Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 107C (2017) 47332-47338

ISSN: 2229-712X

Conformal Lagrangian Dynamics on Contact 9- Manifolds

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ARTICLE INFO
Article history:
Received: 15 May 2017;
Received in revised form: 12 June 2017;
Accepted: 24 June 2017;

Keywords
Differential geometry,
Contact 9- Manifolds,
Lagrangian Dynamics.

ABSTRACT
In this study, we concluded the Conformal Lagrangian Dynamics on $(TM, \xi, J_{)}$, being a model. Finally introduce, some geometrical and physical results on the related mechanic systems have been discussed.

1. Introduction

Differential geometry is a rich and beautiful field in pure mathematics whose origins lie in classical physics. Specifically, Contact spaces arose as the natural setting in which to study Lagrangian mechanics . A Contact structure is precisely what is needed to associate a dynamical system on the space to each energy function.

It is the most important studies on the subject of paper is a study entitled (Lagrangian Mechanics on Contact 5- Manifolds)

In the study entitled conformal Euler-Lagrange mechanical on contact 5- Manifolds ,presents Lagrangian Mechanics on contact 5- manifolds. In the end, the some results related to contact 5- manifolds. dynamical systems are also discussed

The paper is structured as follows. In second 2, we review contact 5- manifolds. In second 3 we introduce Lagrangian equations for dynamical systems on contact 5- manifold .In conclusion, we discuss some geometric-physical results about Lagrangian equations and fields constructed on the base manifolds

2. Contact 9- Manifolds

Definition 2-1 [2]

Let M be a manifold of odd dimension (2n+1) and ξ ***field*** The pair (M, ξ) Is called a contact manifold

Theorem 2-2. A conformal manifold is a differentiable manifold equipped with an equivalence class of Riemann metric tensors, in which two$\mathbf{f_1}$ metrics$\mathbf{f_2}$ and are equivalent if and only if

$$f_2 = B^2 f_1 \tag{1}$$

where is a smooth positive function. $B > 0$

Theorem 2.3

.[1] A conformal transformation is a change of coordinates such that the metric changes by $\sigma^a \to \sigma^b$

$$f_{ab}(\sigma) \to B^2(\sigma) f_{ab}(\sigma) \tag{2}$$

Theorem 2.4

Let $\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}, \frac{\partial}{\partial x_7}\right)$ bases on manifold , J Conformal to the structure coefficient

$$J\left(\frac{\partial}{\partial x_1}\right) = B^2 \cos\theta \left(\frac{\partial}{\partial x_5}\right) + B^2 \sin\theta \left(\frac{\partial}{\partial x_6}\right) \qquad ,J\left(\frac{\partial}{\partial x_2}\right) = B^2 \cos\theta \left(\frac{\partial}{\partial x_6}\right) + B^2 \sin\theta \left(\frac{\partial}{\partial x_5}\right)$$

$$J\left(\frac{\partial}{\partial x_3}\right) = -B^2 \cos\theta \left(\frac{\partial}{\partial x_7}\right) + B^2 \sin\theta \left(\frac{\partial}{\partial x_8}\right), \qquad J\left(\frac{\partial}{\partial x_4}\right) = -B^2 \cos\theta \left(\frac{\partial}{\partial x_8}\right) + B^2 \sin\theta \left(\frac{\partial}{\partial x_7}\right)$$

$$J\left(\frac{\partial}{\partial x_5}\right) = -B^{-2} \cos\theta \left(\frac{\partial}{\partial x_1}\right) - B^{-2} \sin\theta \left(\frac{\partial}{\partial x_2}\right), \quad J\left(\frac{\partial}{\partial x_6}\right) = B^{-2} \cos\theta \left(\frac{\partial}{\partial x_2}\right) - B^{-2} \sin\theta \left(\frac{\partial}{\partial x_1}\right)$$

$$J\left(\frac{\partial}{\partial x_7}\right) = -B^{-2} \cos\theta \left(\frac{\partial}{\partial x_3}\right) - B^{-2} \sin\theta \left(\frac{\partial}{\partial x_4}\right), \quad J\left(\frac{\partial}{\partial x_8}\right) = B^{-2} \cos\theta \left(\frac{\partial}{\partial x_4}\right) - B^{-2} \sin\theta \left(\frac{\partial}{\partial x_3}\right)$$

3. Lagrangian Dynamics

Definition 3.1 [1].

