Mariapresenti. L et al./ Elixir Appl. Math. 109C (2017) 47800-47806

A Note on Interval Valued Vague Volterra Spaces

Mariapresenti. L , Arockiarani. I and Arokia lancy. A

Nirmala College for women, Coimbatore- 641018, Tamilnadu, India.

ABSTRACT
In this paper we present a new class of interval valued vague sets and interval valued vague Volterra space. We obtain several properties and some of their characterizations concerning Volterra space.

1. Introduction

The theory of fuzzy set, pioneered by Zadeh[11], has achieved many successful applications in practice. As a generalization of fuzzy set, Zadeh[12, 13, 14] introduced the concept of interval-valued fuzzy set, after that, some authors investigated the topic and obtained some meaningful conclusions.The theory of fuzzy topological spaces was introduced and developed by Chang[2]. Topology of interval valued fuzzy set has been investigated by Tapas kumar mondal and S. K. Samanta[9] in the year 1999. In 1993 Gau and Buehrer [3] introduced the concept of vague set which was the generalization of fuzzy set with truth membership and false membership function. The vague set theory has been investigated by many authors and has been applied in different fields. In classical topology, the concept of generalized Volterra spaces was initiated and studied by Milan matejdes [6,7].Later Thangaraj and Sundarajan[10] established the concept of fuzzy Volterra spaces. The purpose of this paper is to further extend the concept of vague set theory by introducing the notion of Interval valued vague set and Inteval valued vague Volterra spaces along with some interesting properties. Several examples are given to illustrate the concept.

2. Preliminaries:

Definition 2.1: [11] Let X be a non-empty set. A fuzzy set A drawn from X is defined as $A = \{< x, \mu_A(x) >: x \in X\}$, where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A.

Definition 2.2: [4] Let [I] be the set of all closed subintervals of the interval [0,1] and $\mu = [\mu_L, \mu_U] \in [I]$, where μ_L and μ_U are the lower extreme and the upper extreme, respectively. For a set X, an IVFS A is given by equation $A = \{< x, \mu_A(x) > / x \in X\}$ where the function $\mu_A : X \to [I]$ defines the degree of membership of an element x to A, and $\mu_A(x) = [\mu_{AL}(x), \mu_{AU}(x)]$ is called an interval valued fuzzy number.

Definition 2.3: [3] A vague set A in the universe of discourse U is characterized by two membership functions given by:
(i) A true membership function $t_A : U \to [0,1]$ and
(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. this indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$.The vague set A is written as $A = \left\{\left\langle x, [t_A(x), 1 - f_A(x)]\right\rangle / u \in U \right\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Tele:
E-mail address: presentimaria88@gmail.com

Definition 2.4: [1] Let A and B be VSs of the form $A = \left\{ \left\langle x, \left[t_A(x), 1 - f_A(x) \right] \right\rangle / x \in X \right\}$ and $B = \left\{ \left\langle x, \left[t_B(x), 1 - f_B(x) \right] \right\rangle / x \in X \right\}$ Then

(i) $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$ for all $x \in X$

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $A^C = \left\{ \left\langle x, f_A(x), 1 - t_A(x) \right\rangle / x \in X \right\}$

(iv) $A \cap B = \left\{ \left\langle x, \min\left(t_A(x), t_B(x) \right), \min\left(1 - f_A(x), 1 - f_B(x) \right) \right\rangle / x \in X \right\}$

(v) $A \cup B = \left\{ \left\langle x, \left(t_A(x) \vee t_B(x) \right), \left(1 - f_A(x) \vee 1 - f_B(x) \right) \right\rangle / x \in X \right\}$

For the sake of simplicity, we shall use the notation $A = \left\langle x, t_A, 1 - f_A \right\rangle$ instead of $A = \left\{ \left\langle x, \left[t_A(x), 1 - f_A(x) \right] \right\rangle / x \in X \right\}$.

Definition 2.5: [5] A vague topology (VT in short) on X is a family τ of VSs in X satisfying the following axioms.

(i) $0, 1 \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\left\{ G_i / i \in J \right\} \subseteq \tau$.

In this case the pair (X, τ) is called a Vague topological space (VTS in short) and any VS in τ is known as a Vague open set(VOS in short) in X. The complement A^c of a VOS A in a VTS (X, τ) is called a vague closed set (VCS in short) in X.

