

Performance Evaluation of Two Way Relaying with Hardware Impairments

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ABSTRACT

Practical transceivers have hardware impairments, which may affect the data rate of a system. In this paper, two way relaying with hardware losses is considered and is different from previous methods, which considers only ideal hardware case. Here both hops are subjected to independent and non-identical distributed nakagami- m fading variants with Amplify-and-Forward (AF) protocol and Decode-and-forward (DF) protocol. Hardware impairments are calculated at source, relay and destination. The outage probability is calculated with a function effective end-to-end signal-to-noise-and-distortion ratio (SNDR). In a similar way ergodic capacity also achieved. For low SNR values losses will be less but substantial. And for high SNR values losses will increase without bounds. But for SNDR case, increasing SNR makes some constant value of threshold, which is inversely proportional to the SNDR, called SNDR ceiling. Finally, this paper suggests the design guidelines for selecting hardware equipment to overcome hardware impairments with maximum extent.

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I. Introduction

In recent days, relay is a hot topic in telecommunication. Research on this topic is made importance by both industry [2] and academic institutions [3]. By using relay, many researchers concentrated on coverage and reliability with low cost hardware. Unlike macro base stations, relay is limited cost equipment and will provide good coverage. But till now, all researches conducted on relay are assumed to be ideal hardware. Means that all equipment used is distortion less. In practical point of view, physical transceivers have hardware impairments, which may affect system performance. All previous papers concentrated on these impairments as a single type of loss. Which are mainly phase loss, I/Q imbalance and non-linear HPA (High Power Amplifiers) [4]. Some papers concentrated only on I/Q imbalance, which make the signal to be attenuated and causes phase shift. It creates an image signal from mirror sub-carrier and causes symbol error rate to be increased. Another paper considers non-linear HPA as impairment in hardware. It results increase in bit error rate and reduces efficient data rate. All those noises are irreducible in nature. Even after applying noise compensation techniques, it has residual effects. Those effects are more severe in high SNR regime and less at low SNR case. First these impairments are considered for single hop systems with bit error simulations for Amplify and forward relaying [1] and derived expressions for them with non-linear HPAs or I/Q imbalance [5]. In this paper, the new methodology was introduced as general structure of system shown in figure. 1. For this system aggregate impairments are calculated for dual-hop relaying with AF and DF protocols.

A. Symbolic Notations

$CN(x,y)$ denotes circularly symmetric complex Gaussian distributed random variable with x is mean and $y>0$ is variance. Gamma (α, β) is gamma distributed function with $\alpha \geq 0$ is integer shape parameter and $\beta > 0$ is scaling parameter. $E\{\cdot\}$ is expectation operator.

II. General model and various calculations

Consider the generalized system [5] which is modelled below. First take single hop with source and relay only.

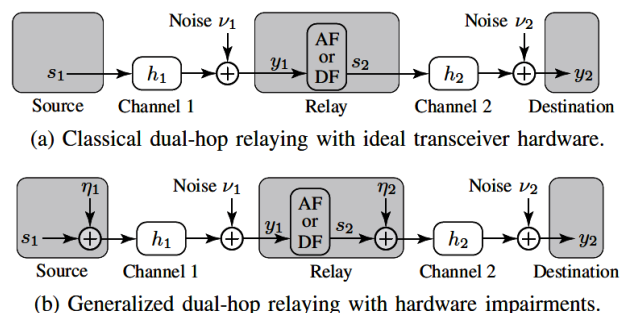


Fig. (1). Block diagram of dual hop relaying system with AF/DF relaying. For (a) ideal hardware and (b) non-ideal hardware with transceiver impairments modelled by aggregate distortion noises η_1, η_2, η_3 .

Consider 's' be the intended signal to be transmitted from source. Channel should be flat fading with impulse response of 'h' and 'v' is the additive noise. At relay the received signal should be 'y' and is conveyed as,

$$y = hs + v; \quad (1)$$

The above mentioned parameters s , h and v respectively are statistically independent.

And equation is for ideal hardware case. But due to impairments at hardware, several types of losses may occur [5]. a) Which creates mismatch between intended to transmit signal and originally transmitted signal. b) Adds noise at different levels of system and makes the signal to be distorted at relay receiver. So as reduces performance.