A Lagrangian function for a Hamiltonian vector field ξ on M is a smooth function $L : TM \to R$ such that

$$i_\xi \omega_L = dE_L \tag{3}$$

Kinetic energy given $T: M \to R$ such that

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$$T=\frac{1}{2}m_i\left(\dot{x}_1^2+\dot{x}_2^2+\dot{x}_3^2+\dot{x}_4^2+\dot{x}_5^2+\dot{x}_6^2+\dot{x}_7^2+\dot{x}_8^2\right) \quad (4)$$

Potential energy $P: M\to R$ such that

$$P=m_i gh \quad (5)$$

$m_i = mass$ and $h = stand$, $g = gravity\ acceleration$

The Lagrangian function $\mathbf{L}:\mathbf{R}^{10n}\to\mathbf{R}$ is map that satisfies the condition then

$$\mathbf{L}=\mathbf{T}-\mathbf{P} \quad (6)$$

Definition 3.2 [2].

A Lagrangian system is a triple $(M;\xi)$, where $(\omega;L)$ is a Symplectic manifold and $L\in C^\infty(M)$ is a function, called the Hamiltonian function.

Theorem 3.3

Let M be m-real dimensional configuration manifold .A tensor field J on TM is called an almost complex structure on TM if at every point p of TM , J is endomorphism of the tangent space $T_p(M)$ such that $J^2=-1$ are complex is

$$J^2\left(\frac{\partial}{\partial x_i}\right)=-\frac{\partial}{\partial x_i} \qquad i=1,2,3,4,5,6,7,8 \quad (7)$$

J is called almost complex manifold

Proposition 3.4

Let λ be the vector field characterized by

$$\lambda=\sum_{i=1}^{8}\left(X^i\frac{\partial}{\partial x_i}\right) \qquad X^i=\dot{x}^i \quad (8)$$

$(\mathrm{T}M,g,J)$.then vector field defined by

$$J(\lambda)=J\left(\sum_{i=1}^{8}\left(X^i\frac{\partial}{\partial x_i}\right)\right)=\sum_{i=1}^{8}X^iJ\left(\frac{\partial}{\partial x_i}\right)$$

$$J(\lambda)=X^1J\left(\frac{\partial}{\partial x_1}\right)+X^2J\left(\frac{\partial}{\partial x_2}\right)+X^3J\left(\frac{\partial}{\partial x_3}\right)+X^4J\left(\frac{\partial}{\partial x_4}\right)+X^5J\left(\frac{\partial}{\partial x_5}\right)+X^6J\left(\frac{\partial}{\partial x_6}\right)+X^7J\left(\frac{\partial}{\partial x_7}\right)+X^8J\left(\frac{\partial}{\partial x_8}\right)$$

$$J(\lambda)= X^1J\left(\mathrm{B}^2\cos\theta\left(\frac{\partial}{\partial x_5}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial}{\partial x_6}\right)\right)+X^2J\left(\mathrm{B}^2\cos\theta\left(\frac{\partial}{\partial x_6}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial}{\partial x_5}\right)\right)+X^3J\left(-\mathrm{B}^2\cos\theta\left(\frac{\partial}{\partial x_7}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial}{\partial x_8}\right)\right)+$$
$$X^4J\left(-\mathrm{B}^2\cos\theta\left(\frac{\partial}{\partial x_8}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial}{\partial x_7}\right)\right)+X^5J\left(-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial}{\partial x_1}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial}{\partial x_2}\right)\right)+$$
$$X^6J\left(\mathrm{B}^{-2}\cos\theta\left(\frac{\partial}{\partial x_2}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial}{\partial x_1}\right)\right)+X^7J\left(-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial}{\partial x_3}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial}{\partial x_4}\right)\right)+$$
$$X^8J\left(\mathrm{B}^{-2}\cos\theta\left(\frac{\partial}{\partial x_4}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial}{\partial x_3}\right)\right)$$

