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Notes on T-fuzzy subfields of a field

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of fuzzy subfield of a field with respect to T-norm.

Introduction

Most of the problems in engineering, medical science, economics, environments, etc. have various uncertainties. To exceed these uncertainties, some kind of theories were given like theory of fuzzy sets, intuitionistic fuzzy sets and so on. Fuzzy set was introduction by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[5, 6]. In this paper, we introduce the some theorems in fuzzy subfield of a field with respect to T-norm. It is denoted as T-fuzzy subfield of a field.

1. Preliminaries

1.1 Definition

A T-norm is a binary operations T: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;

(i)$T(0, x) = 0$, $T(1, x) = x$ (boundary condition)

(ii)$T(x, y) = T(y, x)$ (commutativity)

(iii)$T(x, T(y, z)) = T(T(x,y), z)$(associativity)

(iv)if $x \leq y$ and $w \leq z$, then $T(x, w) \leq T(y, z)$(monotonicity).

1.2 Definition

Let X be a non-empty set. A **fuzzy subset** A of X is a function $A : X \rightarrow [0, 1]$.

1.3 Definition

Let (F, +, ·) be a field. A fuzzy subset A of F is said to be a **T-fuzzy subfield** of F if the following conditions are satisfied:

(i) $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in F,

(ii) $\mu_A(-x) \geq \mu_A(x)$, for all x in F,

(iii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in F,

(iv) $\mu_A(x^{-1}) \geq \mu_A(x)$, for all $x \neq 0$ in F,

1.4 Definition

Let (F, +, ·) and (F^{l}, +, ·) be any two fields. Let $f : F \rightarrow F^{l}$ be any function and A be a T-fuzzy subfield in F, V be a T-fuzzy subfield in $f(F) = F^{l}$, defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$, for all x in F and y in F^{l}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.5 Definition

Let A and B be any two fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{ \langle (x, y), \mu_{A\times B}(x, y) \rangle / \text{ for all x in G and y in H } \}$, where $\mu_{A\times B}(x, y) = \min \{ \mu_A(x), \mu_B(y))$, for all x in G and y in H.

1.6 Definition

Let A be a fuzzy subset in a set S, the strongest fuzzy relation on S, that is a fuzzy relation on A is $V = \{ \langle (x,y), \mu_V(x,y) \rangle / x \text{ and } y \text{ in } S\}$ given by $\mu_V(x, y) = \min\{ \mu_A(x), \mu_A(y)\}$, for all x and y in S.

1.7 Definition

A fuzzy subset A of a set X is said to be normalized if there exist x in X such that $\mu_A(x) = 1$.

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1.8 Definition

Let A be a T-fuzzy subfield of a field (F, +, ·) and a in F. Then the pseudo T-fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$, for every x in F and for some p in P.

2.Properties of T-Fuzzy Subfields of A Field

2.1 Theorem

If A is a T-fuzzy subfield of a field (F, +, ·), then $\mu_A(-x) = \mu_A(x)$, for all x in F and $\mu_A(x^{-1}) = \mu_A(x)$, for all x≠ 0 in F and $\mu_A(x) \le \mu_A(0)$, for all x in F and $\mu_A(x) \le \mu_A(1)$, for all x≠ 0 in F, where 0 and 1 are identity elements in F.

Proof

For x in F and 0, 1 are identity elements in F. Now, $\mu_A(x) = \mu_A(-(-x)) \ge \mu_A(-x) \ge \mu_A(x)$. Therefore, $\mu_A(-x) = \mu_A(x)$, for all x in F. Now, $\mu_A(x) = \mu_A((x^{-1})^{-1}) \ge \mu_A(x^{-1}) \ge \mu_A(x)$. Therefore, $\mu_A(x^{-1}) = \mu_A(x)$, for all x≠ 0 in F. Now, $\mu_A(0) = \mu_A(x-x) \ge T(\mu_A(x), \mu_A(-x)) = \mu_A(x)$. Therefore, $\mu_A(0) \ge \mu_A(x)$, for all x in F. Now $\mu_A(1) = \mu_A(xx^{-1}) \ge T(\mu_A(x), \mu_A(x^{-1})) = \mu_A(x)$. Therefore, $\mu_A(1) \ge \mu_A(x)$, for all x≠ 0 in F.