A. Calculations with hardware impairments

In practical cases impairments causes different types of distortions like phase noise, I/Q Imbalance and non-linear HPA. For the above purpose the ideal system is modified with inclusion of hardware impairments at source and relay. So the model changes like fig. 1(b). And equation can be rewrite as,

$$y = h(s + \eta_t) + \eta_r + v \quad (2)$$

Here η_t and η_r are distortion noises at transmitter and receiver respectively. Distortion noises [6] are defined as,

$$\eta_t \triangleq \text{CN}(0, k_t^2 P), \eta_r \triangleq \text{CN}(0, k_r^2 P |h|^2). \quad (3)$$

Here k_t and k_r are level of impairments with, $k_t, k_r \geq 0$. The joint gaussianity for both distortion noises are simply derived as

$$\mathbb{E}_{\eta_t, \eta_r} \{|h\eta_t + \eta_r|^2\} = P|h|^2(k_t^2 + k_r^2) \quad (4)$$

Here average power is defined as $P = \mathbb{E}\{|s|^2\}$ and also instantaneous channel gain is $|h|^2 = \rho$. The parameters $k_t, k_r \geq 0$ are used to calculate Error Vector Magnitude (EVM). EVM [7] is measured as a ratio of $k = \sqrt{k_t^2 + k_r^2}$. And hence overall distortion noise at relay and destination are defined as $\eta = \text{CN}(0, k^2 P)$

And modified general equation from (2) is

$$y = (s + \eta)h + v \quad (6)$$

From above equation, if the system is ideal model without any impairments. i.e., $\eta = 0$ ($k_t = k_r = 0$). This is reduced simple equation (1)

A. Two way relaying with Hardware Impairments

Consider the model, which is generalised for non-ideal hardware case, it is defined in equation (6). Which is applicable to both hops of relaying is given by

$$y_i = (s_i + \eta_i)h_i + v_i, \quad i=1, 2. \quad (7)$$

From above suffix with '1' indicates transmission between source and relay. Like, suffix with '2' between relay and destination. Recollect all parameters of equation with definitions, $P_i = \mathbb{E}\{|s_i|^2\}$; $\rho_i = |h_i|^2$; $\eta_i \triangleq \text{CN}(0, k_i^2 P_i)$; $v_i \triangleq \text{CN}(0, N_i)$; $i = 1, 2$.

From above ρ_i is the instantaneous channel gain and is gamma distributed. $\rho_i \sim \text{Gamma}(\alpha_i, \beta_i)$, $\alpha_i \geq 0$ is integer shape parameter and $\beta_i > 0$ is scaling parameter. h_i is modelled as independent and non-identical distributed nakagami- m fading variant. For gamma distribution, cumulative distribution function (cdf) and probability distribution function (pdf) for ρ_i are defined as

$$F_{\rho_i}(x) = 1 - \sum_{j=0}^{\alpha_i-1} \frac{e^{-x} x^j}{j!} \left(\frac{x}{\beta_i}\right)^j, \quad x \geq 0. \quad (8)$$

$$f_{\rho_i}(x) = \frac{x^{\alpha_i-1} e^{-x/\beta_i}}{\Gamma(\alpha_i) \beta_i^{\alpha_i}}, \quad x \geq 0 \quad (9)$$

For $i = 1, 2$. The selection of nakagami- m fading is to get the closed form expressions. Here, $\alpha_i = 0$ and $\beta_i = \Omega_i$ then it is equivalent to the Rayleigh fading with variance Ω_i . So, Nakagami- m fading gives more degree of freedom for any channel.

For signal to noise ratio (SNR) with both hops is defined as

$$\text{SNR}_i = \frac{P_i \mathbb{E}_{\rho_i} \{\rho_i\}}{N_i}, \quad i = 1, 2. \quad (10)$$

SNR_1 is from source to relay and SNR_2 is from relay to destination and $\mathbb{E}_{\rho_i} \{\rho_i\} = \alpha_i \beta_i$ for nakagami- m fading. In SNR equation, to increase SNR value it is required to increase either signal power or fading channel power. Increase of signal power makes the power level to cross upper bounds and led to non-linear HPA noise. So alternative is to increase fading channel power (means decrease propagation distance). This does not affect the power bound.