Definition 2.6: [8] An interval valued vague sets $\tilde{A}^V$ over a universe of discourse X is defined as an object of the form $\tilde{A}^V = \{ < x_i, [T_{\tilde{A}^V}(x_i), F_{\tilde{A}^V}(x_i)] >, x_i \in X \}$ where $T_{\tilde{A}^V} : X \to D([0,1])$ and $F_{\tilde{A}^V} : X \to D([0,1])$ are called " truth membership function" and "false membership function" respectively and where D[0,1] is the set of all intervals within [0,1], or in other word an interval valued vague set can be represented by $\tilde{A}^V = < [(x_i), [\mu_1, \mu_2], [v_1, v_2]] >, x_i \in X$ where $0 \leq \mu_1 \leq \mu_2 \leq 1$ *and* $0 \leq v_1 \leq v_2 \leq 1$. For each interval valued vague set $\tilde{A}^V$, $\pi_{1\tilde{A}^V}(x_i) = 1 - \mu_{1\tilde{A}^V}(x_i) - v_{1\tilde{A}^V}(x_i)$ and are called degree of hesitancy of x_i in $\tilde{A}^V$ respectively.

3. Interval valued vague topological spaces:

Definition 3.1: An interval valued vague topology (IVT in short) on X is a family τ of interval valued vague sets(IVS) in X satisfying the following axioms.

(i) $0, 1 \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\left\{ G_i / i \in J \right\} \subseteq \tau$.

In this case the pair (X, τ) is called an interval valued vague topological space (IVTS in short) and any IVS in τ is known as a Interval valued vague open set(IVOS in short) in X.

The complement $\overline{A}$ of a IVOS A in a IVTS (X, τ) is called an interval valued vague closed set (IVCS in short) in X.

Example 3.2: Let X={a,b}, $A = \left\{ \left\langle x, \left[[0.3,0.4], [0.5,0.6] \right], \left[[0.4,0.5], [0.7,0.8] \right] \right\rangle \right\}$ $B = \left\{ \left\langle x, \left[[0.4,0.5], [0.6,0.7] \right], \left[[0.5,0.6], [0.7,0.8] \right] \right\rangle \right\}$ Then the family $\tau = \{0, A, B, 1\}$ of an interval valued vague sets in X is an interval valued vague topology on X.

Definition 3.3: Let $A = \{ \left\langle x, [t_A^L(x), t_A^U(x)], [1 - f_A^L(x), 1 - f_A^U(x)] \right\rangle \}$ and $B = \{ \left\langle x, [t_B^L(x), t_B^U(x)], [1 - f_B^L(x), 1 - f_B^U(x)] \right\rangle \}$ be two interval valued vague sets then their union, intersection and complement are defined as follows:

(i) $A \cup B = \{ < x, [t_{A \cup B}^L(x), t_{A \cup B}^U(x)], [1 - f_{A \cup B}^L(x), 1 - f_{A \cup B}^U(x)] > / x \in X \}$ where

$t_{A \cup B}^L(x) = \max\{ t_A^L(x), t_B^L(x) \}$, $t_{A \cup B}^U(x) = \max\{ t_A^U(x), t_B^U(x) \}$ *and*

$1 - f_{A \cup B}^L(x) = \max\{ 1 - f_A^L(x), 1 - f_B^L(x) \}$, $1 - f_{A \cup B}^U(x) = \max\{ 1 - f_A^U(x), 1 - f_B^U(x) \}$

(ii) $A \cap B = \{ < x, [t_{A \cap B}^L(x), t_{A \cap B}^U(x)], [1 - f_{A \cap B}^L(x), 1 - f_{A \cap B}^U(x)] > / x \in X \}$ where

$t_{A \cap B}^L(x) = \min\{ t_A^L(x), t_B^L(x) \}$, $t_{A \cap B}^U(x) = \min\{ t_A^U(x), t_B^U(x) \}$ *and*

$1 - f_{A \cap B}^L(x) = \min\{ 1 - f_A^L(x), 1 - f_B^L(x) \}$, $1 - f_{A \cap B}^U(x) = \min\{ 1 - f_A^U(x), 1 - f_B^U(x) \}$

(iii) $A^C = \{ < x, [f_A^L(x), f_A^U(x)], [1 - t_A^L(x), 1 - t_A^U(x)] > / x \in X \}$.

Theorem 3.4: Let (X, τ) be a IVS and let $A, B \in IV(X)$. Then the following properties hold.