2.2 Theorem

If A is an T-fuzzy subfield of a field (F, +, ·), then (i) $\mu_A(x-y) = \mu_A(0)$ gives $\mu_A(x) = \mu_A(y)$, for all x and y in F, (ii) $\mu_A(xy^{-1}) = \mu_A(1)$ gives $\mu_A(x) = \mu_A(y)$, for all x and y≠ 0 in F, where 0 and 1 are identity elements in F.

Proof

Let x and y in F and 0, 1 are identity elements in F. (i) Now, $\mu_A(x) = \mu_A(x-y+y) \ge T(\mu_A(x-y), \mu_A(y)) = T(\mu_A(0), \mu_A(y)) = \mu_A(y) = \mu_A(x-(x-y)) \ge T(\mu_A(x-y), \mu_A(x)) = T(\mu_A(0), \mu_A(x)) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$, for all x and y in F. (ii) Now, $\mu_A(x) = \mu_A(xy^{-1}y) \ge T(\mu_A(xy^{-1}), \mu_A(y)) = T(\mu_A(1), \mu_A(y)) = \mu_A(y) = \mu_A((xy^{-1})^{-1}x) \ge T(\mu_A(xy^{-1}), \mu_A(x)) = T(\mu_A(1), \mu_A(x)) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$, for all x and y ≠ 0 in F.

2.3 Theorem

Let A be a fuzzy subset of a field (F, +, ·). If $\mu_A(e) = \mu_A(e^l) = 1$ and $\nu_A(e) = \nu_A(e^l) = 0$ and $\mu_A(x-y) \ge T(\mu_A(x), \mu_A(y)$, for all x and y in F and $\mu_A(xy^{-1}) \ge T(\mu_A(x), \mu_A(y))$, for all x and y≠ e in F, then A is a T-fuzzy subfield of F, where e and e^l are identity elements of F.

Proof

Let x and y in F and e, e^l are identity elements of F. Now $\mu_A(-x) = \mu_A(e-x) \ge T(\mu_A(e), \mu_A(x)) = T(1, \mu_A(x)) = \mu_A(x)$. Therefore, $\mu_A(-x) \ge \mu_A(x)$, for all x in F. Now $\mu_A(x^{-1}) = \mu_A(e^l x^{-1}) \ge T(\mu_A(e^l), \mu_A(x)) = T(1, \mu_A(x)) = \mu_A(x)$. Therefore, $\mu_A(x^{-1}) \ge \mu_A(x)$, for all x≠ e in F. Now, $\mu_A(x+y) = \mu_A(x-(-y)) \ge T(\mu_A(x), \mu_A(-y)) \ge T(\mu_A(x), \mu_A(y))$. Therefore, $\mu_A(x+y) \ge T(\mu_A(x), \mu_A(y))$, for all x and y in F. Now, $\mu_A(xy) = \mu_A(x(y^{-1})^{-1}) \ge T(\mu_A(x), \mu_A(y^{-1})) \ge T(\mu_A(x), \mu_A(y))$. Therefore, $\mu_A(xy) \ge T(\mu_A(x), \mu_A(y))$, for all x and y≠ e in F. Hence A is a T-fuzzy subfield of F.

2.4 Theorem

If A is a T-fuzzy subfield of a field (F, +, ·), then H = { x / x∈F: $\mu_A(x) = 1$} is either empty or a subfield of F.

Proof

If no element satisfies this condition, then H is empty. If x and y in H, then $\mu_A(x-y) \ge T(\mu_A(x), \mu_A(-y)) \ge T(\mu_A(x), \mu_A(y)) = T(1, 1) = 1$. Therefore, $\mu_A(x-y) = 1$, for all x and y in H. And $\mu_A(xy^{-1}) \ge T(\mu_A(x), \mu_A(y^{-1})) \ge T(\mu_A(x), \mu_A(y)) = T(1, 1) = 1$. Therefore, $\mu_A(xy^{-1}) = 1$, for all x and y≠ e in H. We get x–y, xy^{-1} in H. Therefore, H is a subfield of F. Hence H is either empty or a subfield of F.

2.5 Theorem

If A is a T-fuzzy subfield of a field (F, +, ·), then H ={ x∈F: $\mu_A(x) = \mu_A(e) = \mu_A(e^l)$ } is either empty or a subfield of F, where e and e^l are identity elements of F.

Proof

It is trivial.

