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Solution of Telegraph Equation by Using Double Aboodh Transform

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ABSTRACT

In this paper, we apply Double Aboodh transform to solve the general linear telegraph equation. The applicability of this new transform is demonstrated using some functions.

Introduction

Partial differential equations are very important in mathematical physic [1], the wave equation is known as one of the fundamental equations in mathematical physics is occur in many branches of physics, for example, in applied mathematics and engineering.

A lot of problems have been solved by integral transforms such as Laplace [1], Fourier, Mellin, and Sumudu [2, 3]. Also these problems have been solved by differential transform method [4-11].

In this paper we derive, we believe for the first time and solve telegraph and wave equations by using Double Aboodh transform.

We write that Aboodh transform defined for function of exponential order.We consider functions in the set A, defined by:

$$A = \{f(t): \exists M, k_1, k_2 > 0\,, |f(t)| < Me^{-vt}\} \qquad (1)$$

By

$$A[f(t)] = K(v) = \frac{1}{v}\int_0^\infty f(t)\ e^{-vt} dt\,, t \geq 0\,, k_1 \leq v \leq k_2 \qquad (2)$$

1.1 Double Aboodh Transform

Definition

Let $f(x,t)$, where $t, x \in R^+$ be a function, which can be expressed as a convergent infinite series then, its double Aboodh transform given by:

$$A_2[f(x,t), u, v] = K(u,v) = \frac{1}{uv}\int_0^\infty \int_0^\infty f(x,t)e^{-(ux+vt)} dx\, dt \quad , \quad x, t > 0$$

Where, u & v are complex values to find the solution of telegraph and wave equations by double Aboodh transform, first we must find double Aboodh transform for first and second order partial derivatives are in the form.

$$(i) A_2\left\{\frac{\partial f}{\partial x}\right\} = uK(u,v) - \frac{1}{u} K(0,v).$$

$$(ii) A_2\left\{\frac{\partial^2 f}{\partial x^2}\right\} = u^2 K(u,v) - K(0,v) - \frac{1}{u}\frac{\partial[K(0,v)]}{\partial x}.$$

$$(iii) A_2\left\{\frac{\partial f}{\partial t}\right\} = vK(u,v) - \frac{1}{v} K(u,0).$$

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$$(iv)A_2\left\{\frac{\partial^2 f}{\partial t^2}\right\} = v^2K(u,v) - K(u,0) - \frac{1}{v}\frac{\partial[K(u,0)]}{\partial t}.$$

$$(v)A_2\left\{\frac{\partial^2 f}{\partial x\partial t}\right\} = \frac{1}{uv}f(0,0) - \frac{u}{v}K(u,0) + uvK(u,v) - \frac{v}{u}K(0,v).$$

2. Applications

In this section we establish the validity of the double Aboodh transform by applying it to solve the general linear telegraph equations. To solve partial differential equations by double Aboodh transform, we need the following steps.

(i) Take the double Aboodh transform of partial differential equations.

(ii) Take the single Aboodh transform of the conditions.

(iii) Substitute (ii) in (i) and solve the algebraic equation.

(iv) Take the double inverse of Aboodh transform to get the solution

Here we need the main equation:

$$A_2(e^{ax+bt}) = \frac{1}{uv(u-a)(v-b)}$$

Example 2.1

Consider the telegraph equation

$$U_{xx} = U_{tt} + U_t + U \qquad (2-1)$$

With the boundary conditions

$$U(0,t) = e^{-t}, \qquad U_x(0,t) = e^{-t} \qquad (2-2)$$

And the initial conditions

$$U(x,0) = e^{x}, \qquad U_t(x,0) = -e^{x} \qquad (2-3)$$

The exact solution is $U(x,t) = e^{x-t}$

Solution

Take the double Aboodh transform of equation $(2-1)$, and single Aboodh transform of conditions $(2-2)$, $(2-3)$, and then we have:

$$u^2K(u,v) - K(0,v) - \frac{1}{u}\frac{\partial[K(0,v)]}{\partial x} = v^2K(u,v) - K(u,0) - \frac{1}{v}\frac{\partial[K(u,0)]}{\partial t} + vK(u,v) - \frac{1}{v}K(u,0) + K(u,v) \qquad (2-4)$$

And

$$K(0,v) = \frac{1}{v(v+1)}, \qquad \frac{\partial[K(0,v)]}{\partial x} = \frac{1}{v(v+1)} \qquad (2-5)$$

$$K(u,0) = \frac{1}{u(u-1)}, \qquad \frac{\partial[K(u,0)]}{\partial t} = \frac{-1}{u(u-1)} \qquad (2-6)$$