Liouville vector field on contact 9-manifold $(\mathrm{T}M,g,J)$.then vector field defined by energy function given by

$$E_L^{G_1}=U_{G_1}(L)-L$$

is vertical derivation (differentiation) $\mathbf{d}_{\mathbf{G_i}}$

is defined

$$d_{G_1}=[i_{G_1},d]=i_{G_1}d-di_1$$

$$d=\sum_{i=1}^{8}\left(\frac{\partial}{\partial x_i}\,dx_i\right)=\frac{\partial}{\partial x_1}\,dx_1+\frac{\partial}{\partial x_2}\,dx_2+\frac{\partial}{\partial x_3}\,dx_3+\frac{\partial}{\partial x_4}\,dx_4+\frac{\partial}{\partial x_5}\,dx_5+\frac{\partial}{\partial x_6}\,dx_6+\frac{\partial}{\partial x_7}\,dx_7+\frac{\partial}{\partial x_8}\,dx_8$$

The $d_J: F(M)\to\wedge^1 M$

$$d_J=J(d)=\sum_{i=1}^{8}J\left(\frac{\partial}{\partial x_i}\right)dx_i$$
$$=J\left(\frac{\partial}{\partial x_1}\right)dx_1+\mathrm{J}\left(\frac{\partial}{\partial x_2}\right)dx_2+\mathrm{J}\left(\frac{\partial}{\partial x_3}\right)dx_3+\mathrm{J}\left(\frac{\partial}{\partial x_4}\right)dx_4+\mathrm{J}\left(\frac{\partial}{\partial x_5}\right)dx_5+\mathrm{J}\left(\frac{\partial}{\partial x_6}\right)dx_6+J\left(\frac{\partial}{\partial x_7}\right)dx_7$$
$$+\mathrm{J}\left(\frac{\partial}{\partial x_8}\right)dx_8$$

$$d_JL=$$
$$\left(\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_5}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_6}\right)\right)dx_1+\left(\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_6}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_5}\right)\right)dx_2+\left(-\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_7}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_8}\right)\right)dx_3+$$
$$\left(-\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_8}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_7}\right)\right)d4+\left(-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial L}{\partial x_1}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial L}{\partial x_2}\right)\right)dx_5+$$
$$\left(\mathrm{B}^{-2}\cos\theta\left(\frac{\partial L}{\partial x_2}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial L}{\partial x_1}\right)\right)dx_6+\left(-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial L}{\partial x_3}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial L}{\partial x_4}\right)\right)dx_7+$$
$$\left(\mathrm{B}^{-2}\cos\theta\left(\frac{\partial L}{\partial x_4}\right)-\mathrm{B}^{-2}\sin\theta\left(\frac{\partial L}{\partial x_3}\right)\right)dx_8$$

Let $\emptyset_L=-\mathbf{d}(\mathbf{Jd})=-\mathbf{d}\left(d_J\mathrm{L}\right)=-\mathbf{d}\Bigg(\left(\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_5}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_6}\right)\right)dx_1+\left(\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_6}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_5}\right)\right)dx_2+$

$$\left(-\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_7}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_8}\right)\right)\mathrm{d}x_3+\left(-\mathrm{B}^2\cos\theta\left(\frac{\partial L}{\partial x_8}\right)+\mathrm{B}^2\sin\theta\left(\frac{\partial L}{\partial x_7}\right)\right)d4+$$

$\left(-B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_1}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_2}\right)\right)dx_5+\left(B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_2}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_1}\right)\right)dx_6+$
$\left(-B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_3}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_4}\right)\right)dx_7+\left(B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_4}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_3}\right)\right)dx_8\Big)$

$$d=\sum_{i=1}^{8}\left(\frac{\partial}{\partial x_i}\,dx_i\right)=\frac{\partial}{\partial x_1}\,dx_1+\frac{\partial}{\partial x_2}\,dx_2+\frac{\partial}{\partial x_3}\,dx_3+\frac{\partial}{\partial x_4}\,dx_4+\frac{\partial}{\partial x_5}\,dx_5+\frac{\partial}{\partial x_6}\,dx_6+\frac{\partial}{\partial x_7}\,dx_7+\frac{\partial}{\partial x_8}\,dx_8$$