2.6 Theorem

Let A be a T-fuzzy subfield of a field (F, +, ·). Then (i) if $\mu_A(x-y) = 1$, then $\mu_A(x) = \mu_A(y)$, for all x and y in F, (ii) if $\mu_A(xy^{-1}) = 1$, then $\mu_A(x) = \mu_A(y)$, for all x and y≠ e in F, where e and e^l are identity elements of F.

Proof

Let x and y in F. (i) Now, $\mu_A(x) = \mu_A(x-y+y) \ge T(\mu_A(x-y), \mu_A(y)) = T(1, \mu_A(y)) = \mu_A(y) = \mu_A(-y) = \mu_A(-x+x-y) \ge T(\mu_A(-x), \mu_A(x-y)) = T(\mu_A(-x), 1) = \mu_A(-x) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$, for all x and y in F.

(ii) Now $\mu_A(x) = \mu_A(xy^{-1}y) \ge T(\mu_A(xy^{-1}), \mu_A(y)) = T(1, \mu_A(y)) = \mu_A(y) = \mu_A(y^{-1}) = \mu_A(x^{-1}xy^{-1}) \ge T(\mu_A(x^{-1}), \mu_A(xy^{-1})) = T(\mu_A(x^{-1}), 1) = \mu_A(x^{-1}) = \mu_A(x)$.

Therefore, $\mu_A(x) = \mu_A(y)$, for all x≠ e and y≠ e in F.

2.7 Theorem

If A be a T-fuzzy subfield of a field (F, +, ·), then if $\mu_A(x-y) = 0$, then either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y in F and if $\mu_A(xy^{-1}) = 0$, then either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y≠ e in F, where e and e^l are identity elements of F.

Proof

Let x and y in F. By the definition $\mu_A(x-y) \ge T(\mu_A(x), \mu_A(y))$, which implies that $0 \ge T(\mu_A(x), \mu_A(y))$. Therefore, either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y in F. And, by the definition $\mu_A(xy^{-1}) \ge T(\mu_A(x), \mu_A(y))$, which implies that $0 \ge T(\mu_A(x), \mu_A(y))$. Therefore, either $\mu_A(x) = 0$ or $\mu_A(y) = 0$, for all x and y≠ e in F.

2.8 Theorem

Let (F, +, ·) be a field. If A is a T-fuzzy subfield of F, then $\mu_A(x+y) = T(\mu_A(x), \mu_A(y))$, for all x and y in F and $\mu_A(xy) = T(\mu_A(x), \mu_A(y))$, for all x and y ≠ 0 in F, where 0 and 1 are identity elements of F.

Proof

Let x and y belongs to F. Assume that $\mu_A(x) > \mu_A(y)$ and $\nu_A(x) < \nu_A(y)$.

Now $\mu_A(y) = \mu_A(-x+x+y) \geq T(\mu_A(-x), \mu_A(x+y)) \geq T(\mu_A(x), \mu_A(x+y)) = \mu_A(x+y) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(y)$. Therefore, $\mu_A(x+y) = \mu_A(y) = T(\mu_A(x), \mu_A(y))$, for all x and y in F. Now $\mu_A(y) = \mu_A(x^{-1}xy) \geq T(\mu_A(x^{-1}), \mu_A(xy)) \geq T(\mu_A(x), \mu_A(xy)) = \mu_A(xy) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(y)$. Therefore, $\mu_A(xy) = \mu_A(y) = T(\mu_A(x), \mu_A(y))$, for all $x \neq 0$ and y in F.

2.9 Theorem

If A and B are any two T-fuzzy subfields of a field $(F, +, \cdot)$, then their intersection $A \cap B$ is a T-fuzzy subfield of F.

Proof

Let x and y belong to F, $A = \{ \langle x, \mu_A(x) \rangle / x \in F \}$ and $B = \{ \langle x, \mu_B(x) \rangle / x \in F \}$. Let $C = A \cap B$ and $C = \{ \langle x, \mu_C(x) \rangle / x \in F \}$, where $\mu_C(x) = \min \{\mu_A(x), \mu_B(x)\}$. Now $\mu_C(x-y) = \min \{\mu_A(x-y), \mu_B(x-y)\} \geq \min \{ T(\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y)) \} \geq T(\min(\mu_A(x), \mu_B(x)), \min(\mu_A(y), \mu_B(y))) = T(\mu_C(x), \mu_C(y))$. Therefore, $\mu_C(x-y) \geq T(\mu_C(x), \mu_C(y))$, for all x and y in F. And $\mu_C(xy^{-1}) = \min \{ \mu_A(xy^{-1}), \mu_B(xy^{-1}) \} \geq \min \{ T(\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y)) \} \geq T(\min(\mu_A(x), \mu_B(x)), \min(\mu_A(y), \mu_B(y))) = T(\mu_C(x), \mu_C(y))$. Therefore, $\mu_C(xy^{-1}) \geq T(\mu_C(x), \mu_C(y))$, for all x and $y \neq 0$ in F. Hence $A \cap B$ is a T-fuzzy subfield of a field F.