B. SNDR Calculation for AF Relaying

From the generalised model, that is derived from previous section of equation (7) is

$$y_1 = (s_1 + \eta_1)h_1 + v_1 \quad \text{From source to relay}$$

$$y_2 = (s_2 + \eta_2)h_2 + v_2 \quad \text{From relay to destination}$$

So, for amplify-and-Forward relaying the output at relay is just amplified version of received signal. Consider the amplification factor $G > 0$.

$$s_2 = G y_1 \quad (11)$$

s_2 is transmitted signal at relay and y_1 is received signal at relay.

$$y_1 = (s_2 + \eta_2)h_2 + v_2 = (G y_1 + \eta_2)h_2 + v_2 = G s_1 h_1 h_2 + G \eta_1 h_1 h_2 + G v_1 h_2 + \eta_2 h_2 + v_2 \quad (12)$$

And amplification factor G is defined as $G_{ni}^f \triangleq \sqrt{P_2 / \mathbb{E}_{s_1, \eta_1, v_1} \{|y_1|^2\}}$ for fixed gain relaying. Incase of relay doesn't know the information about channel, and then variable gain can be used.

$$G_{ni}^v \triangleq \sqrt{\frac{P_2}{\mathbb{E}_{s_1, \eta_1, v_1} \{|y_1|^2\}}}, \quad G_{ni}^f \triangleq \sqrt{\frac{P_2}{(P_1 \mathbb{E}_{\rho_1} \{\rho_1\} (1 + k_1^2) + N_1)}} \quad (13)$$

Here, $P_2 = \mathbb{E}\{|G y_1|^2\}$ and $\mathbb{E}\{|y_1|^2\} = k_2^2 P_2$.

By solving the equation (12) for the SNDR equation is given by.

$$\gamma_{ni}^{AF-f} = \frac{\rho_1 \rho_2}{\rho_1 \rho_2 d + \rho_2 (1 + k_2^2) \frac{N_1}{P_1} + \frac{N_2}{P_1 (G_{ni}^f)^2}} \quad (14)$$

$$\gamma_{ni}^{AF-v} = \frac{\rho_1 \rho_2}{\rho_1 \rho_2 d + \rho_1 (1 + k_1^2) \frac{N_2}{P_2} + \rho_2 (1 + k_2^2) \frac{N_1}{P_1} + \frac{N_4 N_2}{P_1 P_2}} \quad (15)$$

From above, d is defined as $k_1^2 + k_2^2 + k_1^2 k_2^2$, which is distortion noise parameter.

For ideal hardware case equations (13) – (15) are changed to

$$G_{ni}^f \triangleq \sqrt{\frac{P_2}{(P_1 \mathbb{E}_{\rho_1} \{\rho_1\} + N_1)}}, \quad G_{ni}^v \triangleq \sqrt{\frac{P_2}{(P_1 \rho_1 + N_1)}} \quad (16)$$

And also for SNDR,

$$\gamma_{id}^{AF-f} = \frac{\rho_1 \rho_2}{\rho_2 \frac{N_1}{P_1} + P_1 (G_{ni}^f)^2}, \quad \gamma_{id}^{AF-v} = \frac{\rho_1 \rho_2}{\rho_1 \frac{N_2}{P_2} + \rho_2 \frac{N_1}{P_1} + \frac{N_4 N_2}{P_1 P_2}} \quad (17)$$

When compare equation (14), (15) with (17), ideal case SNDR is less complex than in (14), (15) impairment case. Those impairment case SNDRs are complicated because, the parameter $\rho_1 \rho_2$ was appeared in numerator and denominator.

B. SNDR Calculation for DF Relaying

Unlike in AF relaying, the DF relaying is a method, which just compares the signal at source and relay. If both signals are equal then only the decoding is correct. In case of SNDR, it is evident that the minimum of SNDRs between 1) Source and relay and 2) Relay and destination only be selected. It means both h_1 and h_2 are known to relay.

So for non-ideal case DF Relaying

$$\gamma_{ni}^{DF} = \min\left(\frac{P_1 \rho_1}{(P_1 \rho_1 k_1^2 + N_1)}, \frac{P_2 \rho_2}{(P_2 \rho_2 k_2^2 + N_2)}\right) \quad (18)$$

And for ideal case $k_1 = k_2 = 0$. So, above equation can be modified as

$$\gamma_{id}^{DF} = \min\left(\frac{P_1 \rho_1}{N_1}, \frac{P_2 \rho_2}{N_2}\right) \quad (19)$$

When compared to AF relaying case DF relaying with impairments or non-ideal is more complicated than DF relaying ideal case.