The $d_J: F(M)\to\Lambda^1 M$

$$d_J=J(d)=\sum_{i=1}^{8}J\left(\frac{\partial}{\partial x_i}\right)dx_i$$
$$=J\left(\frac{\partial}{\partial x_1}\right)dx_1+J\left(\frac{\partial}{\partial x_2}\right)dx_2+J\left(\frac{\partial}{\partial x_3}\right)dx_3+J\left(\frac{\partial}{\partial x_4}\right)dx_4+J\left(\frac{\partial}{\partial x_5}\right)dx_5+J\left(\frac{\partial}{\partial x_6}\right)dx_6+J\left(\frac{\partial}{\partial x_7}\right)dx_7+J\left(\frac{\partial}{\partial x_8}\right)dx_8$$

$$d_JL=$$
$\left(B^{2}\cos\theta\left(\frac{\partial L}{\partial x_5}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_6}\right)\right)dx_1+\left(B^{2}\cos\theta\left(\frac{\partial L}{\partial x_6}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_5}\right)\right)dx_2+\left(-B^{2}\cos\theta\left(\frac{\partial L}{\partial x_7}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_8}\right)\right)dx_3+$
$\left(-B^{2}\cos\theta\left(\frac{\partial L}{\partial x_8}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_7}\right)\right)d4+\left(-B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_1}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_2}\right)\right)dx_5+$
$\left(B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_2}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_1}\right)\right)dx_6+\left(-B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_3}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_4}\right)\right)dx_7+$
$\left(B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_4}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_3}\right)\right)dx_8$

Let $\emptyset_L=-d(Jd)=-d\left(d_JL\right)=-d\Big(\left(B^{2}\cos\theta\left(\frac{\partial L}{\partial x_5}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_6}\right)\right)dx_1+\left(B^{2}\cos\theta\left(\frac{\partial L}{\partial x_6}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_5}\right)\right)dx_2+$
$\left(-B^{2}\cos\theta\left(\frac{\partial L}{\partial x_7}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_8}\right)\right)dx_3+\left(-B^{2}\cos\theta\left(\frac{\partial L}{\partial x_8}\right)+B^{2}\sin\theta\left(\frac{\partial L}{\partial x_7}\right)\right)d4+$
$\left(-B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_1}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_2}\right)\right)dx_5+\left(B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_2}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_1}\right)\right)dx_6+$
$\left(-B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_3}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_4}\right)\right)dx_7+\left(B^{-2}\cos\theta\left(\frac{\partial L}{\partial x_4}\right)-B^{-2}\sin\theta\left(\frac{\partial L}{\partial x_3}\right)\right)dx_8\Big)$

$$\emptyset_L=-d(Jd)=-d\left(d_JL\right)$$
$$=\sum_{i=1}^{8}\left(B^{2}\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\right)dx_1\wedge dx_i$$
$$+\sum_{i=1}^{8}\left(-B^{2}\cos\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)+B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)\right)dx_2\wedge dx_i$$
$$+\sum_{i=1}^{8}\left(B^{2}\cos\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)+B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)\right)dx_3\wedge dx_i$$
$$+\sum_{i=1}^{8}\left(-B^{2}\cos\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)+B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)\right)dx_4\wedge dx_i$$
$$+\sum_{i=1}^{8}\left(-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)+2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)-B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)+2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)\right)dx_5\wedge dx_i$$
$$+\sum_{i=1}^{8}\left(B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)+B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)\right)dx_6\wedge dx_i$$
$$+\sum_{i=1}^{8}\left(-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)-B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)\right)dx_7\wedge dx_i$$
$$+\sum_{i=1}^{8}\left(B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)-B^{2}\sin\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)\right)dx_8\wedge dx_i$$