2.10 Theorem

The intersection of a family of T-fuzzy subfields of a field $(F, +, \cdot)$ is a T-fuzzy subfield of F.

Proof

It is trivial.

2.11 Theorem

Let A be a T-fuzzy subfield of a field $(F, +, \cdot)$. If $\mu_A(x) < \mu_A(y)$, for some x and y in F, then $\mu_A(x+y) = \mu_A(x) = \mu_A(y+x)$, for all x and y in F and $\mu_A(xy) = \mu_A(x) = \mu_A(yx)$, for all x and $y \neq 0$ in F.

Proof

Let A be a T-fuzzy subfield of a field F. Also we have $\mu_A(x) < \mu_A(y)$, for some x and y in F, $\mu_A(x+y) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(x)$; and $\mu_A(x) = \mu_A(x+y-y) \geq T(\mu_A(x+y), \mu_A(-y)) \geq T(\mu_A(x+y), \mu_A(y)) = \mu_A(x+y)$. Therefore, $\mu_A(x+y) = \mu_A(x)$, for all x and y in F. Hence $\mu_A(x+y) = \mu_A(x) = \mu_A(y+x)$, for all x and y in F. And, $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y)) = \mu_A(x)$; and $\mu_A(x) = \mu_A(xyy^{-1}) \geq T(\mu_A(xy), \mu_A(y^{-1})) \geq T(\mu_A(xy), \mu_A(y)) = \mu_A(xy)$. Therefore, $\mu_A(xy) = \mu_A(x)$, for all x and $y \neq 0$ in F. Hence $\mu_A(xy) = \mu_A(x) = \mu_A(yx)$, for all x and $y \neq 0$ in F.

2.12 Theorem

Let A be a T-fuzzy subfield of a field $(F, +, \cdot)$. If $\mu_A(x) > \mu_A(y)$, for some x and y in F, then $\mu_A(x+y) = \mu_A(y) = \mu_A(y+x)$, for all x and y in F and $\mu_A(xy) = \mu_A(y) = \mu_A(yx)$, for all x and $y \neq 0$ in F.

Proof: It is trivial.

2.13 Theorem

If A and B are T-fuzzy subfields of the fields G and H respectively, then $A \times B$ is a T-fuzzy subfield of $G \times H$.

Proof

Let A and B be T-fuzzy subfields of the fields G and H respectively. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in $G \times H$. Now, $\mu_{A\times B}[(x_1, y_1) - (x_2, y_2)] = \mu_{A\times B}(x_1-x_2, y_1-y_2) = \min \{ \mu_A(x_1-x_2), \mu_B(y_1-y_2) \} \geq \min \{ T(\mu_A(x_1), \mu_A(x_2)), T(\mu_B(y_1), \mu_B(y_2)) \} \geq T(\min(\mu_A(x_1), \mu_B(y_1)), \min(\mu_A(x_2), \mu_B(y_2))) = T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$.

Therefore, $\mu_{A\times B}[(x_1, y_1) - (x_2, y_2)] \geq T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$, for all x_1, x_2 in G and y_1, y_2 in H.Now, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)^{-1}] = \mu_{A\times B}(x_1x_2^{-1}, y_1y_2^{-1}) = \min \{ \mu_A(x_1x_2^{-1}), \mu_B(y_1y_2^{-1})\} \geq \min \{ T(\mu_A(x_1), \mu_A(x_2)), T(\mu_B(y_1), \mu_B(y_2)) \} \geq T(\min(\mu_A(x_1), \mu_B(y_1)), \min(\mu_A(x_2), \mu_B(y_2))) = T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$.Therefore, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)^{-1}] \geq T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$, for all x_1 and $x_2 \neq 0$ in G and y_1 and $y_2 \neq 0^{I}$ in H. Hence $A \times B$ is a T-fuzzy subfield of $G \times H$.

