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On pair resolving sets in graphs

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ABSTRACT
The concept of pair resolving sets in graphs are defined as a generalization of resolving sets. Some properties of pair resolving sets are discussed.

1. Introduction

The idea of resolving sets and minimum resolving sets has appeared in the literature previously in 1976 and 1979. In [5] and later in [6], Slater introduced the concept of a resolving set for a connected graph G under the term “locating set”. He referred to a minimum resolving set as a reference set for G. He called the cardinality of a minimum resolving set (reference set) the “Location number” of G.

Slater described the usefulness of these ideas when working with sonar and loran stations. Independently, Harary and Melter [3] discovered these concepts as well but used the term metric dimension, rather than location number. We adopt the terminology of Harary and Melter. Consequently, the metric dimension or, more simply, the dimension dim (G) of a connected graph G is the cardinality of a minimum resolving set. Because of the suggestiveness of this terminology to linear algebra, we also refer to a minimum resolving set as a basis for G. Hence, the vertices of G have distinct representations with respect to the basis vertices.

We denote the standard distance between two vertices u and v in a connected graph G by $d(u,v)$. By an ordered set of vertices, we mean a set $\boldsymbol{W} = \{\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_k}\}$ on which the ordering $(\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_k})$ has been imposed. For an ordered subset $\boldsymbol{W} = \{\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_k}\}$ of V(G), we refer to the k-vector (ordered k-tuple). r(v /$\boldsymbol{W}$)= $(d(v,w_1), d(v,w_2), \dots\dots d(v,w_k))$ as the (metric) representation of v with respect to $\boldsymbol{W}$. The set W is called a resolving set for G if r(u /$\boldsymbol{W}$) = r(v /$\boldsymbol{W}$) implies that $u = v$ for all $u, v \in V(G)$.

Hence if $\boldsymbol{W}$ is a resolving set of cardinality k for a graph G of order n, then the set {r(v /$\boldsymbol{W}$)/ $v \in V(G)$ } consists of n distinct k-vectors. A resolving set of minimum cardinality for a graph G is called a minimum resolving set or basis for G.

2. Pair Resolving Sets

Introduction

We define pair resolving sets as a generalization of resolving sets and discuss some properties of pair resolving sets.

Definition 2.1

Let an ordered set of vertices $W = \{w_1, w_2, \dots, w_k\}$ on which the ordering $(w_1, w_2, \dots, w_k)$ has been imposed. For an ordered subset $W = \{w_1, w_2, \dots, w_k\}$ of V(G), we refer to the k-vector (ordered k-tuple). r(v /W) = $(d(v,w_1), d(v,w_2), \dots, d(v,w_k))$ as the representation of v with respect to W. The set W is called a pair resolving set for G if $u \in V(G)$ then r(u/ W) = r(v /W) for at most one v such that $v \in V(G)$.

Remark 2.2

If W is a pair resolving set of cardinality k for a graph G of order n, than the set

{r(v /W) /$v \in V(G)$} consists of $(\dfrac{n+k}{2})$ distinct k-vectors.

Definition 2.3

A pair resolving set of minimum cardinality for a graph G is called a minimum pair resolving set.