(i) $0 \subseteq A \subseteq 1$,

(ii) $A \cup B = B \cup A$; $A \cap B = B \cap A$,

(iii) $A, B \subset A \cup B$; $A \cap B \subset A, B$,

(iv) $A \cap (\cup_i B_i) = \cup_i (A \cap B_i)$ *and* $A \cup (\cap_i B_i) = \cap_i (A \cup B_i)$,

(v) $0^c = 1$; $1^c = 0$,

(vi) $(A^c)^c = A$,

(vii) $(\cup_i A_i)^c = \cap_i A_i^c$ *and* $(\cap_i A_i)^c = \cup_i A_i^c$

Proof: Obvious.

Definition 3.5: Let (X, τ) be an interval valued vague topological space and $A = \{\langle x, [t_A^L, t_A^U], [1 - f_A^L, 1 - f_A^U] \rangle\}$ be a IVS in X.

Then the interval valued vague interior and an interval valued vague closure are defined by

$IV \text{int}(A) = \cup \{G/G \text{ is an IVOS in X and } G \subseteq A\}$

$IVcl(A) = \cap \{K/K \text{ is an IVCS in X and } A \subseteq K\}$

Note that for any IVS A in (X, τ), we have $IVcl(A^c) = (IV \text{int}(A))^c$ and $V \text{int}(A^c) = (IVcl(A))^c$. and $IVcl(A)$ is an IVCS and $IV \text{int}(A)$ is an IVOS in X. Further we have, if A is an IVCS in X then $IVcl(A)=A$ and if A is an IVOS in X then $IVint(A)=A$.

Example 3.6: Let X={a,b} and let $\tau = \{0, A, B, 1\}$ is an IVT on X, where $A = \left\{\left\langle x, [[0.2,0.3],[0.4,0.5]], [[0.3,0.5],[0.6,0.7]] \right\rangle\right\}$ and $B = \left\{\left\langle x, [[0.3,0.4],[0.5,0.6]], [[0.4,0.5],[0.7,0.8]] \right\rangle\right\}$ Here   the   open   sets   are   0,1,A   and   B.   If $F = \left\{\left\langle x, [[0.2,0.5],[0.6,0.8]], [[0.3,0.4],[0.5,0.3]] \right\rangle\right\}$ is a IVT on X then,

$IV \text{int}(A) = \cup \{G/G \text{ is an IVOS in X and } G \subseteq F\} = 0$

$IVcl(A) = \cap \{K/K \text{ is an IVCS in X and } F \subseteq K\} = 1$

Theorem 3.7: Let (X, τ) be a IVS and let $A, B \in IV(X)$. Then the following properties hold.

(i) $IV \text{int}(A) \subset A$

(ii) $A \subset B \Rightarrow IV \text{int}(A) \subset IV \text{int}(B)$

(iii) A is a interval valued vague open set $\Longleftrightarrow IV \text{int}(A) = A$

(iv) $IV \text{int}(IV \text{int}(A)) = IV \text{int}(A)$

(v) $IV \text{int}(0) = 0$, $IV \text{int}(1) = 1$

(vi) $IV \text{int}(A \cap B) = IV \text{int}(A) \cap IV \text{int}(B)$

Proof: The proof is obvious.

Theorem 3.8: Let (X, τ) be a IVS and let $A, B \in IV(X)$. Then the following properties hold. $(A) \subset IVcl(A)$

(i) $A \subset B \Rightarrow IVcl(A) \subset IVcl(B)$

(ii) A is a vague Closed set $\Longleftrightarrow IVcl(A) = A$

(iii) $IVcl(IVcl(A)) = IVcl(A)$

(iv) $IVcl(0) = 0$, $IVcl(1) = 1$

(v) $IVcl(A \cup B) = IVcl(A) \cup IVcl(B)$

Proof: The proof is obvious.

4. Interval valued vague Volterra space:

Definition 4.1: An interval valued vague set A in an interval valued vague topological space (X, τ) is called an interval valued vague dense if there exists no interval valued vague closed set B in (X, τ) such that $A \subset B \subset 1$.

Definition 4.2: An interval valued vague set A in an interval valued vague topological space (X, τ) is called an interval valued vague nowhere dense set if there exists no interval valued vague open set B in (X, τ) such that $B \subset IVcl(A)$. That is, $IV \text{int}(IVcl(A)) = 0$.

