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Level Subset of Bipolar Valued Fuzzy Subsemirings of a Semiring

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ABSTRACT

In this paper, we study some of the properties of (α,β)-level subsets of bipolar valued fuzzy subsemiring and prove some results on these.

Introduction

In 1965, Zadeh [15] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc [7]. Lee [9] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [9,10]. Anitha.M.S., Muruganantha Prasad & K.Arjunan[1] defined as bipolar valued fuzzy subgroups of a group. We introduce the concept of (α, β)-level subsets of bipolar valued fuzzy subsemirings of a semiring are discussed. Using these concepts, some results are established.

1. PRELIMINARIES:

1.1 Definition:

A bipolar valued fuzzy set (BVFS) An X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) >/ x\in X\}$, where $A^+ : X\to [0, 1]$ and $A^- : X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.2 Example:

$A = \{ < a, 0.5, -0.3 >, < b, 0.1, -0.7 >, < c, 0.5, -0.4 >\}$ is a bipolar valued fuzzy subset of $X = \{a, b, c \}$.

1.3 Definition:

Let R be a semiring. A bipolar valued fuzzy subset A of R is said to be a bipolar valued fuzzy subsemiring of R (BVFSSR) if the following conditions are satisfied,

(i) $A^+(x+y) \geq \min\{ A^+(x), A^+(y) \}$

(ii) $A^+(xy) \geq \min\{ A^+(x), A^+(y) \}$

(iii) $A^-(x+y) \leq \max\{ A^-(x), A^-(y) \}$

(iv) $A^-(xy) \leq \max\{ A^-(x), A^-(y) \}$ for all x and y in R.

1.4 Example:

Let $R = Z_3 = \{ 0, 1, 2 \}$ be a semiring with respect to the ordinary addition and multiplication. Then $A = \{ < 0, 0.5, -0.6 >, < 1, 0.4, -0.5 >, < 2, 0.4, -0.5 > \}$ is a bipolar valued fuzzy subsemiring of R.

1.5 Definition:

Let $A = \langle A^+, A^- \rangle$ be a bipolar valued fuzzy subset of X. For α in [0, 1] and β in [−1, 0], the (α, β)-level subset of A is the set $A_{(\alpha,\beta)} = \{ x\in X: A^+(x) \geq \alpha \text{ and } A^-(x) \leq \beta \}$.

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1.6 Example:

Consider the set X = { 0, 1, 2, 3, 4 }.

Let A = { (0, 0.5, –0.1), (1, 0.4, –0.3), (2, 0.6, –0.05), (3, 0.45, –0.2), (4, 0.2, –0.5) } be a bipolar valued fuzzy subset of X and $\alpha = 0.4$, $\beta = -0.1$. Then (0.4, –0.1)-level subset of A is $A_{(0.4,\,-0.1)}$ = { 0, 1, 3 }.

1.7 Definition:

Let A = $\langle A^+, A^- \rangle$ be a bipolar valued fuzzy subset of X. For α in [0, 1], the A^+-level α-cut of A is the set $P(A^+, \alpha) = \{ x\in X: A^+(x) \geq \alpha \}$.

1.8 Example:

Consider the set X = { 0, 1, 2, 3, 4 }.

Let A = { (0, 0.5, –0.1), (1, 0.4, –0.3), (2, 0.6, –0.05), (3, 0.45, –0.2), (4, 0.2, –0.5) } be a bipolar valued fuzzy subset of X and $\alpha = 0.4$. Then A^+-level 0.4-cut of A is $P(A^+, 0.4)$ = { 0, 1, 2, 3 }.

1.9 Definition:

Let A = $\langle A^+, A^- \rangle$ be a bipolar valued fuzzy subset of X. For β in [–1, 0], the A^--level β-cut of A is the set $N(A^-, \beta) = \{ x\in X: A^-(x) \leq \beta \}$.

1.10 Example:

Consider the set X = { 0, 1, 2, 3, 4 }.

Let A = { (0, 0.5, –0.1), (1, 0.4, –0.3), (2, 0.6, –0.05), (3, 0.45, –0.2), (4, 0.2, –0.5) } be a bipolar valued fuzzy subset of X and $\beta = -0.1$. Then A^--level –0.1-cut of A is $N(A^-, -0.1)$ = { 0, 1, 3, 4 }.

1.11 Definition:

Let A = $\langle A^+, A^- \rangle$ be a bipolar valued fuzzy subset of X and α in [0, 1– sup { $A^+(x)$}], β in [–1– inf { $A^-(x)$}, 0]. Then T = $\langle T^+, T^- \rangle$ is called a bipolar valued fuzzy translation of A if $T^+(x) = T^{+A}{}_{\alpha_i}(x) = A^+(x)+ \alpha$, $T^-(x) = T_{\beta}^{-A}(x) = A^-(x) + \beta$, for all x in X.

1.12 Example:

Consider the set X = { 0, 1, 2, 3, 4 }.

Let A = { (0, 0.5, –0.1), (1, 0.4, –0.3), (2, 0.6, –0.05), (3, 0.45, –0.2), (4, 0.2, –0.5) } be a bipolar valued fuzzy subset of X and $\alpha = 0.1$, $\beta = -0.1$. Then the bipolar valued fuzzy translation of A is T = $T^A{}_{(0.1,\, -0.1)}$ = { (0, 0.6, –0.2), (1, 0.5, –0.4), (2, 0.7, –0.15), (3, 0.55, –0.3), (4, 0.3, –0.6) }.