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Example 2.4

Consider the following graph G

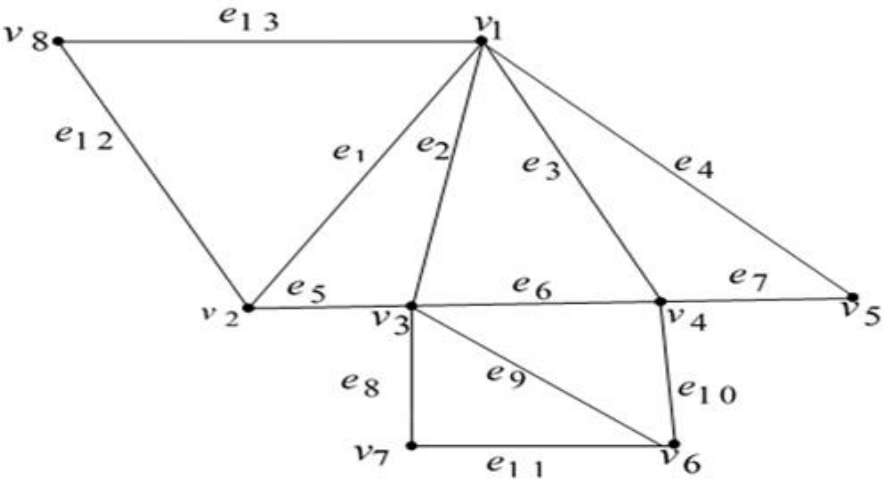

Fig.1

The set $W = \{v_1, v_2\}$ is a pair resolving set for G.Since,
r(v_1/ W) = (0,1), r(v_2/ W) = (1,0), r(v_3/ W) = (1,1), r(v_4/ W) = (1,2), r(v_5/ W) = (1,2),
r(v_6/ W) = (2,2), r(v_7/ W) = (2,2), r(v_8/ W) = (1,1).
r(v_3/ W)=(1,1)=r(v_8/ W), r(v_4/ W)=(1,2) = r(v_5/ W), r(v_6/ W) = (2,2)=r(v_7/ W).

Example 2.5

Consider the following graph

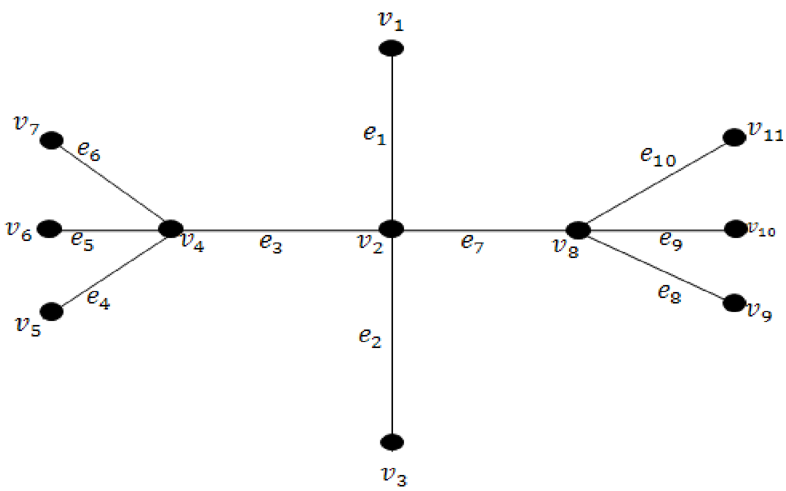

Fig.2

The set $W = \{v_2, v_4, v_{6,} v_8, v_{10}\}$ is a pair resolving set for *G*. Since,
r(v_1/ W) = (1,2,3,2,3), r(v_2/ W) = (0,1,2,1,2), r(v_3/ W) = (1,2,3,2,3), r(v_4/ W) = (1,0,1,2,3), r(v_5/ W) = (2,1,2,3,4), r(v_6/ W) = (2,1,0,3,4), r(v_7/ W) = (2,1,2,3,4), r(v_8/ W) = (1,2,3,0,1), r(v_9/ W) = (2,3,4,1,2), r(v_{10}/ W) = (2,3,4,1,0), r(v_{11}/ W) = (2,3,4,1,2).
r(v_1/ W) = (1,2,3,2,3) = r(v_3/ W), r(v_5/ W) = (2,1,2,3,4) = r(v_7/ W), r(v_9/ W) = (2,3,4,1,2) = r(v_{11}/ W)
Since, no single vertex constitutes a pair resolving set for this graph, it follow that W is a minimum pair resolving sets.

Theorem 2.6

If n is even then the cardinality of minimum pair resolving set is even.

Proof

Let G be a graph with *n* vertices and *n* is even. Let *k* be the cardinality of a pair resolving set W. Since *(n – k)* is even, *k* must also be even.