Theorem 4.3: If A is an interval valued vague dense and interval valued vague open set in an interval valued vague topological space (X, τ) then A^c is a interval valued vague nowhere dense set in(X, τ).

Proof: Let A be an interval valued vague dense and interval valued vague open set in(X, τ). Then we have $IVcl(A) = 1$ and $IV\,int(A) = A$. Now we have to show that $IV\,int(IVcl(A^C)) = 0$. Let $IVcl(A^C) = (IV\,int(A))^C = A^C$ which implies that $IV\,int(IVcl(A^C)) = IV\,int(A^C) = (IVcl(A))^C = 1^C = 0$ That is, $IV\,int(IVcl(A^C)) = 0$. Hence A^C is interval valued vague nowhere dense set in(X, τ).

Theorem 4.4: Let A be an interval valued vague set. If A is an interval valued vague closed set in (X, τ) with $IV\,int(A) = 0$, then A is an interval valued vague nowhere dense set in (X, τ).

Proof: Let A is an interval valued vague closed set in (X, τ). Then $IVcl(A) = A$. Now $IV\,int(IVcl(A)) = IV\,int(A) = 0$ .and hence A is an interval valued vague nowhere dense set in (X, τ).

Theorem 4.5: Let A be an interval valued vague closed set in (X, τ), then A is an interval valued vague nowhere dense set in (X, τ) if and only if $IV\,int(A) = 0$.

Proof: Let A be an interval valued vague closed set in(X, τ), with $IV\,int(A) = 0$. Then by theorem 4.4, A is an interval valued vague nowhere dense set in(X, τ). Conversely, let A be an interval valued vague nowhere dense set in (X, τ).Then $IV\,int(IVcl(A)) = 0$ which implies that $IV\,int(A) = 0$. Since A is an interval valued vague closed, $IVcl(A) = A$.

Theorem 4.6: If A is an interval valued vague nowhere dense set in interval valued vague topological space (X, τ), then $IVcl(A)$ is also a interval valued vague nowhere dense set in (X, τ).

Proof: Let A be an interval valued vague nowhere dense set in a (X, τ). Then, $IV\,int(IVcl(A)) = 0$. Now $IVcl(IVcl(A)) = IVcl(A)$. Hence $IV\,int(IVcl(IVcl(A))) = IV\,int(IVcl(A)) = 0$. Therefore $IVcl(A)$ is also a interval valued vague nowhere dense set in (X, τ).

Definition 4.7: An interval valued vague topological space (X, τ) is called an interval valued vague first category set if $A = \bigcup\limits_{i=1}^{\infty} (A_i)$, where A_i's are interval valued vague nowhere dense sets in (X, τ). Any other interval valued vague set in (X, τ) is said to be of interval valued vague second category.

Definition 4.8: An interval valued vague set A in an interval valued vague topological space(X, τ) is called an interval valued vague G_δ-sets in (X, τ) if $A = \bigcap\limits_{i=1}^{\infty} (A_i)$ where $A_i \in \tau$,for $i \in I$.

Definition 4.9: An interval valued vague set A in an interval valued vague topological space(X, τ) is called an interval valued vague F_σ-sets in (X, τ) if $A = \bigcup\limits_{i=1}^{\infty} (A_i)$ where $A_i^c \in \tau$,for $i \in I$.

Definition 4.10: An interval valued vague topological space (X, τ) is called an interval valued vague Volterra space if $IVcl(\bigcap\limits_{i=1}^{N} A_i) = 1$, where A_i's are interval valued vague dense and interval valued vague G_δ-sets in (X, τ).

Example 4.11: Let X={a,b}. Define the interval valued vague sets A, B, C and D as follows, $A = \{< x, \left[[0.4,0.6],[0.8,0.9]\right], \left[[0.2,0.3],[0.6,0.7]\right] >\}$, $B = \{< x, \left[[0.4,0.5],[0.7,0.8]\right], \left[[0.3,0.4].[0.6,0.8]\right] >\}$, $C = \{< x, \left[[0.4,0.5],[0.7,0.8]\right], \left[[0.2,0.3],[0.6,0.7]\right] >\}$ and $D = \{< x, \left[[0.4,0.6],[0.8,0.9]\right], \left[[0.3,0.4],[0.6,0.8]\right] >\}$.
Clearly $\tau = \{0,1, A, B, C, D\}$ is an interval valued vague topology on X.
thus (X, τ) is an Interval valued vague topological space. Let $E = \{A \bigcap B \bigcap C\}$, $F = \{A \bigcap B \bigcap D\}$ and $G = \{A \bigcap B \bigcap C \bigcap D\}$ where E, F and G are Interval valued vague G_δ-sets in (X, τ). Also, we have $IVcl(E) = 1$, $IVcl(F) = 1$ $IVcl(G) = 1$. Also we have $IVcl(E \bigcap F \bigcap G) = 1$. Therefore, is an Interval valued vague Volterra space.