Let $i_\lambda(\emptyset_L)=\emptyset_L(\lambda)=\emptyset_L\left(\sum_{i=1}^{8}X^i\frac{\partial}{\partial x_i}\right)$

$$\sum_{i=1}^{8} X^{i}\Bigg[\left(B^{2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{5}\partial x_{i}}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{5}}\right)+B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{6}\partial x_{i}}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{6}}\right)\right)dx_{1}$$
$$+\left(-B^{2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{6}\partial x_{i}}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{6}}\right)+B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{5}\partial x_{i}}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{5}}\right)\right)dx_{2}$$
$$+\left(B^{2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{7}\partial x_{i}}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{7}}\right)+B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{8}\partial x_{i}}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{8}}\right)\right)dx_{3}$$
$$+\left(-B^{2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{8}\partial x_{i}}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{8}}\right)+B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{7}\partial x_{i}}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{7}}\right)\right)dx_{4}\left(-B^{-2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{1}\partial x_{i}}\right)\right.$$
$$\left.+2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{1}}\right)-B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{2}\partial x_{i}}\right)+2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{2}}\right)\right)dx_{5}$$
$$+\left(B^{-2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{2}\partial x_{i}}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{2}}\right)+B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{1}\partial x_{i}}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{1}}\right)\right)dx_{6}$$
$$+\left(-B^{-2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{3}\partial x_{i}}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{3}}\right)-B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{4}\partial x_{i}}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{4}}\right)\right)dx_{7}$$
$$+\left(B^{-2}\cos\theta\left(\frac{\partial^{2}L}{\partial x_{4}\partial x_{i}}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{4}}\right)-B^{2}\sin\theta\left(\frac{\partial^{2}L}{\partial x_{3}\partial x_{i}}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_{i}}\right)\left(\frac{\partial}{\partial x_{3}}\right)\right)dx_{8}\Bigg]$$

The energy function $\boldsymbol{E_L = J(\lambda) - L}$

$$\boldsymbol{E_L = X^1\left(B^2\cos\theta\left(\frac{\partial}{\partial x_5}\right)+B^2\sin\theta\left(\frac{\partial}{\partial x_6}\right)\right)+X^2\left(B^2\cos\theta\left(\frac{\partial}{\partial x_6}\right)+B^2\sin\theta\left(\frac{\partial}{\partial x_5}\right)\right)}$$
$$\boldsymbol{+X^3\left(-B^2\cos\theta\left(\frac{\partial}{\partial x_7}\right)+B^2\sin\theta\left(\frac{\partial}{\partial x_8}\right)\right)+X^4\left(-B^2\cos\theta\left(\frac{\partial}{\partial x_8}\right)+B^2\sin\theta\left(\frac{\partial}{\partial x_7}\right)\right)}$$
$$\boldsymbol{+X^5\left(-B^{-2}\cos\theta\left(\frac{\partial}{\partial x_1}\right)-B^{-2}\sin\theta\left(\frac{\partial}{\partial x_2}\right)\right)+X^6\left(B^{-2}\cos\theta\left(\frac{\partial}{\partial x_2}\right)-B^{-2}\sin\theta\left(\frac{\partial}{\partial x_1}\right)\right)}$$
$$\boldsymbol{+X^7\left(-B^{-2}\cos\theta\left(\frac{\partial}{\partial x_3}\right)-B^{-2}\sin\theta\left(\frac{\partial}{\partial x_4}\right)\right)+X^8\left(B^{-2}\cos\theta\left(\frac{\partial}{\partial x_4}\right)-B^{-2}\sin\theta\left(\frac{\partial}{\partial x_3}\right)\right)-L}$$

differential energy function

$$\boldsymbol{dE_L = \sum_{i=1}^{8}\Bigg[X^1\left[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\right]}$$
$$\boldsymbol{+X^2\left[-B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)\right]}$$
$$\boldsymbol{+X^3\left[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)\right]}$$
$$\boldsymbol{+X^4\left[-B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)\right]}$$
$$\boldsymbol{+X^5\left[-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)+2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)+2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)\right]}$$
$$\boldsymbol{+X^6\left[B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)\right]}$$
$$\boldsymbol{+X^7\left[-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)\right]}$$
$$\boldsymbol{+X^8\left[B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)\right]\Bigg]-\frac{\partial L}{\partial x_3}\,dx_i}$$