2.14 Theorem

Let A and B be fuzzy subsets of the fields G and H respectively. Suppose that 0, 1and 0^{I},1^{I} are the identity elements of G and H respectively. If $A \times B$ is a T-fuzzy subfield of $G \times H$, then at least one of the following two statements must hold.

(i) $\mu_B(0^{I}) \geq \mu_A(x)$, for all x in G and $\mu_B(1^{I}) \geq \mu_A(x)$, for all $x \neq 0$ in G,

(ii) $\mu_A(0) \geq \mu_B(y)$, for all y in H and $\mu_A(1) \geq \mu_B(y)$, for all $y \neq 0^{I}$ in H.

Proof

Let $A \times B$ is a T-fuzzy subfield of $G \times H$. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in G and b in H such that $\mu_A(a) > \mu_B(0^{I})$ and $\mu_B(b) > \mu_A(0)$ and we can find $a \neq 0$ in G and $b \neq 0^{I}$ in H such that $\mu_A(a) > \mu_B(1^{I})$ and $\mu_B(b) > \mu_A(1)$. We have $\mu_{A\times B}(a, b) = \min \{\mu_A(a), \mu_B(b)\} > \min \{ \mu_A(0), \mu_B(0^{I}) \} = \mu_{A\times B}(0, 0^{I})$. And, $\mu_{A\times B}(a, b) = \min \{\mu_A(a), \mu_B(b)\} > \min \{ \mu_A(1), \mu_B(1^{I}) \} = \mu_{A\times B}(1, 1^{I})$. Thus $A \times B$ is not a T-fuzzy subfield of $G \times H$. Hence either $\mu_B(0^{I}) \geq \mu_A(x)$, for all x in G and $\mu_B(1^{I}) \geq \mu_A(x)$, for all $x \neq 0$ in G or $\mu_A(0) \geq \mu_B(y)$, for all y in H and $\mu_A(1) \geq \mu_B(y)$, for all $y \neq 0^{I}$ in H.

2.15 Theorem

Let A and B be fuzzy subsets of the fields G and H, respectively and $A \times B$ is a T-fuzzy subfield of $G \times H$. Then the following are true:

(i) if $\mu_A(x) \leq \mu_B(0^{I})$, for all x in G and $\mu_A(x) \leq \mu_B(1^{I})$, for all $x \neq 0$ in G, then A is a T-fuzzy subfield of G, where 0^{I}, $1^{I}$are identity elements of H.

(ii) if $\mu_B(x) \leq \mu_A(0)$ for all x in H and $\mu_B(x) \leq \mu_A(1)$, for all $x \neq 0^{I}$ in H, then B is a T-fuzzy subfield of H, where 0, 1 are identity elements of G.

(iii) either A is a T-fuzzy subfield of G or B is a T-fuzzy subfield of H, where 0, 1and 0^{I}, 1^{I} are the identity elements of G and H respectively.

Proof

Let A×B be a T-fuzzy subfield of G×H and x and y in G. Then $(x, 0')$, $(x, 1')$ and $(y, 0')$, $(y, 1')$ are in G×H. Now, using the property if $\mu_A(x) \leq \mu_B(0')$, for all x in G and $\mu_A(x) \leq \mu_B(1')$, for all $x \neq 0$ in G, where 0 and 1 are identity elements of G and $0'$ and $1'$ are identity elements of H, we get, $\mu_A(x-y) = \min\{\mu_A(x-y), \mu_B(0'+0')\} = \mu_{A\times B}((x-y), (0'+0')) = \mu_{A\times B}[(x, 0')+(-y, 0')] \geq T(\mu_{A\times B}(x, 0'), \mu_{A\times B}(-y, 0')) = T(\min(\mu_A(x), \mu_B(0')), \min(\mu_A(-y), \mu_B(0')))$ $T(\mu_A(x), \mu_A(-y)) \geq T(\mu_A(x), \mu_A(y))$. Therefore $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in G. And $\mu_A(xy^{-1}) = \min\{\mu_A(xy^{-1}), \mu_B(1'1')\} = \mu_{A\times B}((xy^{-1}), (1'1')) = \mu_{A\times B}[(x, 1')(y^{-1}, 1')] \geq T(\mu_{A\times B}(x, 1'), \mu_{A\times B}(y^{-1}, 1')) = T(\min(\mu_A(x), \mu_B(1')), \min(\mu_A(y^{-1}), \mu_B(1'))) = T(\mu_A(x), \mu_A(y^{-1})) \geq T(\mu_A(x), \mu_A(y))$. Therefore $\mu_A(xy^{-1}) \geq T(\mu_A(x), \mu_A(y))$, for all x and $y \neq 0$ in G. Hence A is a T-fuzzy subfield of G. Thus (i) is proved.