1.13 Definition:

Let X and X^I be any two sets. Let $f : X\rightarrow X^I$ be any function and let A be a bipolar valued fuzzy subset in X, V be a bipolar valued fuzzy subset in $f(X) = X^I$, defined by $V^+(y) = \sup\limits_{x\in f^{-1}(y)} A^+(x)$ and $V^-(y) = \inf\limits_{x\in f^{-1}(y)} A^-(x)$, for all x in X and y in X^I. A is called a preimage of V under f and is defined as $A^+(x) = V^+(f(x))$, $A^-(x) = V^-(f(x))$ for all x in X and is denoted by $f^{-1}(V)$.

2. Properties:

2.1 Theorem: Let R and R^I be any two semirings. The homomorphic image of a bipolar valued fuzzy subsemiring of R is a bipolar valued fuzzy subsemiring of R^I.

2.2 Theorem: Let R and R^I be any two semirings. The homomorphic preimage of a bipolar valued fuzzy subsemiring of R^I is a bipolar valued fuzzy subsemiring of R.

2.3 Theorem: Let R and R^I be any two semirings. The antihomomorphic image of a bipolar valued fuzzy subsemiring of R is a bipolar valued fuzzy subsemiring of R^I.

2.4 Theorem: Let R and R^I be any two semirings. The antihomomorphic preimage of a bipolar valued fuzzy subsemiring of R^I is a bipolar valued fuzzy subsemiring of R.

2.5 Theorem: Let A = $\langle A^+, A^- \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R. Then for α in [0, 1] and β in [–1, 0] such that $\alpha \leq A^+(e)$ and $\beta \geq A^-(e)$, $A_{(\alpha,\beta)}$ is a (α, β)-level subsemiring of R.

Proof: For all x and y in $A_{(\alpha,\beta)}$, we have, $A^+(x) \geq \alpha$ and $A^-(x) \leq \beta$ and $A^+(y) \geq \alpha$ and $A^-(y) \leq \beta$.

Now $A^+(x+y) \geq \min\{A^+(x), A^+(y) \} \geq \min \{\alpha, \alpha\}= \alpha$,

which implies that $A^+(x+y) \geq \alpha$.

And $A^+(xy) \geq \min\{A^+(x), A^+(y)\} \geq \min\{\alpha, \alpha\} = \alpha$,

which implies that $A^+(xy) \geq \alpha$.

Also $A^-(x+y) \leq \max \{ A^-(x), A^-(y) \} \leq \max \{ \beta, \beta \} = \beta$,

which implies that $A^-(x+y) \leq \beta$.

And $A^-(xy) \leq \max \{ A^-(x), A^-(y) \} \leq \max\{ \beta, \beta \} = \beta$,

which implies that $A^-(xy) \leq \beta$.

Therefore x+y, xy in $A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}$ is a (α, β))-level subsemiring of R.

2.6 Theorem: Let A = $\langle A^+, A^- \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R. Then for α, δ in [0, 1], β, ϕ in [–1, 0], $\alpha \leq A^+(e)$, $\delta \leq A^+(e)$, $\beta \geq A^-(e)$, $\phi \geq A^-(e)$, $\delta < \alpha$ and $\beta < \phi$, the two (α, β)-level subsemirings $A_{(\alpha,\beta)}$ and $A_{(\delta,\phi)}$ of A are equal if and only if there is no x in R such that $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$.

Proof: Assume that $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$.

Suppose there exists x in R such that $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$.

Then $A_{(\alpha,\beta)} \subseteq A_{(\delta,\phi)}$ implies x belongs to $A_{(\delta,\phi)}$, but not in $A_{(\alpha,\beta)}$.

This is contradicTon to $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$.

Therefore there is no x in R such that $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$.
Conversely, if there is no x in R such that $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$.
Then $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$ (By the definition of (α, β)-level subset).

2.7 Theorem: Let $A = \langle A^+, A^- \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R. If any two (α, β)-level subsemirings of A belongs to R, then their intersection is also (α, β)-level subsemiring of A in R.

Proof: Let α, δ in [0, 1], β, ϕ in [−1, 0], $\alpha \leq A^+(e)$, $\delta \leq A^+(e)$,
$\beta \geq A^-(e)$, $\phi \geq A^-(e)$.

Case (i): If $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\alpha,\beta)} \subseteq A_{(\delta,\phi)}$.

Therefore $A_{(\alpha,\beta)} \cap A_{(\delta,\phi)} = A_{(\alpha,\beta)}$, but $A_{(\alpha,\beta)}$ is a (α, β)-level subsemiring of A.

Case (ii): If $\alpha < A^+(x) < \delta$ and $\beta > A^-(x) > \phi$, then $A_{(\delta,\phi)} \subseteq A_{(\alpha,\beta)}$.

Therefore $A_{(\alpha,\beta)} \cap A_{(\delta,\phi)} = A_{(\delta,\phi)}$, but $A_{(\delta,\phi)}$ is a (α, β)-level subsemiring of A.

Case (iii): If $\alpha < A^+(x) < \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\delta,\beta)} \subseteq A_{(\alpha,\phi)}$.

Therefore $A_{(\delta,\beta)} \cap A_{(\alpha,\phi)} = A_{(\delta,\beta)}$, but $A_{(\delta,\beta)}$ is a (α, β)-level subsemiring of A.

Case (iv): If $\alpha > A^+(x) > \delta$ and $\beta > A^-(x) > \phi$, then $A_{(\alpha,\phi)} \subseteq A_{(\delta,\beta)}$.

Therefore $A_{(\alpha,\phi)} \cap A_{(\delta,\beta)} = A_{(\alpha,\phi)}$, but $A_{(\alpha,\phi)}$ is a (α, β)-level subsemiring of A.

Case (v): If $\alpha = \alpha$ and $\beta = \beta$, then $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$.

In other cases are true,
So, in all the cases, intersection of any two (α, β)-level subsemirings is a (α, β)-level subsemiring of A.