Derivations for Outage Probability

The outage probability is defined as the probability that the fading channel makes effective end-to-end SNDR value fall below certain threshold 'x' of acceptable communication. This includes hardware impairments but, most of previous papers analysed only for ideal hardware and both are different in terms of complexity and accuracy.

$P_{out}(x) = \Pr(\gamma \leq x)$. Here γ – SNDR.

A. Outage Probability for AF Relaying

Outage probability will provide closed form expressions for any of the fading distribution and channel gains. Here both the complexity is only due to $\rho_1 \rho_2$ is also available in denominator of SNDR equation. So the model formula given for reduction of derivation is

Formulization

Consider c_1, c_2 and c_3 are positive constants and ρ_i be the random variable of non-negative type. If cdf is $F_\rho(\cdot)$ and pdf is $f_\rho(\cdot)$,

$$\Pr\left\{\frac{c_1 \rho}{c_2 \rho + c_3} \leq x\right\} = \begin{cases} F_\rho\left(\frac{c_3 x}{c_1 - c_2 x}\right), & 0 \leq x < \frac{c_1}{c_2} \\ 1, & x \geq \frac{c_1}{c_2} \end{cases}$$

It is simply derived as

$$\Pr\left\{\frac{c_1 \rho}{c_2 \rho + c_3} \leq x\right\} = \Pr\{c_1 \rho \leq x(c_2 \rho + c_3)\} = \Pr\{\rho \leq \frac{c_3 x}{c_1 - c_2 x}\}$$

Apply same functionality on equation (15), it is modified as

$$\gamma_{ni}^{AF-v} = \frac{\rho_1 \rho_2}{\rho_1 \rho_2 d + \rho_1 b_1 + \rho_2 b_2 + c}; \quad (20)$$

$$b_1 = (1 + k_1^2) \frac{N_2}{P_2}; b_2 = (1 + k_2^2) \frac{N_1}{P_1}; c = \frac{N_1 N_2}{P_1 P_2};$$

After some mathematical calculations the outage probability is

$$P_{out}^{AF-ni}(x) = 1 - \int_0^\infty (1 - \Pr(\gamma \leq x/\rho_2)) f_{\rho_2}(\rho_2) d\rho_2 \quad (21)$$

$$P_{out}^{AF-ni}(x) = 1 - \int_0^\infty \left(1 - F_{\rho_1}\left(\frac{b_2 x}{1-dx} + \frac{b_1 b_2 x^2 + cx}{z(1-dx)}\right)\right) f_{\rho_2}(z + \frac{b_1 x}{1-dx}) dz$$

Where the value of z is $\rho_2 - \frac{b_1 x}{1-dx}$, considered for simplicity. This is the final equation for outage probability

B. Outage Probability for DF Relaying

Using formulization in previous section outage probability can be derived as

$$\Pr\left(\frac{P_1 \rho_1}{(P_1 \rho_1 k_1^2 + N_1)} \leq x\right) \text{ and } \Pr\left(\frac{P_2 \rho_2}{(P_2 \rho_2 k_2^2 + N_2)} \leq x\right) \quad (22)$$

Using probability formula in (22)

$$P_{out}^{DF-ni}(x) = 1 - \prod_{i=1,2} (1 - F_{\rho_i}\left(\frac{N_i x}{P_i(1-k_i^2 x)}\right)) \quad (23)$$

With $x < \frac{1}{\delta}$ and for $x \geq \frac{1}{\delta}$, $P_{out}^{DF-ni}(x) = 1$

And $\delta = \max(k_1^2, k_2^2)$

$$P_{out}^{DF-id}(x) = 1 - \prod_{i=1,2} (1 - F_{\rho_i}\left(\frac{N_i x}{P_i}\right)) \quad (24)$$

Here, in both AF and DF relaying cases outage probability $P_{out}(x)$ is always 1 for $x \geq \frac{1}{\delta}$ (AF) and $x \geq \frac{1}{\delta}$ (DF).

Those are cases for SNDR ceiling.

C. Channel Characteristics with Nakagami-m fading

Previous research was about Ops with AF and DF relaying, those use ideal hardware case.