Now, using the property $\mu_B(x) \leq \mu_A(0)$ for all x in H and $\mu_B(x) \leq \mu_A(1)$, for all $x \neq 0'$ in H, we get, $\mu_B(x-y) = \min\{\mu_B(x-y), \mu_A(0+0)\} = \mu_{A\times B}((0+0), (x-y)) = \mu_{A\times B}[(0, x)+(0, -y)] \geq T(\mu_{A\times B}(0, x), \mu_{A\times B}(0, -y)) = T(\min(\mu_A(0), \mu_B(x)), \min(\mu_A(0), \mu_B(-y))) = T(\mu_B(x), \mu_B(-y)) \geq T(\mu_B(x), \mu_B(y))$. Therefore $\mu_B(x-y) \geq T(\mu_B(x), \mu_B(y))$, for all x and y in H. And $\mu_B(xy^{-1}) = \min\{\mu_B(xy^{-1}), \mu_A(1.1)\} = \mu_{A\times B}((1.1), (xy^{-1})) = \mu_{A\times B}[(1, x)(1, y^{-1})] \geq T(\mu_{A\times B}(1, x), \mu_{A\times B}(1, y^{-1})) = T(\min(\mu_A(1), \mu_B(x)), \min(\mu_A(1), \mu_B(y^{-1}))) = T(\mu_B(x), \mu_B(y^{-1})) \geq T(\mu_B(x), \mu_B(y))$. Therefore, $\mu_B(xy^{-1}) \geq T(\mu_B(x), \mu_B(y))$, for all x and $y \neq 0'$ in H. Hence B is a T-fuzzy subfield of H. Thus (ii) is proved. Hence (iii) is clear.

2.16 Theorem

Let A be a fuzzy subset of a field (F, +, .) and V be the strongest fuzzy relation of F. Then A is a T-fuzzy subfield of F if and only if V is a T-fuzzy subfield of F×F.

Proof

Suppose that A is a T-fuzzy subfield of F. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in F×F. We have, $\mu_V(x-y) = \mu_V[(x_1, x_2)-(y_1, y_2)] = \mu_V(x_1-y_1, x_2-y_2) = \min\{\mu_A(x_1-y_1), \mu_A(x_2-y_2)\} \geq \min\{T(\mu_A(x_1), \mu_A(y_1)), T(\mu_A(x_2), \mu_A(y_2))\} \geq T(\min(\mu_A(x_1), \mu_A(x_2)), \min(\mu_A(y_1), \mu_A(y_2))) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\mu_V(x), \mu_V(y))$. Therefore $\mu_V(x-y) \geq T(\mu_V(x), \mu_V(y))$, for all x and y in F×F. And $\mu_V(xy^{-1}) = \mu_V[(x_1, x_2)(y_1, y_2)^{-1}] = \mu_V(x_1y_1^{-1}, x_2y_2^{-1}) = \min\{\mu_A(x_1y_1^{-1}), \mu_A(x_2y_2^{-1})\} \geq \min\{T(\mu_A(x_1), \mu_A(y_1)), T(\mu_A(x_2), \mu_A(y_2))\} \geq T(\min(\mu_A(x_1), \mu_A(x_2)), \min(\mu_A(y_1), \mu_A(y_2))) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\mu_V(x), \mu_V(y))$. Therefore $\mu_V(xy^{-1}) \geq T(\mu_V(x), \mu_V(y))$, for all x and $y \neq (0, 0)$ in F×F. This proves that V is a T-fuzzy subfield of F×F. Conversely, assume that V is a T-fuzzy subfield of F×F, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in F×F, we have $\min\{\mu_A(x_1-y_1), \mu_A(x_2-y_2)\} = \mu_V(x_1-y_1, x_2-y_2) = \mu_V[(x_1, x_2)-(y_1, y_2)] = \mu_V(x-y) \geq T(\mu_V(x), \mu_V(y)) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\min(\mu_A(x_1), \mu_A(x_2)), \min(\mu_A(y_1), \mu_A(y_2)))$, if we put $x_2 = y_2 = 0$, we get, $\mu_A(x_1-y_1) \geq T(\mu_A(x_1), \mu_A(y_1))$, for all x_1 and y_1 in F. And $\min\{\mu_A(x_1y_1^{-1}), \mu_A(x_2y_2^{-1})\} = \mu_V(x_1y_1^{-1}, x_2y_2^{-1}) = \mu_V[(x_1, x_2)(y_1, y_2)^{-1}] = \mu_V(xy^{-1}) \geq T(\mu_V(x), \mu_V(y)) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\min(\mu_A(x_1), \mu_A(x_2)), \min(\mu_A(y_1), \mu_A(y_2)))$, if we put $x_2 = y_2 = 1$, we get, $\mu_A(x_1y_1^{-1}) \geq T(\mu_A(x_1), \mu_A(y_1))$, for all x_1 and $y_1 \neq 0$ in F. Hence A is an T-fuzzy subfield of F.