2.8 Theorem: Let $A = \langle A^+, A^- \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R. The intersection of a collection of (α, β)-level subsemirings of A is also a (α, β)-level subsemiring of A.

2.9 Theorem: Let $A = \langle A^+, A^- \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R. If any two (α, β)-level subsemirings of A belongs to R, then their union is also (α, β)-level subsemiring of A in R.

Proof: Let α, δ in [0, 1], β, ϕ in [−1, 0], $\alpha \leq A^+(e)$, $\delta \leq A^+(e)$,
$\beta \geq A^-(e)$, $\phi \geq A^-(e)$.

Case (i): If $\alpha > A^+(x) > \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\alpha,\beta)} \subseteq A_{(\delta,\phi)}$

Therefore $A_{(\alpha,\beta)} \cup A_{(\delta,\phi)} = A_{(\delta,\phi)}$, but $A_{(\delta,\phi)}$ is a (α, β)-level subsemiring of A.

Case (ii): If $\alpha < A^+(x) < \delta$ and $\beta > A^-(x) > \phi$, then $A_{(\delta,\phi)} \subseteq A_{(\alpha,\beta)}$.

Therefore $A_{(\alpha,\beta)} \cup A_{(\delta,\phi)} = A_{(\alpha,\beta)}$, but $A_{(\alpha,\beta)}$ is a (α, β)-level subsemiring of A.

Case (iii): If $\alpha < A^+(x) < \delta$ and $\beta < A^-(x) < \phi$, then $A_{(\delta,\beta)} \subseteq A_{(\alpha,\phi)}$.

Therefore $A_{(\delta,\beta)} \cup A_{(\alpha,\phi)} = A_{(\alpha,\phi)}$, but $A_{(\alpha,\phi)}$ is a (α, β)-level subsemiring of A.

Case (iv): If $\alpha > A^+(x) > \delta$ and $\beta > A^-(x) > \phi$, then $A_{(\alpha,\phi)} \subseteq A_{(\delta,\beta)}$.

Therefore $A_{(\alpha,\phi)} \cup A_{(\delta,\beta)} = A_{(\delta,\beta)}$, but $A_{(\delta,\beta)}$ is a (α, β)-level subsemiring of A.

Case (v): If $\alpha = \delta$ and $\beta = \phi$, then $A_{(\alpha,\beta)} = A_{(\delta,\phi)}$.

In other cases are true,
so, in all the cases, union of any two (α, β)-level subsemirings is a (α, β)-level subsemiring of A.

2.10 Theorem: Let $A = \langle A^+, A^- \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R. The union of a collection of (α, β)-level subsemirings of A is also a (α, β)-level subsemiring of A.

2.11 Theorem: The homomorphic image of a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R is a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R^I.

Proof: Let V = f(A). Here $A = \langle A^+, A^- \rangle$ is a bipolar valued fuzzy subsemiring of R. By Theorem 2.1, $V = \langle V^+, V^- \rangle$ is a bipolar valued fuzzy subsemiring of R^I. Let x and y in R. Then f(x) and f(y) in R^I.
Let $A_{(\alpha, \beta)}$ be a (α, β)-level subsemiring of A.
That is, $A^+(x) \geq \alpha$ and $A^-(x) \leq \beta$; $A^+(y) \geq \alpha$ and $A^-(y) \leq \beta$;
$A^+(x+y) \geq \alpha$, $A^-(x+y) \leq \beta$, $A^+(xy) \geq \alpha$, $A^-(xy) \leq \beta$.
We have to prove that $f(A_{(\alpha, \beta)})$ is a (α, β)-level subsemiring of V.
Now $V^+(f(x)) \geq A^+(x) \geq \alpha$ which implies that $V^+(f(x)) \geq \alpha$;
and $V^+(f(y)) \geq A^+(y) \geq \alpha$ which implies that $V^+(f(y)) \geq \alpha$.

Then $V^+(f(x)+f(y)) = V^+(f(x+y)) \geq A^+(x+y) \geq \alpha$,
which implies that $V^+(f(x)+f(y)) \geq \alpha$.
And $V^+(f(x)f(y)) = V^+(f(xy)) \geq A^+(xy) \geq \alpha$,
which implies that $V^+(f(x)f(y)) \geq \alpha$.
And $V^-(f(x)) \leq A^-(x) \leq \beta$ which implies that $V^-(f(x)) \leq \beta$;
and $V^-(f(y)) \leq A^-(y) \leq \beta$ which implies that $V^-(f(y)) \leq \beta$.
Then $V^-(f(x)+f(y)) = V^-(f(x+y)) \leq A^-(x+y) \leq \beta$,
which implies that $V^-(f(x)+f(y)) \leq \beta$.
And $V^-(f(x)f(y)) = V^-(f(xy)) \leq A^-(xy) \leq \beta$,
which implies that $V^-(f(x)f(y)) \leq \beta$.
Hence $f(A_{(\alpha, \beta)})$ is a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring V of R^l.

2.12 Theorem: The homomorphic pre-image of a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R^l is a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R.