Example 2.7

Consider the following graph

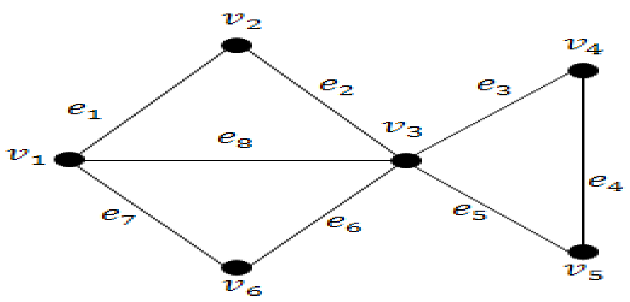

Fig. 3

Let $\boldsymbol{W} = \{\boldsymbol{v_1}, \boldsymbol{v_3}\}$, Now

r($\boldsymbol{v_1}/\boldsymbol{W}$) = (0,1), r($\boldsymbol{v_2}/\boldsymbol{W}$) = (1,1), r($\boldsymbol{v_3}/\boldsymbol{W}$) = (1,0), r($\boldsymbol{v_4}/\boldsymbol{W}$) = (2,1), r($\boldsymbol{v_5}/\boldsymbol{W}$) = (2,1), r($\boldsymbol{v_6}/\boldsymbol{W}$) = (1,1).

r($\boldsymbol{v_2}/\boldsymbol{W}$)=(1,1)=r($\boldsymbol{v_6}/\boldsymbol{W}$), r($\boldsymbol{v_4}/\boldsymbol{W}$)=(2,1)=r($\boldsymbol{v_5}/\boldsymbol{W}$).

Hence W is a pair resolving set

Theorem 2.8

If P_n is a path on odd vertices then it has single vertex as pair resolving set. Then the element of this set is the mid vertex.

Proof

Let P_n be a path of even length and has odd number of vertices. Let $\boldsymbol{v_1}, \boldsymbol{v_2}, \ldots, \boldsymbol{v_n}$ be the vertices of P_n. Consider the vertex $\boldsymbol{V}_{\left(\frac{n+1}{2}\right)}$, all other vertices can be grouped as

$$\left(\boldsymbol{v}_{\left(\frac{n+1}{2}\right)-1}, \boldsymbol{v}_{\left(\frac{n+1}{2}\right)+1}\right)\left(\boldsymbol{v}_{\left(\frac{n+1}{2}\right)-2}, \boldsymbol{v}_{\left(\frac{n+1}{2}\right)+2}\right), \ldots, (\boldsymbol{v_1}, \boldsymbol{v_n})$$

where each pairs are at distances 1,2,…($\dfrac{n-1}{2}$). Hence all paths of even length has pair resolving sets of cardinality one and the set contains the mid vertex ie, $\left(\dfrac{n+1}{2}\right)^{th}$ vertex of the path.

Example 2.9

Consider the following graph

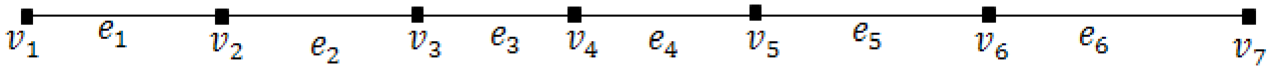

Fig. 4

The set $\boldsymbol{W} = \{\boldsymbol{v_4}\}$ is a pair resolving sets for G. Since,

r($\boldsymbol{v_1}/\boldsymbol{W}$) = 3, r($\boldsymbol{v_2}/\boldsymbol{W}$) = 2, r($\boldsymbol{v_3}/\boldsymbol{W}$) = 1, r ($\boldsymbol{v_4}/\boldsymbol{W}$) = 0, r ($\boldsymbol{v_5}/\boldsymbol{W}$) = 1, r($\boldsymbol{v_6}/\boldsymbol{W}$) = 2, r($\boldsymbol{v_7}/\boldsymbol{W}$) = 3.

r($\boldsymbol{v_1}/\boldsymbol{W}$) = 3 = r($\boldsymbol{v_7}/\boldsymbol{W}$), r($\boldsymbol{v_2}/\boldsymbol{W}$) = 2 = r($\boldsymbol{v_6}/\boldsymbol{W}$), r($\boldsymbol{v_3}/\boldsymbol{W}$) = 1 = r($\boldsymbol{v_5}/\boldsymbol{W}$).