Let= $\boldsymbol{i_\lambda(\emptyset_L)}$t $\boldsymbol{dE_L}$

$$= \sum_{i=1}^{8} X^i \Bigg[\Bigg(\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\Bigg) dx_1$$
$$+ \Bigg(-\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_6 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_5 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)\Bigg) dx_2$$
$$+ \Bigg(\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_7 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_8 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)\Bigg) dx_3$$
$$+ \Bigg(-\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_8 \partial x_i}\right)$$
$$+ 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_7 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)\Bigg) dx_4 \Bigg(-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_1 \partial x_i}\right)$$
$$+ 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right) - \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_2 \partial x_i}\right) + 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)\Bigg) dx_5$$
$$+ \Bigg(\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_2 \partial x_i}\right) - 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_1 \partial x_i}\right) - 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)\Bigg) dx_6$$
$$+ \Bigg(-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_3 \partial x_i}\right) - 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right) - \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_4 \partial x_i}\right) - 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)\Bigg) dx_{67}$$
$$+ \Bigg(\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_4 \partial x_i}\right) - 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right) - \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_3 \partial x_i}\right) - 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)\Bigg) dx_8\Bigg]$$

$$= \sum_{i=1}^{8} \Bigg[X^1\left[\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\right]$$
$$+ X^2\left[-\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_6 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_5 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)\right]$$
$$+ X^3\left[\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_7 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_8 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)\right]$$
$$+ X^4\left[-\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_8 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_7 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)\right]$$
$$+ X^5\left[-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_1 \partial x_i}\right) + 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right) - \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_2 \partial x_i}\right) + 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)\right]$$
$$+ X^6\left[\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_2 \partial x_i}\right) - 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_1 \partial x_i}\right) - 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)\right]$$
$$+ X^7\left[-\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_3 \partial x_i}\right) - 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right) - \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_4 \partial x_i}\right) - 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)\right]$$
$$+ X^8\left[\mathrm{B}^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_4 \partial x_i}\right) - 2\mathrm{B}^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right) - \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_3 \partial x_i}\right) - 2\mathrm{B}^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)\right]\Bigg]$$
$$- \frac{\partial L}{\partial x_i}\,\mathrm{d}x_i$$

$$X^1\left[\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\right]$$
$$= X^1\left[\mathrm{B}^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5 \partial x_i}\right) + 2\mathrm{B}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right) + \mathrm{B}^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6 \partial x_i}\right) + 2\mathrm{B}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\right] - \frac{\partial L}{\partial x_i}\,\mathrm{d}x_i$$

differential energy function

$$dE_L=\sum_{i=1}^{8}\Big[X^1\Big[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\Big]$$
$$+X^2\Big[-B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)\Big]$$
$$+X^3\Big[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)\Big]$$
$$+X^4\Big[-B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)\Big]$$
$$+X^5\Big[-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)+2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)+2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)\Big]$$
$$+X^6\Big[B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)\Big]$$
$$+X^7\Big[-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)\Big]$$
$$+X^8\Big[B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)\Big]\Big]$$
$$-\frac{\partial L}{\partial x_3}\,dx_i$$

$$=\sum_{i=1}^{8}\Big[X^1\Big[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\Big]$$
$$+X^2\Big[-B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)\Big]$$
$$+X^3\Big[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)\Big]$$
$$+X^4\Big[-B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_8\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_8}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_7\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_7}\right)\Big]$$
$$+X^5\Big[-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)+2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)+2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)\Big]$$
$$+X^6\Big[B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_2\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_2}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_1\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_1}\right)\Big]$$
$$+X^7\Big[-B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)\Big]$$
$$+X^8\Big[B^{-2}\cos\theta\left(\frac{\partial^2 L}{\partial x_4\partial x_i}\right)-2B^{-3}\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_4}\right)-B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_3\partial x_i}\right)-2B^{-3}\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_3}\right)\Big]\Big]-\frac{\partial L}{\partial x_i}\,dx_i$$

$$X^1\Big[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\Big]$$
$$=X^1\Big[B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)\Big]-\frac{\partial L}{\partial x_i}\,dx_i$$