2.17 Theorem

Let (F, +, ·) and (F', +, ·) be any two fields. The homomorphic image of a T-fuzzy subfield of F is a T-fuzzy subfield of F'.

Proof

Let (F, +, ·) and (F', +, ·) be any two fields and f : F→F' be a homomorphism. That is f(x+y) = f(x)+f(y) for all x and y in F, f(xy) = f(x)f(y), for all x and y in F. Let V = f(A), where A is a T-fuzzy subfield of F. We have to prove that V is a T-fuzzy subfield of F'. Now, for f(x) and f(y) in F', we have $\mu_V(f(x)-f(y)) = \mu_V(f(x-y)) \geq \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$, which implies that $\mu_V(f(x)-f(y)) \geq T(\mu_V(f(x)), \mu_V(f(y)))$, for all f(x) and f(y) in F'. And $\mu_V(f(x)(f(y))^{-1}) = \mu_V(f(xy^{-1})) \geq \mu_A(xy^{-1}) \geq T(\mu_A(x), \mu_A(y))$, which implies that $\mu_V(f(x)(f(y))^{-1}) \geq T(\mu_V(f(x)), \mu_V(f(y)))$, for all f(x) and $f(y) \neq 0'$ in F'. Hence V is a T-fuzzy subfield of a field F'.

2.18 Theorem

Let (F, +, ·) and (F', +, ·) be any two fields. The homomorphic pre-image of a T-fuzzy subfield of F' is a T-fuzzy subfield of F.

Proof: Let (F, +, ·) and (F', +, ·) be any two fields and f : F→F' be a homomorphism. That is f(x+y) = f(x)+f(y), for all x and y in F and f(xy) = f(x)f(y), for all x and y in F. Let V = f(A), where V is a T-fuzzy subfield of F'.

We have to prove that A is a T-fuzzy subfield of F. Let x and y in F. Then $\mu_A(x-y) = \mu_V(f(x-y)) = \mu_V(f(x)-f(y)) \geq T(\mu_V(f(x)), \mu_V(f(y))) = T(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in F. And $\mu_A(xy^{-1}) = \mu_V(f(xy^{-1})) = \mu_V(f(x)(f(y^{-1})) = \mu_V(f(x)(f(y))^{-1}) \geq T(\mu_V(f(x)), \mu_V(f(y))) = T(\mu_A(x), \mu_A(y))$, which implies that $\mu_A(xy^{-1}) \geq T(\mu_A(x), \mu_A(y))$, for all x and $y \neq 0$ in F. Hence A is a T-fuzzy subfield of a field F.

2.19 Theorem

A T-fuzzy subfield A of a field (F, +, .) is normalized if and only if $\mu_A(e) = \mu_A(e^1) = 1$, where e and e^1 are identity elements of the field F.

Proof: If A is normalized, then there exists $x \in F$ such that $\mu_A(x) = 1$, but by properties of a T-fuzzy subfield A of F, $\mu_A(x) \leq \mu_A(e)$, for all x in F and $\mu_A(x) \leq \mu_A(e')$, for all $x \neq e$ in F, where e and e' are identity elements of the field F. Since $\mu_A(x) = 1$ and and $\mu_A(x) \leq \mu_A(e)$, for all x in F and $\mu_A(x) \leq \mu_A(e')$, for all $x \neq e$ in F. Therefore $1 \leq \mu_A(e)$, $1 \leq \mu_A(e')$. But $1 \geq \mu_A(e)$, $1 \geq \mu_A(e')$. Hence $\mu_A(e) = \mu_A(e') = 1$. Conversely, if $\mu_A(e) = \mu_A(e') = 1$, then by the definition of normalized fuzzy subset, A is normalized.