Proof: Let V = f(A). Here $V = \langle V^+, V^- \rangle$ is a bipolar valued fuzzy subsemiring of R^l. By Theorem 2.2, $A = \langle A^+, A^- \rangle$ is a bipolar valued fuzzy subsemiring of R. Let f(x) and f(y) in R^l. Then x and y in R. Let $f(A_{(\alpha, \beta)})$ be a (α, β)-level subsemiring of V. That is $V^+(f(x)) \geq \alpha$ and $V^-(f(x)) \leq \beta$; $V^+(f(y)) \geq \alpha$ and $V^-(f(y)) \leq \beta$; $V^+(f(x)+f(y)) \geq \alpha$, $V^-(f(x)+f(y)) \leq \beta$, $V^+(f(x)f(y)) \geq \alpha$, $V^-(f(x)f(y)) \leq \beta$. We have to prove that $A_{(\alpha, \beta)}$ is a (α, β)-level subsemiring of A. Now $A^+(x) = V^+(f(x)) \geq \alpha$ implies that $A^+(x) \geq \alpha$;
$A^+(y) = V^+(f(y)) \geq \alpha$ implies that $A^+(y) \geq \alpha$.
Then $A^+(x+y) = V^+(f(x+y)) = V^+(f(x)+f(y)) \geq \alpha$,
which implies that $A^+(x+y) \geq \alpha$.
And $A^+(xy) = V^+(f(xy)) = V^+(f(x)f(y)) \geq \alpha$,
which implies that $A^+(xy) \geq \alpha$.
And $A^-(x) = V^-(f(x)) \leq \beta$ implies that $A^-(x) \leq \beta$;
$A^-(y) = V^-(f(y)) \leq \beta$ implies that $A^-(y) \leq \beta$.
Also $A^-(x+y) = V^-(f(x+y)) = V^-(f(x)+f(y)) \leq \beta$,
which implies that $A^-(x+y) \leq \beta$.
And $A^-(xy) = V^-(f(xy)) = V^-(f(x)f(y)) \leq \beta$,
which implies that $A^-(xy) \leq \beta$.
Hence $A_{(\alpha, \beta)}$ is a (α, β)-level subsemiring of bipolar valued fuzzy subsemiring A of R.

2.13 Theorem: The anti-homomorphic image of a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R is a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R^l.

Proof: Let V = f(A). Here $A = \langle A^+, A^- \rangle$ is a bipolar valued fuzzy subsemiring of R. By Theorem 2.3, $V = \langle V^+, V^- \rangle$ is a bipolar valued fuzzy subsemiring of R^l. Let x and y in R. Then f(x) and f(y) in R^l. Let $A_{(\alpha, \beta)}$ be a (α, β)-level subsemiring of A.
That is $A^+(x) \geq \alpha$ and $A^-(x) \leq \beta$; $A^+(y) \geq \alpha$ and $A^-(y) \leq \beta$.
And $A^+(y+x) \geq \alpha$ and $A^-(y+x) \leq \beta$, $A^+(yx) \geq \alpha$ and $A^-(yx) \leq \beta$.
We have to prove that $f(A_{(\alpha, \beta)})$ is a (α, β)-level subsemiring of V.
Now $V^+(f(x)) \geq A^+(x) \geq \alpha$ which implies that $V^+(f(x)) \geq \alpha$;
and $V^+(f(y)) \geq A^+(y) \geq \alpha$ which implies that $V^+(f(y)) \geq \alpha$.
Also $V^+(f(x)+f(y)) = V^+(f(y+x)) \geq A^+(y+x) \geq \alpha$,
which implies that $V^+(f(x)+f(y)) \geq \alpha$.
And $V^+(f(x)f(y)) = V^+(f(yx)) \geq A^+(yx) \geq \alpha$,
which implies that $V^+(f(x)f(y)) \geq \alpha$.
And $V^-(f(x)) \leq A^-(x) \leq \beta$ which implies that $V^-(f(x)) \leq \beta$
and $V^-(f(y)) \leq A^-(y) \leq \beta$ which implies that $V^-(f(y)) \leq \beta$.
Also $V^-(f(x)+f(y)) = V^-(f(y+x)) \leq A^-(y+ x) \leq \beta$,
which implies that $V^-(f(x)+f(y)) \leq \beta$.
And $V^-(f(x)f(y)) = V^-(f(yx)) \leq A^-(yx) \leq \beta$,
which implies that $V^-(f(x)f(y)) \leq \beta$.
Hence $f(A_{(\alpha, \beta)})$ is a (α, β)-level subsemiring of bipolar valued fuzzy subsemiring V of R^l.

2.14 Theorem: The anti-homomorphic pre-image of a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R^l is a (α, β)-level subsemiring of a bipolar valued fuzzy subsemiring of a semiring R.

Proof: Let V = f(A). Here $V = \langle V^+, V^- \rangle$ is a bipolar valued fuzzy subsemiring of R^l. By Theorem 2.4, $A = \langle A^+, A^- \rangle$ is a bipolar valued fuzzy subsemiring of R. Let f(x) and f(y) in R^l. Then x and y in R.
Let $f(A_{(\alpha, \beta)})$ be a (α, β)-level subsemiring of V.
That is $V^+(f(x)) \geq \alpha$ and $V^-(f(x)) \leq \beta$; $V^+(f(y)) \geq \alpha$ and $V^-(f(y)) \leq \beta$;
$V^+(f(y)+f(x)) \geq \alpha$, $V^-(f(y)+f(x)) \leq \beta$, $V^+(f(y)f(x)) \geq \alpha$, $V^-(f(y)f(x)) \leq \beta$.
We have to prove that $A_{(\alpha, \beta)}$ is a (α, β)-level subsemiring of A.
Now $A^+(x) = V^+(f(x)) \geq \alpha$, which implies that $A^+(x) \geq \alpha$;
and $A^+(y) = V^+(f(y)) \geq \alpha$, which implies that $A^+(y) \geq \alpha$.
Then $A^+(x+y) = V^+(f(x+y)) = V^+(f(y)+f(x)) \geq \alpha$,

which implies that $A^+(x+y) \geq \alpha$.
And $A^+(xy) = V^+(f(xy)) = V^+(f(y)f(x)) \geq \alpha$,
which implies that $A^+(xy) \geq \alpha$.
And $A^-(x) = V^-(f(x)) \leq \beta$ which implies that $A^-(x) \leq \beta$
and $A^-(y) = V^-(f(y)) \leq \beta$ which implies that $A^-(y) \leq \beta$.
Also $A^-(x+y) = V^-(f(x+y)) = V^-(f(y)+f(x)) \leq \beta$,
which implies that $A^-(x+y) \leq \beta$.
And $A^-(xy) = V^-(f(xy)) = V^-(f(y)f(x)) \leq \beta$,
which implies that $A^-(xy) \leq \beta$.
Hence $A_{(\alpha, \beta)}$ is a (α, β)-level subsemiring of bipolar valued fuzzy subsemiring A of R.