Most of the work previously done was with Rayleigh fading and some minor work was with Nakagami-m fading[8].

i. AF Relaying

First consider gamma distribution as defined in (8) and (9) with $\rho_1 \rho_2$ as channels with $\text{gamma}(\alpha_i, \beta_i)$. Using some critical derivation of reduction process, the gamma function can be reduced to,

$$P_{out}^{AF-ni}(x) = 1 - 2e^{-\frac{x}{1-dx}} \frac{(\frac{b_1}{\beta_1}, \frac{b_2}{\beta_2})}{\sum_{j=0}^{\alpha_1-1} \sum_{k=0}^{\alpha_2-1} c(j, n, k)} \times \left(\frac{x}{1-dx}\right)^{\alpha_2+j} \left(b_1 b_2 + \frac{c(1-dx)}{x}\right)^{\frac{n+k+1}{2}} \times K_{n-k+1} \left(2\sqrt{\frac{b_1 b_2 x^2}{\beta_1 \beta_2 (1-dx)^2} + \frac{cx}{\beta_1 \beta_2 (1-dx)}}\right) \quad (25)$$

For $x < \frac{1}{d}$ and $P_{out}^{AF-ni}(x) = 1$ for $x \geq \frac{1}{d}$.

$$\text{Here } c(j, n, k) = \frac{b_1^{\alpha_2-n-1} b_2^{j-k} \beta_1^{k-n-1-2j} \beta_2^{\frac{n-k+1-2\alpha_2}{2}}}{k!(j-k)!n!(\alpha_2-n-1)!} \quad (26)$$

In special case the Nakagami-m fading[9] is converted to Rayleigh fading. i.e., $\alpha_i = 1$ and $\beta_i = \Omega_i$.

$$P_{out}^{AF-ni}(x) = 1 - 2e^{-\frac{x}{1-dx}} \frac{(\frac{b_1}{\beta_1}, \frac{b_2}{\beta_2})}{\sqrt{\frac{b_1 b_2 x^2}{\beta_1 \beta_2 (1-dx)^2} + \frac{cx}{\beta_1 \beta_2 (1-dx)}}} \times K_1\left(\frac{1}{\sqrt{\frac{b_1 b_2 x^2}{\beta_1 \beta_2 (1-dx)^2} + \frac{cx}{\beta_1 \beta_2 (1-dx)}}}\right) \quad (27)$$

For $x < \frac{1}{d}$ and $P_{out}^{AF-ni}(x) = 1$ for $x \geq \frac{1}{d}$.

$K_v(\cdot)$ is Bessel function of v_{th} order.

ii. For DF Relaying

Here also consider the case of gamma distribution, and then the outage probability is

$$P_{out}^{DF}(x) = 1 - \prod_{i=1,2} \left(\sum_{j=0}^{\alpha_i-1} \frac{e^{-\frac{N_i x}{P_i(1-k_i^2 x)}}}{j!} \left(\frac{N_i x}{P_i(1-k_i^2 x)}\right)^j\right) \quad (28)$$

For $x < \frac{1}{\delta}$ and for $x \geq \frac{1}{\delta}$, $P_{out}^{DF-ni}(x) = 1$.

When Rayleigh fading is used the Nakagami-m fading is changed to

$$P_{out}^{DF}(x) = 1 - e^{-\sum_{i=1}^2 \frac{N_i x}{P_i(1-k_i^2 x)}} \text{ for } 0 \leq x < \frac{1}{\delta} \text{ and } P_{out}^{DF}(x) = 0 \text{ for } x \geq \frac{1}{\delta}$$

Derivations for Ergodic Capacity

Consider the case of ergodic channels. The main function in this is to derive ergodic channel capacity. All prior work about ergodic capacity has approved equations for AF and DF relaying [10]. They will be discussed in below parts.

A. For AF Relaying

Ergodic channel capacity for AF is

$$C_{ni}^{AF} \triangleq \frac{1}{2} \mathbb{E}\{\log_2(1 + \gamma_{ni}^{AF})\} \quad (29)$$

Here, the factor $\frac{1}{2}$ is an indication for entire communication has two time slots and ergodic capacity can be found by numerical integration. To find the closed form expression for ergodic capacity of AF relaying, the equation (29) is of the form

$$C_{ni}^{AF} \leq \frac{1}{2} \log_2\left(1 + \frac{J}{J_{d+1}}\right) \quad (30)$$

And from equation (20) the value of J will be $\frac{\rho_1 \rho_2}{\rho_1 b_1 + \rho_2 b_2 + c}$.