Definition 2.10 S-components of graph

Let S be a vertex cut of a connected graph G, and let the components of G-S have vertex sets $\boldsymbol{V_1}, \boldsymbol{V_2}, \ldots, \boldsymbol{V_n}$. Then the sub graphs $\boldsymbol{G_i} = \boldsymbol{G}[\boldsymbol{V_i} \cup \boldsymbol{S}]$are called the S-components of G.

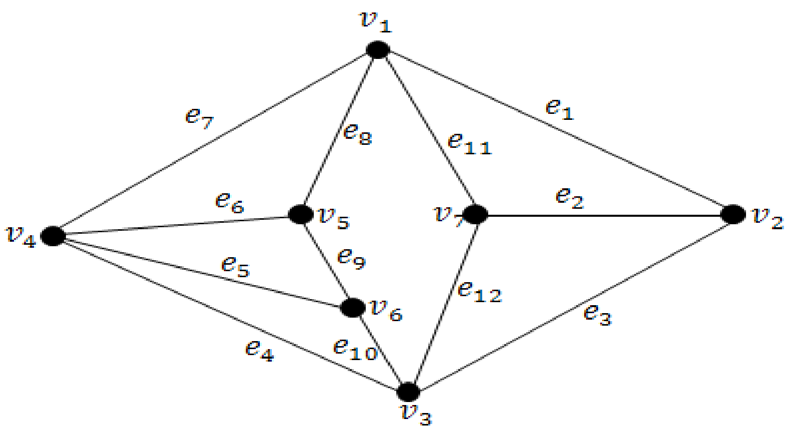

Fig 5a. G

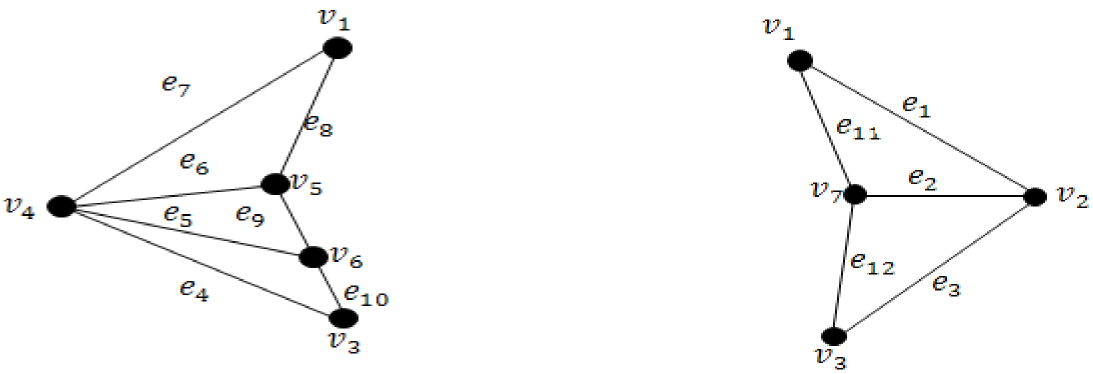

Fig 5b. The $\{v_1, v_3\} -$ Components of G.

Definition 2.11

Let G be a graph. If there is a set of vertices $\boldsymbol{S} = (\boldsymbol{u_1}, \boldsymbol{u_2}, \dots, \boldsymbol{u_n})$ such that the $\boldsymbol{S}$-components of G gives two isomorphic components of G then the graph is said to be symmetric about S.