Theorem 4.12: An interval valued vague topological space (X, τ) is an interval valued vague Volterra space, if and only if $IV\,int(\bigcup\limits_{i=1}^{N} \overline{A_i}) = 0$, where A_i's are interval valued vague dense and interval valued vague G_δ-sets in (X, τ).

Proof: Let (X, τ) be an interval valued vague Volterra space and A_i's are interval valued vague dense and interval valued vague G_δ-sets in (X, τ).Then we have $IVcl(\bigcap\limits_{i=1}^{N} A_i) = 1$ .Now $IV\,int(\bigcup\limits_{i=1}^{N} (A_i)^C) = (IVcl(\bigcap\limits_{i=1}^{N} A_i))^C = 0$ .Conversely, let $IV\,int(\bigcup\limits_{i=1}^{N} (A_i)^C) = 0$, where A_i's are interval valued vague dense and interval valued vague G_δ-sets in (X, τ). Then $(IVcl(\bigcap\limits_{i=1}^{N} A_i))^C = 0$ implies $IVcl(\bigcap\limits_{i=i}^{N} A_i) = 1$. Hence (X, τ) is an Interval valued vague Volterra space.

Theorem 4.13: Let (X, τ) be an interval valued vague topological space. If $IV\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$, A_i's are interval valued vague nowhere dense and an interval valued vague F_σ-sets in (X, τ), then (X, τ) is an interval valued vague Volterra space.

Proof: Let $IV\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$, then this implies that, $(IV\operatorname{int}(\bigcup_{i=1}^{N} A_i))^c = 1$ (i.e.) $IVcl(\bigcap_{i=1}^{N} (A_i)^c) = 1$. A_i's are interval valued vague nowhere dense and an interval valued vague F_σ-sets implies that $\overline{A}_i$'s are interval valued vague dense and an interval valued vague G_δ-sets in(X, τ), and also $IVcl(\bigcap_{i=1}^{N} (A_i)^c) = 1$. Then (X, τ) is an interval valued vague Volterra space.

Definition 4.14: Let A be a vague first category set in an interval valued vague topological space (X, τ). Then A^c is called an interval valued vague residual sets in (X, τ).

Definition 4.15: An interval valued vague topological space (X, τ) is called an interval valued vague ε_r-Volterra space if $IVcl(\bigcap_{i=1}^{N} A_i) = 1$, where A_i's are interval valued vague dense and interval valued vague residual sets in(X, τ).

Theorem 4.16: Let (X, τ) be an interval valued vague ε_r-Volterra space, then $IV\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$, where A_i's are interval valued vague first category sets such that $IV\operatorname{int}(A_i) = 0$ in (X, τ).

Proof: Let A_i's (i=1 to N) be interval valued vague first category set such that $IV\operatorname{int}(A_i) = 0$ in(X, τ). Then $\overline{A}$ is interval valued vague residual sets such that $IVcl(\overline{A}_i) = 1$ in(X, τ). That is $\overline{A}$'s are interval valued vague residual and interval valued vague dense sets in (X, τ). Since (X, τ) is an interval valued vague ε_r-Volterra space, $IVcl(\bigcap_{i=1}^{N} (\overline{A}_i) = 1$ and hence therefore, we have $IV\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$ where A_i's are interval valued vague first category sets such that, $IV\operatorname{int}(A_i) = 0$ in(X, τ).

Theorem 4.17: If each interval valued vague nowhere dense set is an interval valued vague closed set in an interval valued vague Volterra space in (X, τ), then (X, τ) is an interval valued vague ε_r-Volterra space.