Again, to get closed form expressions for ergodic capacity, consider an approximation as follows,

$$\mathbb{E}\left\{\log_2\left(1 + \frac{x}{y}\right)\right\} \approx \log_2\left(1 + \frac{\mathbb{E}[x]}{\mathbb{E}[y]}\right) \quad (31)$$

Substitute equation (30) in (31) and use Nakagami-m fading distribution for channels [10].

$$C_{ni}^{AF} \approx \frac{1}{2} \log_2\left(1 + \frac{\alpha_1 \alpha_2 \beta_1 \beta_2}{\alpha_1 \alpha_2 \beta_1 \beta_2 d + \alpha_1 b_1 + \alpha_2 b_2 + c}\right) \quad (32)$$

This is the simplified version for ergodic capacity.

B. For DF Relaying

In DF relaying, the ergodic capacity is more complicated than AF relaying. Decoding signal at one hop and encoding at another hop at relay will be a complicated process. For that, prior research was assumed that symmetric process at both hops with same power levels.

So the ergodic capacity can be upper bounded as below using SNDR equation.

$$C_{ni}^{DF} \leq \min_{i=1,2} \frac{1}{2} \left\{ \log \left(1 + \frac{P_i \rho_i}{P_i \rho_i k_i^2 + N_i} \right) \right\} \quad (33)$$

From the above expression the minimum of two ergodic channel capacities is taken as upper bound.

V. Asymptotic SNR

To make the SNDR ceiling to be related to SNDR value, this will be available at high SNDR value. Consider, $SNR_1 = \mu SNR_2$. Here, $0 < \mu < \infty$.

Take a limit and make $SNR_1, SNR_2 \rightarrow \infty$,

$$\lim_{SNR_1, SNR_2 \rightarrow \infty} \gamma_{ni}^{AF} = \frac{1}{d} = \frac{1}{k_1^2 + k_2^2 + k_1^2 k_2^2} \quad (34)$$

And outage probability is as follows,

$$\lim_{SNR_1, SNR_2 \rightarrow \infty} P_{ni}^{AF} = \begin{cases} 0, & x \leq \frac{1}{k_1^2 + k_2^2 + k_1^2 k_2^2} \\ 1, & x > \frac{1}{k_1^2 + k_2^2 + k_1^2 k_2^2} \end{cases} \quad (35)$$

For DF relaying case with asymptotic SNR, SNDR ceiling is,

$$\lim_{SNR_1, SNR_2 \rightarrow \infty} \gamma_{ni}^{DF} = \frac{1}{\delta} = \frac{1}{\max(k_1^2, k_2^2)} \quad (36)$$

And outage probability for DF relaying as follows,

$$\lim_{SNR_1, SNR_2 \rightarrow \infty} P_{ni}^{DF} = \begin{cases} 0, & x \leq \frac{1}{\max(k_1^2, k_2^2)} \\ 1, & x > \frac{1}{\max(k_1^2, k_2^2)} \end{cases} \quad (37)$$

So, finally the SNDR ceiling for AF and DF relaying is observed and is for high SNR values only. But in ideal hardware case $P_{out}(x)$ cannot be 1 at any value of x . In this case, if ceiling is below threshold value $P_{out}(x) = 0$ and at high SNR $P_{out}(x) = 1$. So for both cases SNDR ceiling can be defined as,

$$\gamma^* \triangleq \begin{cases} \frac{1}{k_1^2 + k_2^2 + k_1^2 k_2^2}, & \text{for AF protocol} \\ \frac{1}{\max(k_1^2, k_2^2)}, & \text{for DF Protocol} \end{cases} \quad (38)$$

Apply this ceiling effect to ergodic capacity with high SNR values and ρ_1, ρ_2 to be independent and positive channel distributions.

Channel capacity for AF relaying,

$$\lim_{SNR_1, SNR_2 \rightarrow \infty} C_{ni}^{AF} = \log_2 \left(1 + \frac{1}{k_1^2 + k_2^2 + k_1^2 k_2^2} \right) \quad (39)$$

Channel capacity for DF relaying,

$$\lim_{SNR_1, SNR_2 \rightarrow \infty} C_{ni}^{DF} = \log_2 \left(1 + \frac{1}{\max(k_1^2, k_2^2)} \right) \quad (40)$$

VI. Design Guidelines

Consider the aggregate level of impairments k_i for $i=1, 2$ as defined in previous section.

$$k_i = \sqrt{k_{i,t}^2 + k_{i,r}^2}, \quad \text{for } i = 1, 2.$$

To make cost estimation, consider $\sum_{i=1}^2 \zeta(k_{i,t}) + \zeta(k_{i,r}) \leq T_{\max}$ with a cost $T_{\max} \geq 0$ with all impairments equal SNDR ceiling will be maximized and also cost will be reduced. So, from above discussion, it is better to have same level of impairments at every node. This gives better results than mixing low and high quality hardware.