Remark 2.12

Let G be a graph and let $\boldsymbol{S} = \{\boldsymbol{u_1}, \boldsymbol{u_2}, \dots, \boldsymbol{u_k}\}$ be a vertex cut such that G has only two $\boldsymbol{S}$ -components $\boldsymbol{G_1}$ ***and*** $\boldsymbol{G_2}$, then the vertices of S in $\boldsymbol{G_1}$ ***and*** $\boldsymbol{G_2}$ can be represented as $\boldsymbol{S_1} = \{\boldsymbol{u'_1}, \boldsymbol{u'_2}, \dots, \boldsymbol{u'_k}\}$ and $\boldsymbol{S_2} = \{\boldsymbol{u''_1}, \boldsymbol{u''_2}, \dots, \boldsymbol{u''_k}\}$ respectively.

Example 2.13

Consider the following symmetric graph

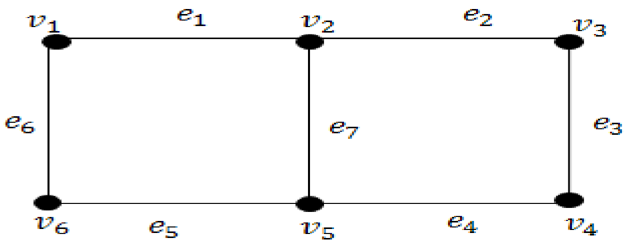

Fig 6.

If $\boldsymbol{S} = (\boldsymbol{v_2}, \boldsymbol{v_5})$ is a vertex cut

The S-components of G are isomorphism

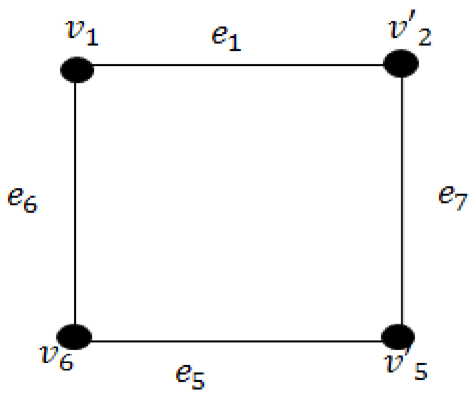

Fig 6a. $\boldsymbol{G_1}$

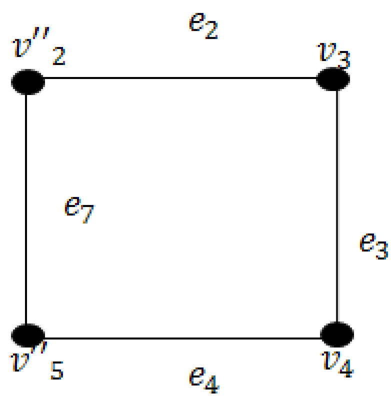

Fig 6b. $\boldsymbol{G_2}$

Theorem 2.14

Symmetric graphs have pair resolving set.

Proof

Let G be a symmetric graph and $\boldsymbol{S} = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \dots, \boldsymbol{v_k}\}$ be a vertex cut such that G is symmetric with respect to *S*.

Now the two S-components $\boldsymbol{G_1}$ ***and*** $\boldsymbol{G_2}$ of *G* are isomorphic with respect to *S*.

Since $\boldsymbol{G_1}$ ***and*** $\boldsymbol{G_2}$ are isomorphic with respect to *S*, if v is a vertex of $\boldsymbol{G_1}$ with the *k* vector $\boldsymbol{r}\left({}^{\boldsymbol{v}}/_{\boldsymbol{S}}\right) = \{\boldsymbol{d_1}, \boldsymbol{d_2}, \dots, \boldsymbol{d_k}\}$ then we can find a vertex *u* in $\boldsymbol{G_2}$ with the *k* vector $\boldsymbol{r}\left({}^{\boldsymbol{u}}/_{\boldsymbol{S}}\right) = \{\boldsymbol{d_1}, \boldsymbol{d_2}, \dots, \boldsymbol{d_k}\}$.

Therefore $\boldsymbol{S}$ is a pair resolving set for G.

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