2.15 Theorem: Let A be a bipolar valued fuzzy subsemiring of a semiring R. Then for α in [0, 1], A^+-level α-cut $P(A^+, \alpha)$ is a A^+-level α-cut subsemiring of R.

Proof: For all x and y in $P(A^+, \alpha)$, we have $A^+(x) \geq \alpha$ and $A^+(y) \geq \alpha$.
Now $A^+(x+y) \geq \min \{ A^+(x), A^+(y) \} \geq \min \{ \alpha, \alpha \} = \alpha$,
which implies that $A^+(x+y) \geq \alpha$.
And $A^+(xy) \geq \min \{ A^+(x), A^+(y) \} \geq \min \{ \alpha, \alpha \} = \alpha$,
which implies that $A^+(xy) \geq \alpha$.
Therefore x+y, xy in $P(A^+, \alpha)$.
Hence $P(A^+, \alpha)$ is a A^+-level α-cut subsemiring of R.

2.16 Theorem: Let A be a bipolar valued fuzzy subsemiring of a semiring R. Then for β in $[-1, 0]$, A^--level β-cut $N(A^-, \beta)$ is a A^--level β-cut subsemiring of R.

Proof: For all x and y in $N(A^-, \beta)$, we have $A^-(x) \leq \beta$ and $A^-(y) \leq \beta$.
Now $A^-(x+y) \leq \max \{ A^-(x), A^-(y) \} \leq \max \{ \beta, \beta \} = \beta$,
which implies that $A^-(x+y) \leq \beta$.
And $A^-(xy) \leq \max \{ A^-(x), A^-(y) \} \leq \max \{ \beta, \beta \} = \beta$,
which implies that $A^-(xy) \leq \beta$.
Therefore x+y, xy in $N(A^-, \beta)$.
Hence $N(A^-, \beta)$ is a A^--level β-cut subsemiring of R.

3 - Propertes of Bipolar Valued Fuzzy Translatons:

3.1 Theorem: If M and N are two bipolar valued fuzzy translations of bipolar valued fuzzy subsemiring A of a semiring R, then their intersection M□N is also a bipolar valued fuzzy translation of A

Proof: Let x and y belong to R.
Let $M = T^A_{(\alpha,\beta)} = \{ \langle x, A^+(x) + \alpha, A^-(x) + \beta \rangle / x \in R \}$ and
$N = T^A_{(\gamma,\delta)} = \{ \langle x, A^+(x) + \gamma, A^-(x) + \delta \rangle / x \in R \}$ be two bipolar valued fuzzy translations of bipolar valued fuzzy subsemiring $A = \langle A^+, A^- \rangle$ of R.
Let $C = M \cap N$ and $C = \{ \langle x, C^+(x), C^-(x) \rangle / x \in R \}$, where $C^+(x) = \min \{ A^+(x) + \alpha, A^+(x) + \gamma \}$ and $C^-(x) = \max \{ A^-(x) + \beta, A^-(x)+\delta \}$.

Case (i): $\alpha \leq \gamma$ and $\beta \leq \delta$.
Now $C^+(x) = \min\{M^+(x), N^+(x)\}$
$= \min\{A^+(x)+\alpha, A^+(x)+\gamma\} = A^+(x)+\alpha = M^+(x)$ for all x in R.
And $C^-(x) = \max \{ M^-(x), N^-(x) \}$
$= \max \{ A^-(x) + \beta, A^-(x) + \delta \} = A^-(x) + \delta = N^-(x)$ for all x in R.
Therefore $C = T^A_{(\alpha,\delta)} = \{ \langle x, A^+(x) + \alpha, A^-(x)+ \delta \rangle / x \in R \}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subsemiring A of R.

Case (ii): $\alpha \geq \gamma$ and $\beta \geq \delta$.
Now $C^+(x) = \min \{ M^+(x), N^+(x) \}$
$= \min \{ A^+(x) + \alpha, A^+(x)+ \gamma \} = A^+(x) + \gamma = N^+(x)$ for all x in R.
And $C^-(x) = \max \{ M^-(x), N^-(x) \}$
$= \max \{ A^-(x) + \beta, A^-(x) + \delta \} = A^-(x) + \beta = M^-(x)$ for all x in R.
Therefore $C = T^A_{(\gamma,\beta)} = \{ \langle x, A^+(x)+ \gamma, A^-(x)+ \beta \rangle / x \in R \}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subsemiring A of R.

Case (iii): $\alpha \leq \gamma$ and $\beta \geq \delta$.
Clearly $C = T^A_{(\alpha,\beta)} = \{ \langle x, A^+(x) + \alpha, A^-(x)+\beta \rangle / x \in R\}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subsemiring A of R.

Case (iv): $\alpha \geq \gamma$ and $\beta \leq \delta$.
Clearly $C = T^A_{(\gamma,\delta)} = \{ \langle x, A^+(x)+ \gamma, A^-(x)+ \delta \rangle / x \in R \}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subs miring A of R.

In other cases are true, so in all the cases, the intersection of any two bipolar valued fuzzy translations of bipolar valued fuzzy subsemiring A of R is a bipolar valued fuzzy translation of A.