Proof: Let A_i's (i=1 to N) be interval valued vague dense set and interval valued vague residual set in (X, τ). Since A_i's are interval valued vague residual set, $(\overline{A}_i)'s$ are interval valued vague first category set in(X, τ). Now $\overline{A}_i = \bigcup_{i=1}^{\infty} B_{ij}$, where B_{ij}'s are interval valued vague nowhere dense set in (X, τ). By hypothesis, an interval valued vague nowhere dense set B_{ij}'s are interval valued vague closed sets and hence $(\overline{A}_i)'s$ are interval valued vague F_σ sets in(X, τ). This implies that A_i's are interval valued vague G_δ-sets in (X, τ). Hence A_i's are interval valued vague dense and interval valued vague G_δ-sets in (X, τ). Since (X, τ) is an interval valued vague Volterra space, $IVcl(\bigcap_{i=1}^{N} A_i) = 1$. Hence $IVcl(\bigcap_{i=1}^{N} A_i) = 1$, where A_i's are interval valued vague dense and interval valued vague residual sets in(X, τ) implies that (X, τ) is an interval valued vague ε_r-Volterra space.

Definition 4.18: Let(X, τ) be an interval valued vague topological space. Then(X, τ) is called an interval valued vague baire space if $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0$ where A_i's are interval valued vague nowhere dense sets in (X, τ).

Theorem 4.19: Let(X, τ) be an interval valued vague topological space. Then the following are equivalent
(i) (X, τ) is an interval valued vague Baire space.
(ii) $IV\operatorname{int}(A) = 0$, for every interval valued vague first category set A in(X, τ).
(iii) $IVcl(B) = 1$, for every interval valued vague residual set B in(X, τ).

Proof: (i) $\Longrightarrow$ (ii) Let A be an interval valued first category set in (X, τ). Then $A = \bigcup_{i=1}^{\infty} A_i$, where $A_i's$ are interval valued vague nowhere dense sets in (X, τ). Now $IV\operatorname{int}(A) = IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0$ Since(X, τ) is a interval valued vague baire space. Therefore $IV\operatorname{int}(A) = 0$.

(ii)$\Rightarrow$(iii) Let A be a interval valued vague residual set in(X, τ). Then B^C is a interval valued vague first category set in(X, τ). By hypothesis $IV \operatorname{int}(B^C) = 0$ which implies that $(IVcl(B))^C = 0$. Hence $IVcl(B) = 1$.

(iii)$\Rightarrow$(i) Let A be a interval valued vague first category set in(X, τ). Then $A = \overset{\infty}{\underset{i=1}{\cup}} A_i$, where $A_i's$ are interval valued vague nowhere dense sets in (X, τ). Now A is a interval valued vague first category set implies that A^C is a interval valued vague residual set in(X, τ). By hypothesis, we have $IVcl(A^C) = 1$ which implies that $(IV \operatorname{int}(A))^C = 1$. Hence $IV \operatorname{int}(A) = 0$. That is, $IV \operatorname{int}(\overset{\infty}{\underset{i=1}{\cup}} A_i) = 0$, where $A_i's$ are interval valued vague nowhere dense sets in (X, τ). Hence(X, τ) is a interval valued vague baire space.

Theorem 4.20: If $\overset{\infty}{\underset{i=1}{\cup}} A_i$, is the interval valued vague set, $A_i's$ are interval valued vague nowhere dense sets in an interval valued vague baire space in(X, τ), then (X, τ) is an interval valued vague ε_r-Volterra space.

Proof: Let (X, τ) be an interval valued vague baire space and A_i's (i=1 to N) be interval valued vague dense set and interval valued vague residual set in (X, τ). Since A_i's are interval valued vague residual set, $(A_i^C)'s$ are interval valued vague first category set in(X, τ). Now $A_i^C = \overset{\infty}{\underset{i=1}{\cup}} B_{ij}$, where B_{ij}'s are interval valued vague nowhere dense set in(X, τ). By hypothesis (A_i^C) is a interval valued vague nowhere dense sets in(X, τ). Let (B_α)'s be an interval valued vague nowhere dense sets in (X, τ) in which the first N interval valued vague nowhere dense sets be (A_i^C). Since (X, τ) is an interval valued vague baire space,

$IV \operatorname{int}(\overset{\infty}{\underset{i=1}{\cup}} (B_\alpha)) = 0$. But $IV \operatorname{int}(\overset{N}{\underset{i=1}{\cup}} A_i^C) \leq IV \operatorname{int}(\overset{\infty}{\underset{\alpha=1}{\cup}} B_\alpha)$ and $IV \operatorname{int}(\overset{\infty}{\underset{\alpha=1}{\cup}} B_\alpha) = 0$. Then $IV \operatorname{int}(\overset{N}{\underset{i=1}{\cup}} A_i^C) = 0$.