2.20 Theorem

Let A be a T-fuzzy subfield of a field H and f is an isomorphism from a field F onto H. Then A∘f is a T-fuzzy subfield of F.

Proof

Let x and y in F and A be a T-fuzzy subfield of a field H. Then we have, $(\mu_A \circ f)(x-y) = \mu_A(f(x-y)) = \mu_A(f(x)+f(-y)) = \mu_A(f(x)-f(y)) \geq T(\mu_A(f(x)), \mu_A(f(y))) \geq T((\mu_A \circ f)(x), (\mu_A \circ f)(y))$, which implies that $(\mu_A \circ f)(x-y) \geq T((\mu_A \circ f)(x), (\mu_A \circ f)(y))$, for all x, y in

F. And $(\mu_A \circ f)(xy^{-1}) = \mu_A(f(xy^{-1})) = \mu_A(f(x)f(y^{-1})) = \mu_A(f(x)(f(y))^{-1}) \geq T(\mu_A(f(x)), \mu_A(f(y))) \geq T((\mu_A \circ f)(x), (\mu_A \circ f)(y))$, which implies that $(\mu_A \circ f)(xy^{-1}) \geq T((\mu_A \circ f)(x), (\mu_A \circ f)(y))$, for all x and $y \neq 0$ in F. Therefore $(A \circ f)$ is a T-fuzzy subfield of a field F.

2.21 Theorem

Let A be a T-fuzzy subfield of a field (F, +, .), then the pseudo T-fuzzy coset $(aA)^p$ is a T-fuzzy subfield of a field F, for every $a \in F$.

Proof

Let A be a T-fuzzy subfield of a field (F, +, .). For every x and y in F, we have, $((a\mu_A)^p)(x-y) = p(a)\mu_A(x-y) \geq p(a)\, T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore $((a\mu_A)^p)(x-y) \geq T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$, for all x, y in F. And for every x and $y \neq 0$ in F, $((a\mu_A)^p)(xy^{-1}) = p(a)\mu_A(xy^{-1}) \geq p(a)\, T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore, $((a\mu_A)^p)(xy^{-1}) \geq T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$, for all x and $y \neq 0$ in F. Hence $(aA)^p$ is a T-fuzzy subfield of a field F.

Reference

1. Akram. M and Dar.K.H, On fuzzy d-algebras, Punjab University Journal of Mathematics, 37, 61-76, (2005).
2. Anthony.J.M. and Sherwood.H, Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69,124 -130 (1979).
3. Asok Kumer Ray, On product of fuzzy subgroups, Fuzzy sets and sysrems, 105 , 181-183 (1999).
4. Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications, 35, 512-517 (1971).
5. Biswas.R, Fuzzy subgroups and Anti-fuzzy subgroups, Fuzzy sets and systems, 35,121-124 (1990).
6. Biswas.R, Fuzzy fields and fuzzy linear spaces redefined, Fuzzy sets and systems, (1989) North Holland.
7. Davvaz.B, Wieslaw.A and Dudek, Fuzzy n-ary groups as a generalization of Rosen feld Fuzzy groups, ARXIV- 0710. 3884VI (MATH.RA), 1-16, (20 OCT 2007).
8. Mustafa Akgul, Some properties of fuzzy groups, Journal of mathematical analysis and applications, 133, 93-100 (1988).
9. Mohamed Asaad , Groups and fuzzy subgroups, fuzzy sets and systems (1991), North-Holland.
10. Nanda. S, Fuzzy fields and fuzzy linear spaces, Fuzzy sets and systems, 19 (1986), 89-94.
11. Palaniappan. N & K.Arjunan, Some properties of intuitionistic fuzzy subgroups, Acta Ciencia Indica, Vol.XXXIII (2) : 321-328 (2007).
12. Rajesh Kumar, Fuzzy Algebra, Volume 1, University of Delhi Publication Division, July -1993.
13. Rosenfeld. A, Fuzzy groups, J. Math. Anal. Appl., 35 (1971), 512-517.
14. Salah Abou-Zaid, On generalized characteristic fuzzy subgroups of a finite group, fuzzy sets and systems , 235-241 (1991).
15. Vasantha kandasamy .W.B, Smarandache fuzzy algebra, American research press , Rehoboth -2003.
16. ZADEH.L.A, Fuzzy sets, Information and control, Vol.8, 338-353 (1965).