3.2 Theorem: The intersection of a family of bipolar valued fuzzy translations of bipolar valued fuzzy subsemiring A of a semiring R is a bipolar valued fuzzy translation of A.

Proof: It is trivial.

3.3 Theorem: Union of any two bipolar valued fuzzy translations of bipolar valued fuzzy subsemiring A of a semiring R is a bipolar valued fuzzy translation of A.

Proof: Let x and y belong to R.

Let M = $T^{A}_{(\alpha,\beta)}$ = { $\langle$ x, $A^+(x) + \alpha$, $A^-(x) + \beta$ $\rangle$ / x$\in$R } and

N = $T^{A}_{(\gamma,\delta)}$ = { $\langle$ x, $A^+(x) + \gamma$, $A^-(x) + \delta$ $\rangle$ / x$\in$R } be two bipolar valued fuzzy translations of bipolar valued fuzzy subs miring A = $\langle A^+, A^- \rangle$ of R.

Let C = M$\cup$N and C = { $\langle$ x, $C^+(x)$, $C^-(x)$ $\rangle$ / x$\in$R }, where $C^+(x)$ = max { $A^+(x) + \alpha$, $A^+(x) + \gamma$ } and $C^-(x)$ = Min { $A^-(x)+ \beta$, $A^-(x)+ \delta$ }.

Case (i): $\alpha \le \gamma$ and $\beta \le \delta$.

Now $C^+(x)$ = max { $M^+(x)$, $N^+(x)$ }

= max{$A^+(x)+\alpha$, $A^+(x)+ \gamma$}= $A^+(x)+ \gamma = N^+(x)$ for all x and y in R.

And $C^-(x)$ = Min { $M^-(x)$, $N^-(x)$ }

= Min { $A^-(x)+ \beta$, $A^-(x)+ \delta$} = $A^-(x)+ \beta = M^-(x)$ for all x in R.

Therefore C = $T^{A}_{(\gamma,\beta)}$ = { $\langle$ x, $A^+(x) + \gamma$, $A^-(x)+ \beta$ $\rangle$ / x$\in$R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subsemiring A of R.

Case (ii): $\alpha \ge \gamma$ and $\beta \ge \delta$.

Now $C^+(x)$ = max { $M^+(x)$, $N^+(x)$ }

= max { $A^+(x) + \alpha$, $A^+(x)+ \gamma$}= $A^+(x) + \alpha = M^+(x)$ for all x in R.

And $C^-(x)$ = min{ $M^-(x)$, $N^-(x)$ }

= min {$A^-(x)+\beta$, $A^-(x)+ \delta$} = $A^-(x)+ \delta = N^-(x)$ for all x in R.

Therefore C = $T^{A}_{(\alpha,\delta)}$ = { $\langle$ x, $A^+(x)+ \alpha$, $A^-(x)+ \delta$ $\rangle$ / x$\in$R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subs miring A of R.

Case (iii): $\alpha \le \gamma$ and $\beta \ge \delta$.

Clearly C = $T^{A}_{(\gamma,\delta)}$ = { $\langle$ x, $A^+(x) + \gamma$, $A^-(x)+\delta$ $\rangle$ / x$\in$R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subs miring A of R.

Case (iv): $\alpha \ge \gamma$ and $\beta \le \delta$.

Clearly C = $T^{A}_{(\alpha,\beta)}$ = { $\langle$ x, $A^+(x) + \alpha$, $A^-(x)+\beta$$\rangle$ / x$\in$R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subs miring A of R.

In other cases are true, so in all the cases, union of any two bipolar valued fuzzy translations of bipolar valued fuzzy subs miring A of R is a bipolar valued fuzzy translation of A.

3.4 Theorem: The union of a family of bipolar valued fuzzy translations of bipolar valued fuzzy subsemiring A of a semiring R is a bipolar valued fuzzy translation of A.

Proof: It is trivial.

3.5 Theorem: A bipolar valued fuzzy translation of a bipolar valued fuzzy subsemiring A of a semiring R is a bipolar valued fuzzy subs miring of R.

Proof: Assume that T = $\langle T^+, T^- \rangle$ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subsemiring A = $\langle A^+, A^- \rangle$ of a semiring R.

Let x and y in R.

We have $T^+(x+y) = A^+(x+y) + \alpha \ge$ min{ $A^+(x)$, $A^+(y)$ } + α

= min { $A^+(x) + \alpha$, $A^+(y) + \alpha$ } = min { $T^+(x)$, $T^+(y)$ }.

Therefore $T^+(x+y) \ge$ min { $T^+(x)$, $T^+(y)$ } for all x and y in R.

And $T^+(xy) = A^+(xy) + \alpha \ge$ min{ $A^+(x)$, $A^+(y)$ } + α

= min { $A^+(x) + \alpha$, $A^+(y) + \alpha$ }= min { $T^+(x)$, $T^+(y)$ }.

Therefore $T^+(xy) \ge$ min { $T^+(x)$, $T^+(y)$ } for all x and y in R.

Also $T^-(x+y) = A^-(x+y) +\beta \le$ max{ $A^-(x)$, $A^-(y)$ }+ β

= max { $A^-(x) + \beta$, $A^-(y) + \beta$ } = max { $T^-(x)$, $T^-(y)$ }.

Therefore $T^-(x+y) \le$ max{ $T^-(x)$, $T^-(y)$ } for all x and y in R.

And $T^-(xy) = A^-(xy) +\beta \le$ max{ $A^-(x)$, $A^-(y)$ }+ β

= max { $A^-(x) + \beta$, $A^-(y) + \beta$ }= max { $T^-(x)$, $T^-(y)$ }.

