensionless variables $X=\frac{x}{L}$ and $T=\frac{k\beta}{\phi\mu_o L^2}t$ and taking $S=1-S_w$, we get

$$\frac{\partial S}{\partial T}=\frac{\partial}{\partial X}\left(S\frac{\partial S}{\partial X}\right)+A$$

$$\therefore \frac{\partial S}{\partial T}=S\frac{\partial^2 S}{\partial X^2}+\left(\frac{\partial S}{\partial X}\right)^2+A \quad (19)$$

where $A=\frac{\mu L^2\mu_o\lambda^2\alpha}{k\beta 4\pi}$ [11] (19a)

and $S(X,0)=e^{-X}; X\geq 0,$ (19b)

$S(0,T)=0; T\geq 0$ (19c)

$\frac{\partial S}{\partial X}(1,T)=0; T\geq 0$ (19d)

Here we assume that the saturation of water S_w is 1 at $X=0$ for any $T\geq 0$, i.e. at distance $X=0$, the medium is fully saturated. Also at the time $T=0$, we assume $S=1-S_w=e^{-X}$, i.e. saturation of water S_w is increasing, which is feasible with the physical phenomenon [5].

Solution of the Problem by HPSTM:

To use Homotopy Perturbation Sumudu transform method (HPSTM) [8], first apply Sumudu transform on both sides of the equation (19) and using initial condition, we get

$$\mathcal{S}\left[\frac{\partial S}{\partial T}\right]=\mathcal{S}\left[S\frac{\partial^2 S}{\partial X^2}+\left(\frac{\partial S}{\partial X}\right)^2\right]+\mathcal{S}[A]$$

$$\therefore \frac{1}{u}[\mathcal{S}[S(X,T)]-S(X,0)]=\mathcal{S}[SS_{XX}+(S_X)^2]+A$$

$$\therefore \mathcal{S}[S(X,T)]=e^{-X}+u\mathcal{S}[SS_{XX}+(S_X)^2]+Au \quad (20)$$

Apply inverse Sumudu transform, we get

$$S(X,T)=e^{-X}+AT+\mathcal{S}^{-1}\left[u\mathcal{S}[SS_{XX}+(S_X)^2]\right] \quad (21)$$

Apply Homotopy Perturbation Method (HPM), we get

$$S(X,T)=\sum_{n=0}^{\infty}p^nS_n=e^{-X}+AT+p\mathrm{S}^{-1}[u\mathrm{S}[\sum_{n=0}^{\infty}p^nH_n(S)]] \quad (22)$$

where $H_n(S)$ is He's polynomials (see [3]) given by

$$H_n(S_0,S_1,...,S_n)=\frac{1}{n!}\frac{\partial^n}{\partial p_n}[N(\sum_{i=0}^{\infty}p^iS_i)]_{p=0}, n=0,1,2,3,...$$

Comparing like powers of '*p*', we get

$$p^0: S_0=e^{-X}+AT$$

$$p^1: S_1=\mathrm{S}^{-1}[u\mathrm{S}[S_0(S_0)_{XX}+((S_0)_X)^2]]=2e^{-2X}T+\frac{A}{2}e^{-X}T^2$$

$$p^2: S_2=\mathrm{S}^{-1}[u\mathrm{S}[S_0(S_1)_{XX}+S_1(S_0)_{XX}+2(S_0)_X(S_1)_X]]$$

$$=9e^{-3X}T^2+6Ae^{-2X}T^3+\frac{A^2}{8}e^{-X}T^4$$

$$p^3: S_3=\mathrm{S}^{-1}[u\mathrm{S}[S_0(S_2)_{XX}+S_2(S_0)_{XX}+((S_1)_X)^2+2(S_0)_X(S_2)_X]]$$

$$=\frac{176}{3}e^{-4X}T^3+36e^{-3X}T^4+5A^2e^{-2X}T^5+\frac{A^2}{48}e^{-X}T^6,... \quad (23)$$

Thus the approximate solution is given by

$$S(X,T)=e^{-X}+AT+2e^{-2X}T+\frac{A}{2}e^{-X}T^2+9e^{-3X}T^2+6Ae^{-2X}T^3++\frac{A^2}{8}e^{-X}T^4+\frac{176}{3}e^{-4X}T^3+36e^{-3X}T^4+5A^2e^{-2X}T^5+\frac{A^2}{48}e^{-X}T^6+\dots \tag{24}$$

Hence the required approximate solution for the saturation of injected water is given by

$$S_w(X,T)=1-S(X,T)$$

$$=1-(e^{-X}+AT+2e^{-2X}T+\frac{A}{2}e^{-X}T^2+9e^{-3X}T^2+6Ae^{-2X}T^3+\frac{A^2}{8}e^{-X}T^4+\frac{176}{3}e^{-4X}T^3+36e^{-3X}T^4+5A^2e^{-2X}T^5+\frac{A^2}{48}e^{-X}T^6+\dots) \tag{25}$$

Numerical and Graphical Solution:

The numerical as well as the graphical solution (25) of saturation of injected water have been discussed using MATLAB. Figure 3 represents the graph of saturation of water (S_w) versus distance (X) for fixed time T = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 and Figure 4 represents the graph of saturation of water (S_w) versus time (T) for fixed distance X=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. Table 1 indicates the numerical values.