In terms of SNDR threshold x , the aggregate level of impairments are related as

$$k_i^2 \leq \sqrt{\frac{1}{x} + 1} - 1 \text{ for AF relay and } k_i^2 \leq \frac{1}{x} \text{ for DF relay}$$

here $k_1 = k_2$. In terms of ergodic capacity the SNDR threshold is defined as $x = 2^{2R} - 1$. Here, R bits/channel use.

VII. Numerical Result Analysis

In this section, all derived equations are validated by the simulation results using Math Toolbox in MATLAB [11] with different thresholds, SNDRs and capacity ceilings.

A. For Channel Fading Variation

In this first consider outage probability with hardware impairments, $P_{out}(x)$ with two thresholds; $x = 2^2 - 1 = 3$ and $x = 2^5 - 1 = 31$. Therefore 1 and 2.5 bits/channel is the threshold level. For fig. 2 and 3 shows the relationship between OP and average SNRs, for AF (fixed and variable gain) and DF relaying with and without impairments. For those figures, $k_1 = k_2 = 0.1$ with nakagami- m fading channels of $\alpha_1 = \alpha_2 = 2$. From results in AF and DF with lower threshold leads smooth degradation and is high for higher threshold.

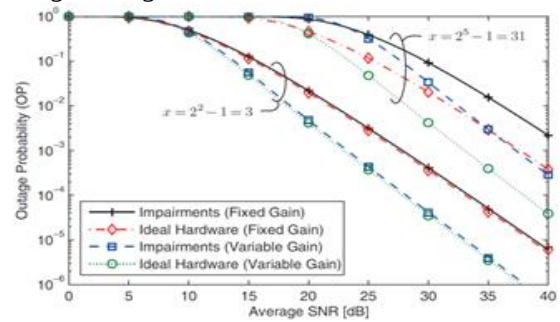


Fig. 2. Outage Probability $P_{out}(x)$ for AF relaying with ideal and non-ideal (hardware impairments) of $k_1 = k_2 = 0.1$.

While fig. 4 shows the relationship between OP and shape parameters with remaining same values and SNRs are $SNR_1 = \mu SNR_2$. For $\mu \in \{\frac{1}{5}, 1, 5\}$, and $\max(SNR_1, SNR_2) = 20$ dB. It shows OP is decreasing function of shape parameters.

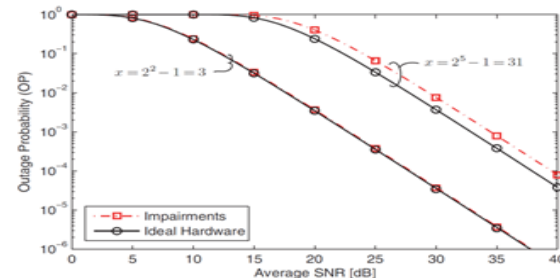


Fig. 3. Outage Probability $P_{out}(x)$ for DF relaying with ideal and impaired hardware of $k_1 = k_2 = 0.1$.

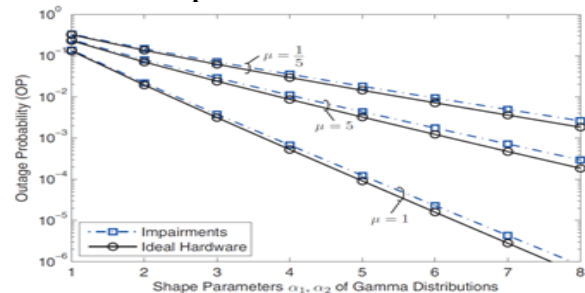


Fig. 4. Outage Probability $P_{out}(3)$ for AF relaying with ideal and impaired hardware of $k_1 = k_2 = 0.1$ and asymmetric SNRs: $SNR_1 = \mu SNR_2$.