Therefore $IVcl(\overset{N}{\underset{i=1}{\cap}} A_i) = 1$ where A_i's (i=1 to N) are interval valued vague dense set and interval valued vague residual set in $(X, \tau$ Therefore (X, τ) is an interval valued vague ε_r-Volterra space.

Theorem 4.21: If an interval valued vague ε_r-Volterra space is an interval valued vague baire space, then $IVcl(\overset{N}{\underset{i=1}{\cap}} A_i) = 1$, where where A_i's (i=1 to N) are interval valued vague residual set in(X, τ).

Proof: Let A_i's (i=1 to N) are interval valued vague residual set in(X, τ). Since(X, τ) is an interval valued vague baire space, (by theorem 4.19) $IVcl(A_i) = 1 \forall i$. Then A_i's are interval valued vague dense set and interval valued vague residual set in (X, τ).Since (X, τ) is an interval valued vague ε_r-Volterra space, $IVcl(\overset{N}{\underset{i=1}{\cap}} A_i) = 1$.Therefore, $IVcl(\overset{N}{\underset{i=1}{\cap}} A_i) = 1$ where A_i's (i=1 to N) are interval valued vague residual set in(X, τ).

Theorem 4.22: If $\overset{N}{\underset{i=1}{\cap}} A_i$, is an interval valued vague residual set in an interval valued vague baire space (X, τ), where A_i's (i=1 to N) are interval valued vague residual sets, then(X, τ) is an interval valued vague ε_r-Volterra space.

Proof: Let A_i's (i=1 to N) be interval valued vague dense and interval valued vague residual sets, then (X, τ). Then, by hypothesis, $\overset{N}{\underset{i=1}{\cap}} A_i$ is an interval valued vague residual set in (X, τ).Since (X, τ) is an interval valued vague baire space, therefore (by theorem 4.19) $IVcl(\overset{N}{\underset{i=1}{\cap}} A_i) = 1$. Hence $IVcl(\overset{N}{\underset{i=1}{\cap}} A_i) = 1$, where A_i's are interval valued vague dense set and interval valued vague residual set in (X, τ). Therefore (X, τ) is an interval valued vague ε_r-Volterra space.

References:.

[1] Borumandsaeid. A and Zarandi. A., Vague set theory applied to BM- Algebras. International journal of algebra, 5, 5 (2011), 207-222.

[2] Chang. C. L., Fuzzy topological spacea, J. Math. Anal. Appl. 24(1968) 182-190.

[3] Gau. W. L, Buehrer. D. J., Vague sets, IEEE Trans, Systems Man and Cybernet, 23 (2) (1993), 610-614.

[4] Hye Mi Choi, Gil Seong mun and Jeong yong ahn, A medical diagnosis based on interval valued fuzzy sets, Biomedical engineering: Applications, Basis and communication, 24(2012), 349-354.

[5] Mariapresenti. L, Arockiarani. I.,2016.Vague generalized alpha closed sets in topological spaces, International journal of mathematical archive,7(8), 23-29

[6] Milan matejdes., Generalized Volterra spaces, Int. J. Pure Appl. Math. 85(5) (2013), 955-963.

[7] Milan matejdes., Characterization of Volterra spaces, Int. J. Pure Appl. Math. 88(3) (2013), 437-442.

[8] Sharma. M. K, Similarity measure using interval valued vague sets in multiple criteria decision making, IOSR journal of mathematics(IOSR-JM), 11(2015), 46-55.

[9] Tapas kumar mondal and Samanta. S. K., Topology of interval valued fuzzy sets, Indian J. pure appl. Math. 30(1) (1999), 23-38.

[10] Thangaraj. G and Soundararajan. S., A note on fuzzy Volterra spaces, Annals of Fuzzy Mathematics and Informatics,8(4) (2014), 505-510.

[11] Zadeh. L. A, Fuzzy sets, Information and control, 8 (1965), 338-353.

[12] Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning, Part 1, Infor. Sci. 8(1975), 199-249.

[13] Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning, Part 2, Infor. Sci. 8(1975), 301-357.

[14] Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning, Part 3, Infor. Sci. 9(1975), 43-80.