Therefore $T^-(xy) \le$ max{ $T^-(x)$, $T^-(y)$ } for all x and y in R.

Hence T is a bipolar valued fuzzy subsemiring of R.

3.6 Theorem: Let (R, +, .) and ($R^{|}$, +, .) be any two semirings and f be a homomorphism. Then the homomorphic image of a bipolar valued fuzzy translation of a bipolar valued fuzzy subsemiring A of R is also a bipolar valued fuzzy subsemiring of $R^{|}$.

Proof: Let V = (V^+, V^-) = f($T^{A}_{(\alpha,\beta)}$), where $T^{A}_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subs miring A = (A^+, A^-) of R.

We have to prove that V is a bipolar valued fuzzy subsemiring of R^{l}.

For all f(x) and f(y) in R^{l},

we have V^+[f(x)+f(y)] = V^+ [f(x+y)] ≥ $T^{+A}{}_{\alpha}$ (x+y)

= A^+(x+y) + α ≥ min{ A^+(x), A^+(y) } + α

= min{A^+(x) +α, A^+(y) +α }= min { $T^{+A}{}_{\alpha}$ (x), $T^{+A}{}_{\alpha}$ (y) }

which implies that V^+[f(x)+f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^{l}.

And V^+[f(x)f(y)] = V^+ [f(xy)] ≥ $T^{+A}{}_{\alpha}$ (xy)

= A^+(xy) + α ≥ min{ A^+(x), A^+(y) } + α

= min { A^+(x) +α, A^+(y) +α }= min { $T^{+A}{}_{\alpha}$ (x), $T^{+A}{}_{\alpha}$ (y) }

which implies that V^+[f(x)f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^{l}.

Also V^-[f(x)+f(y)] = V^-[f(x+y)] ≤ T^{-A}_{β}(x+y)

= A^-(x+y) + β ≤ max { A^-(x), A^-(y) }+ β

= max { A^-(x)+ β, A^-(y)+ β} = max { T^{-A}_{β} (x), T^{-A}_{β} (y) }

which implies that V^-[f(x)+f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^{l}.

And V^-[f(x)f(y)] = V^-[f(xy)] ≤ T^{-A}_{β} (xy)

= A^-(xy) + β ≤ max { A^-(x), A^-(y) }+ β

= max { A^-(x)+ β, A^-(y)+ β} = max { T^{-A}_{β} (x), T^{-A}_{β} (y) }

which implies that V^-[f(x)f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^{l}. Therefore V is a bipolar valued fuzzy subsemiring of R^{l}.

3.7 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two semirings and f be a homomorphism. Then the homomorphic pre-image of bipolar valued fuzzy translation of a bipolar valued fuzzy subsemiring V of R^{l} is a bipolar valued fuzzy subsemiring of R.

Proof: Let T = $T^{V}_{(\alpha,\beta)}$ = f(A), where $T^{V}_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subs miring V = (V^+, V^-) of R^{l}. We have to prove that A = (A^+, A^-) is a bipolar valued fuzzy subsemiring of R.

Let x and y in R.

Then A^+(x+y) = T^{+V}_{α} (f(x+y)) = T^{+V}_{α} (f(x)+f(y))

= V^+[f(x)+f(y)] + α ≥ min { V^+(f(x)), V^+(f(y)) }+ α

= min { V^+(f(x)) + α, V^+(f(y)) + α }

= min { T^{+V}_{α} (f(x)), T^{+V}_{α} (f(y)) } = min { A^+(x), A^+(y) }

which implies that A^+(x+y) ≥ min { A^+(x), A^+(y) } for all x and y in R.

And A^+(xy) = T^{+V}_{α} (f(xy)) = T^{+V}_{α} (f(x)f(y))

= V^+[f(x)f(y)] + α ≥ min { V^+(f(x)), V^+(f(y)) }+ α

= min { V^+(f(x)) + α, V^+(f(y)) + α }

= min { T^{+V}_{α} (f(x)), T^{+V}_{α} (f(y)) } = min { A^+(x), A^+(y) }

which implies that A^+(xy) ≥ Min { A^+(x), A^+(y) } for all x and y in R.

Also A^-(x+y) = T^{-V}_{β} (f(x+y)) = T^{-V}_{β} (f(x)+f(y))

= V^-[f(x)+f(y)] +β ≤ max { V^-(f(x)), V^-(f(y)) } + β

= max { V^-(f(x)) + β, V^-(f(y)) + β }

= max { T^{-V}_{β} (f(x)), T^{-V}_{β} (f(y)) } = max { A^-(x), A^-(y) }

which implies A^-(x+y) ≤ max{ A^-(x), A^-(y) } for all x and y in R.

And A^-(xy) = T^{-V}_{β} (f(xy)) = T^{-V}_{β} (f(x)f(y))

= V^-[f(x)f(y)] +β ≤ max { V^-(f(x)), V^-(f(y)) } + β

= max { V^-(f(x)) + β, V^-(f(y)) + β }

= max { T^{-V}_{β} (f(x)), T^{-V}_{β} (f(y)) } = max { A^-(x), A^-(y) }

which implies A^-(xy) ≤ max{ A^-(x), A^-(y) } for all x and y in R.

Therefore A is a bipolar valued fuzzy subsemiring of R.

3.8 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two semirings and f be a anti-homomorphism. Then the anti-homomorphic image of a bipolar valued fuzzy translation of a bipolar valued fuzzy subsemiring A of R is also a bipolar valued fuzzy subsemiring of R^{l}.