B. For SNDR and Capacity Ceilings

In this, the relation between OP and SNDR threshold for AF and DF relaying with $k_1 = k_2 = 0.15$ and $\alpha_1 = \alpha_2 = 2$ is shown if Fig. 5. Here average SNR is fixed at 30 dB and the

OP is 1 at particular points called SNDR ceilings. Fig. 6 shows the relation between ergodic capacity and average SNR with just for AF relaying. Here also there exists a capacity ceiling at particular SNR values, $\alpha_1 = \alpha_2 = 2$ and level of impairments with $k_1 = k_2 \in \{0.05, 0.15\}$. This result also presents different variations for different level of impairments. For ideal hardware there is no capacity ceiling. Fig. 7 shows the relationship between OP and asymmetric level of impairments with threshold value is 15, $P_{out}(15)$ and $SNR_1 \in \{20, 30\}$ for first hop. Overall level of impairment factors $k_1 + k_2 = 0.3$. The performance is improved at the case of equal level of impairments, $k_1 = k_2 = 0.3/2$. The OP with DF relaying is also minimized by having a slightly higher hardware quality on the weakest hop than on the strongest hop. But it seems to be different from attained guidelines in previous section. When one of the hop is fully ideal, the system is fully outage and thus, having one ideal hop doesn't help if another hop has poor hardware quality.

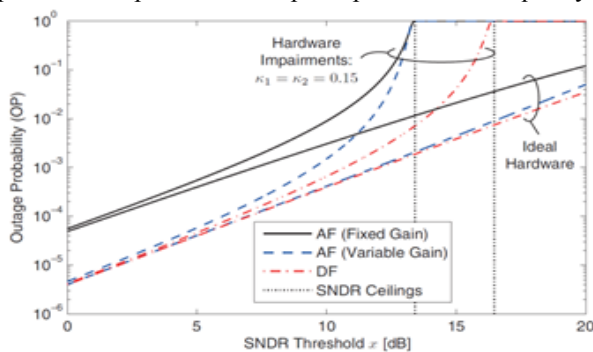


Fig. 5. Outage Probability $P_{out}(x)$ for AF and DF relaying for different thresholds x . Existence of SNDR ceiling for hardware impairments.

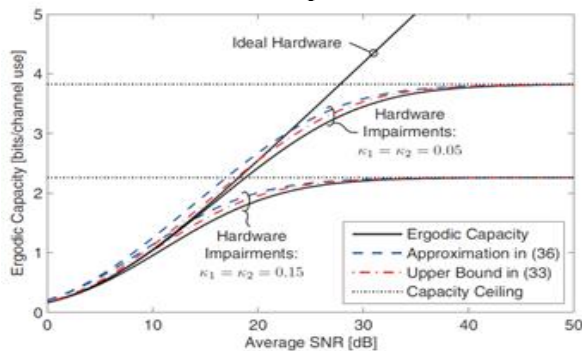


Fig. 6. Ergodic Capacity for variable gain relaying. Existence of Capacity ceilings for hardware impairments.

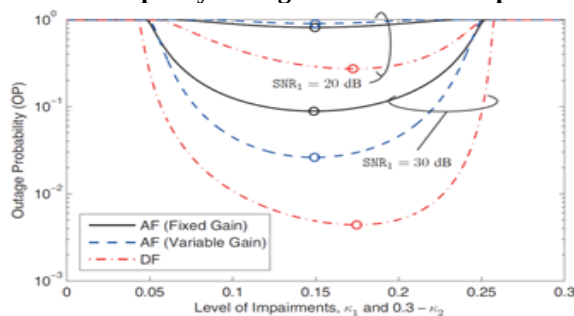


Fig. 7. Outage Probability $P_{out}(15)$ for AF and DF relaying for different levels of impairments k_1, k_2 for which $k_1 + k_2 = 0.3$.

VIII. CONCLUSION

Practical transceivers have hardware impairments; they distort signals at different levels. Different researches were

done on individual impairments with single hop relaying on either AF or DF relaying. In this case, aggregate level of impairment was considered and calculated performance with nakagami- m fading. Derived closed form expressions for effective end-to-end Signal-to-distortion ratio and Ergodic capacities. Proved the existence of upper bounds for SNDR and Ergodic Capacity called ceilings. Finally, provided design guidelines for hardware to be selected, to reduce effect of impairments in transceivers.

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