Table 1. Saturation of Water $S_w(X,T)$ for Different Distance X and for Fixed Time T= 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10.

T	**Saturation of Water: $S_w(X,T)$**				
	X=0.1	***X=0.2***	***X=0.3***	***X=0.4***	***X=0.5***
0.01	0.076966	0.166228	0.246707	0.319296	0.384789
0.02	0.057187	0.150028	0.233385	0.308290	0.375652
0.03	0.035584	0.132508	0.219105	0.296588	0.366007
0.04	0.011917	0.113507	0.203758	0.284117	0.355806
0.05	0.081406	0.092861	0.187235	0.270802	0.344998
0.06	0.064258	0.070407	0.169427	0.256569	0.333534
0.07	0.033739	0.045982	0.150222	0.241345	0.321362
0.08	0.010824	0.019420	0.129509	0.225053	0.308434
0.09	0.008146	0.009441	0.107179	0.207620	0.294697
0.10	0.007187	0.004077	0.083119	0.188970	0.280101

T	**Saturation of Water: $S_w(X,T)$**				
	X=0.6	***X=0.7***	***X=0.8***	***X=0.9***	***X=1***
0.01	0.443897	0.497256	0.545436	0.588950	0.628256
0.02	0.436269	0.490849	0.540018	0.584334	0.624290
0.03	0.428271	0.484171	0.534401	0.579571	0.620217
0.04	0.419869	0.477199	0.528570	0.574652	0.616029
0.05	0.411029	0.469911	0.522510	0.569566	0.611719
0.06	0.401718	0.462284	0.516204	0.564302	0.607280
0.07	0.391902	0.454294	0.509638	0.558850	0.602704
0.08	0.381546	0.445919	0.502794	0.553199	0.597984
0.09	0.370617	0.437134	0.495659	0.547338	0.593113
0.10	0.359079	0.427918	0.488215	0.541256	0.588084

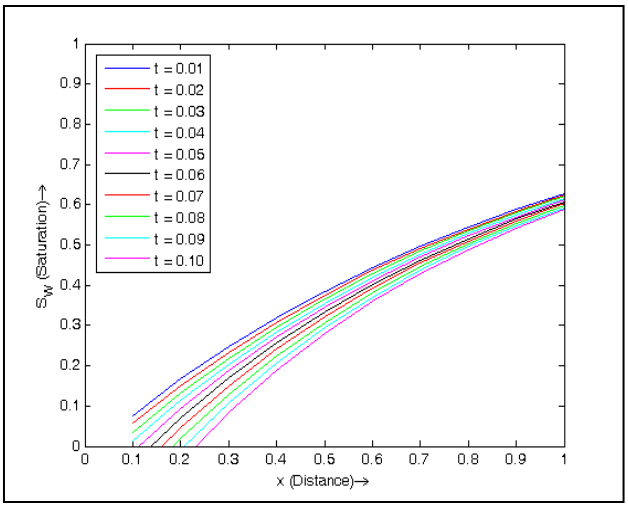

Figure 3. Saturation of injected water $S_w(X,T)$ versus distance X for fixed time T

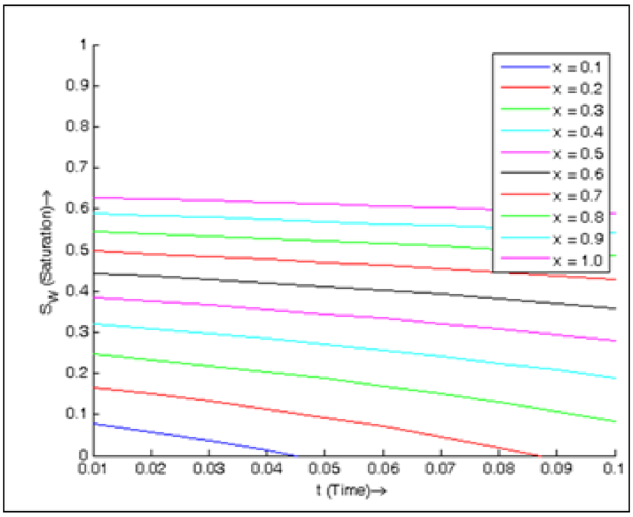

Figure 4. Saturation of injected water $S_w(X,T)$ versus time T for fixed distance X

Conclusion

The solution (25) shows the saturation of injected water during the secondary oil recovery process for any distance, *X* and for any time, $T > 0$. The graph shows that the saturation of injected water is increasing with distance *X* at each fixed time, *T* which is feasible with the physical phenomenon, i.e. more oil can be produced during oil recovery process by this HPSTM. Also, it shows that initially the saturation of water is increasing scatteredly, but at the distance *X = 0.95*, they increase simultaneously.

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