Proof: Let V = (V^+, V^-) = f($T^A_{(\alpha,\beta)}$), where $T^A_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subsemiring A = (A^+, A^-) of R.
We have to prove that V is a bipolar valued fuzzy subsemiring of R^l.
For all f(x) and f(y) in R^l,

we have $V^+[f(x)+f(y)] = V^+ [f(y+x)] \geq T^{+A}{}_{\alpha}(y+x)$

$= A^+(y+x) + \alpha \geq \min\{ A^+(y), A^+(x) \} + \alpha$

$= \min \{ A^+(x) +\alpha, A^+(y) +\alpha \} = \min \{ T^{+A}{}_{\alpha}(x), T^{+A}{}_{\alpha}(y) \}$

which implies that $V^+[f(x)+f(y)] \geq \min \{ V^+(f(x)), V^+(f(y)) \}$ for all f(x) and f(y) in R^l.

And $V^+[f(x)f(y)] = V^+ [f(yx)] \geq T^{+A}{}_{\alpha}(yx)$

$= A^+(yx) + \alpha \geq \min\{ A^+(y), A^+(x) \} + \alpha$

$= \min \{ A^+(x) +\alpha, A^+(y) +\alpha \} = \min \{ T^{+A}{}_{\alpha}(x), T^{+A}{}_{\alpha}(y) \}$

which implies that $V^+[f(x)f(y)] \geq \min \{ V^+(f(x)), V^+(f(y)) \}$ for all f(x) and f(y) in R^l.

Also $V^-[f(x)+f(y)] = V^-[f(y+x)] \leq T_{\beta}^{-A}(y+x)$

$= A^-(y+x) + \beta \leq \max \{ A^-(y), A^-(x) \}+ \beta$

$= \max \{ A^-(x)+ \beta, A^-(y)+ \beta\}= \max \{ T_{\beta}^{-A}(x), T_{\beta}^{-A}(y) \}$

which implies that $V^-[f(x)+f(y)] \leq \max \{ V^-(f(x)), V^-(f(y)) \}$ for all f(x) and f(y) in R^l.

And $V^-[f(x)f(y)] = V^-[f(yx)] \leq T_{\beta}^{-A}(yx)$

$= A^-(yx) + \beta \leq \max \{ A^-(x), A^-(y) \} + \beta$

$= \max \{ A^-(x) + \beta, A^-(y) + \beta\}= \max \{ T_{\beta}^{-A}(x), T_{\beta}^{-A}(y) \}$

which implies that $V^-[f(x)f(y)] \leq \max \{ V^-(f(x)), V^-(f(y)) \}$ for all f(x) and f(y) in R^l. Therefore V is a bipolar valued fuzzy subsemiring of R^l.

3.9 Theorem: Let (R, +,.) and (R^l, +, .) be any two semirings and f be an anti-homomorphism. Then the anti-homomorphic pre-image of bipolar valued fuzzy translation of a bipolar valued fuzzy subsemiring V of R^l is a bipolar valued fuzzy subsemiring of R.

Proof: Let T = $T^V_{(\alpha,\beta)}$ = f(A), where $T^V_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subs miring V = (V^+, V^-) of R^l. We have to prove that A = (A^+, A^-) is a bipolar valued fuzzy subsemiring of R.
Let x and y in R.

Then $A^+(x+y) = T_{\alpha}^{+V}(f(x+y)) = T_{\alpha}^{+V}(f(y)+f(x))$

$= V^+[f(y)+f(x)] + \alpha \geq \min \{ V^+(f(y)), V^+(f(x)) \}+ \alpha$

$= \min \{ V^+(f(x)) + \alpha, V^+(f(y)) + \alpha \}$

$= \min\{ T_{\alpha}^{+V}(f(x)), T_{\alpha}^{+V}(f(y))\} = \min\{A^+(x), A^+(y) \}$

which implies that $A^+(x+y) \geq \min \{ A^+(x), A^+(y) \}$ for all x and y in R.

And $A^+(xy) = T_{\alpha}^{+V}(f(xy)) = T_{\alpha}^{+V}(f(y)f(x))$

$= V^+[f(y)f(x)] + \alpha \geq \min \{ V^+(f(y)), V^+(f(x)) \}+ \alpha$

$= \min \{ V^+(f(x)) + \alpha, V^+(f(y))+ \alpha \}$

$= \min \{ T_{\alpha}^{+V}(f(x)), T_{\alpha}^{+V}(f(y)) \} = \min \{ A^+(x), A^+(y) \}$

which implies that $A^+(xy) \geq \min \{ A^+(x), A^+(y) \}$ for all x and y in R.

Also $A^-(x+y) = T_{\beta}^{-V}(f(x+y)) = T_{\beta}^{-V}(f(y)+f(x))$

$= V^-[f(y)+f(x)] +\beta \leq \max \{ V^-(f(y)), V^-(f(x)) \} + \beta$

$= \max \{ V^-(f(x)) + \beta, V^-(f(y)) + \beta \}$

$= \max \{ T_{\beta}^{-V}(f(x)), T_{\beta}^{-V}(f(y)) \} = \max \{ A^-(x), A^-(y) \}$

which implies $A^-(x+y) \leq \max\{ A^-(x), A^-(y) \}$ for all x and y in R.

And $A^-(xy) = T_{\beta}^{-V}(f(xy)) = T_{\beta}^{-V}(f(y)f(x))$

$= V^-[f(y)f(x)] +\beta \leq \max \{ V^-(f(y)), V^-(f(x)) \} + \beta$

$= \max \{ V^-(f(x)) + \beta, V^-(f(y)) + \beta \}$

$= \max \{ T_{\beta}^{-V}(f(x)), T_{\beta}^{-V}(f(y)) \} = \max \{ A^-(x), A^-(y) \}$

which implies $A^-(xy) \leq \max\{ A^-(x), A^-(y) \}$ for all x and y in R.
Therefore A is a bipolar valued fuzzy subsemiring of R.

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