Or

$$-X^1B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2X^1B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+X^1B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2X^1B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)$$
$$-X^1B^2\cos\theta\left(\frac{\partial^2 L}{\partial x_5\partial x_i}\right)+2X^1B\cos\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_5}\right)+X^1B^2\sin\theta\left(\frac{\partial^2 L}{\partial x_6\partial x_i}\right)+2X^1B\sin\theta\left(\frac{\partial B}{\partial x_i}\right)\left(\frac{\partial}{\partial x_6}\right)+\frac{\partial L}{\partial x_1}\,dx_1=0$$

$$\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_5}$$
$$-\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_6}+\frac{\partial L}{\partial x_1}\,dx_1=0$$

The integral curve of the vector filed G ,let $\rho: R\to R^2$ the curve

$$\gamma(\alpha)=\frac{\partial\alpha}{\partial t}=X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}$$
$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_5}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_6}\right)+\left(\frac{\partial L}{\partial x_1}\right)=0$$

$$\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_6}$$
$$-\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_5}+\frac{\partial L}{\partial x_2}\,dx_2=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_6}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_5}\right)+\left(\frac{\partial L}{\partial x_2}\right)=0$$

$$-\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_7}$$

$$+\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_8}+\frac{\partial L}{\partial x_3}\,dx_3=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_7}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_8}\right)+\left(\frac{\partial L}{\partial x_3}\right)=0$$

$$\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_8}$$

$$+\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_7}+\frac{\partial L}{\partial x_4}\,dx_4=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_8}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_7}\right)+\left(\frac{\partial L}{\partial x_4}\right)=0$$

$$-\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_1}$$

$$-\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_2}+\frac{\partial L}{\partial x_5}\,dx_5=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_1}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_2}\right)+\left(\frac{\partial L}{\partial x_5}\right)=0$$

$$\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_2}$$

$$-\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_1}+\frac{\partial L}{\partial x_6}\,dx_6=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_2}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_1}\right)+\left(\frac{\partial L}{\partial x_6}\right)=0$$

$$-\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_3}$$

$$-\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_4}+\frac{\partial L}{\partial x_7}\,dx_7=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_3}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_4}\right)+\left(\frac{\partial L}{\partial x_7}\right)=0$$

$$\cos\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_4}$$

$$-\sin\theta\left[X^1\frac{\partial}{\partial x_i}+X^2\frac{\partial}{\partial x_i}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}\right]B^2\frac{\partial L}{\partial x_3}+\frac{\partial L}{\partial x_8}\,dx_8=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_4}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_3}\right)+\left(\frac{\partial L}{\partial x_8}\right)=0$$

Thus

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_5}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_6}\right)+\left(\frac{\partial L}{\partial x_1}\right)=0,\qquad \cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_6}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_5}\right)+\left(\frac{\partial L}{\partial x_2}\right)=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_7}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_8}\right)+\left(\frac{\partial L}{\partial x_3}\right)=0,\qquad \cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_8}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^2\frac{\partial L}{\partial x_7}\right)+\left(\frac{\partial L}{\partial x_4}\right)=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_1}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_2}\right)+\left(\frac{\partial L}{\partial x_5}\right)=0,\quad \cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_2}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_1}\right)+\left(\frac{\partial L}{\partial x_6}\right)=0$$

$$\cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_3}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_4}\right)+\left(\frac{\partial L}{\partial x_7}\right)=0,\quad \cos\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_4}\right)+\sin\theta\left(\frac{\partial}{\partial t}\right)\left(B^{-2}\frac{\partial L}{\partial x_3}\right)+\left(\frac{\partial L}{\partial x_8}\right)=0 \quad (9)$$

4. Conclusions

The solutions of the Lagrangian equations (9) are named Conformal Lagrangian Dynamics on Contact 9- Manifolds determined by on the mechanical system triple $(TM, \mathbf{g}, \mathbf{L})$are the paths of vector field **J** on **M**.

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