Substituting $(2-5)$ and $(2-6)$ in $(2-4)$ we obtain

$$u^2K(u,v) - \frac{1}{v(v+1)} - \frac{1}{u}\left(\frac{1}{v(v+1)}\right)$$
$$= v^2K(u,v) - \frac{1}{u(u-1)} - \frac{1}{v}\left(\frac{-1}{u(u-1)}\right) + vK(u,v)$$
$$-\frac{1}{v}\left(\frac{1}{u(u-1)}\right) + K(u,v)$$

$$(u^2 - v^2 - v - 1)K(u,v) = \frac{1}{v(v+1)} + \frac{1}{u}\left(\frac{1}{v(v+1)}\right) - \frac{1}{u(u-1)}$$

$$= \frac{1}{v(v+1)}\left(1 + \frac{1}{u}\right) - \frac{1}{u(u-1)}$$

$$= \frac{u+1}{uv(v+1)} - \frac{1}{u(u-1)} = \frac{u^2-1-v^2-v}{uv(u-1)(v+1)}$$

$$K(u,v) = \frac{1}{uv(u-1)(v+1)} = \frac{1}{u(u-1)}\frac{1}{v(v+1)} \qquad (2-7)$$

Inversion to find the solution of equation in the form

$$U(x,t) = e^{x}e^{-t} = e^{x-t}$$

Example 2.2

Consider the telegraph equation

$$U_{xx} = U_{tt} + U_t - U \qquad (2-8)$$

With the boundary conditions:

$$U(0,t) = e^{-2t}, \qquad U_x(0,t) = e^{-2t} \qquad (2-9)$$

And the initial conditions:

$$U(x,0) = e^{x}, \qquad U_t(x,0) = -2e^{x} \qquad (2-10)$$

The exact solution is $U(x,t) = e^{x-2t}$

Solution

Take the double Aboodh transform of equation $(2-8)$, and single Aboodh transform of conditions $(2-9)$, $(2-10)$, and then we have

$$u^2K(u,v) - K(0,v) - \frac{1}{u}\frac{\partial[K(0,v)]}{\partial x} = v^2K(u,v) - K(u,0) - \frac{1}{v}\frac{\partial[K(u,0)]}{\partial t} +$$
$$vK(u,v) - \frac{1}{v}K(u,0) - K(u,v) \qquad (2-11)$$

And

$$K(0,v) = \frac{1}{v(v+1)} \quad , \quad \frac{\partial[K(0,v)]}{\partial x} = \frac{1}{v(v+1)} \quad (2-12)$$
$$K(u,0) = \frac{1}{u(u-1)} \quad , \quad \frac{\partial[K(u,0)]}{\partial t} = \frac{-2}{u(u-1)} \quad (2-13)$$

Substituting $(2-12)$ and $(2-13)$ in $(2-11)$ to find

$$u^2K(u,v) - \frac{1}{v(v+2)} - \frac{1}{u}\left(\frac{1}{v(v+2)}\right)$$
$$= v^2K(u,v) - \frac{1}{u(u-1)} - \frac{1}{v}\left(\frac{-2}{u(u-1)}\right) + vK(u,v)$$
$$- \frac{1}{v}\left(\frac{1}{u(u-1)}\right) - K(u,v)$$

$$(u^2 - v^2 - v + 1)K(u,v)$$
$$= \frac{1}{v(v+2)} + \frac{1}{u}\left(\frac{1}{v(v+2)}\right) - \frac{1}{u(u-1)} + \frac{1}{u}\left(\frac{2}{v(u-1)}\right)$$
$$- \frac{1}{u}\left(\frac{1}{v(u-1)}\right)$$
$$= \frac{1}{v(v+2)}\left(1 + \frac{1}{u}\right) - \frac{1}{u(u-1)} + \frac{1}{u}\left(\frac{1}{v(u-1)}\right)$$
$$= \frac{u+1}{uv(v+2)} - \frac{v-1}{uv(u-1)} = \frac{u^2-1-v^2-2v+v+2}{uv(u-1)(v+2)}$$
$$= \frac{u^2-v^2-v+1}{uv(u-1)(v+2)}$$

$$(u^2 - v^2 - v + 1)K(u,v) = \frac{u^2 - v^2 - v + 1}{uv(u-1)(v+1)}$$

$$K(u,v) = \frac{u^2-v^2-v+1}{uv(u-1)(v+1)} \cdot \frac{1}{u^2-v^2-v+1} = \frac{1}{u(u-1)} \cdot \frac{1}{v(v+1)} \qquad (2-14)$$

Inversion to find the solution of equation $(2-14)$ in the form:

$$U(x,t) = e^{x}e^{-2t} = e^{x-2t}$$

Example 2.3

Consider the general linear telegraph equation in the form

$$U_{tt} + aU_t + bU = C^2U_{tt} \qquad (2-15)$$

With the boundary conditions

$$U(0,t)=f_1(t)\quad,\quad U_x(0,t)=g_1(t)\qquad(2-16)$$

And the initial conditions

$$U(x,0)=f_2(x)\quad,\quad U_t(x,0)=g_2(x)\qquad(2-17)$$

Solution

Take the double Aboodh transform of equation $(2-15)$, and single Aboodh transform of conditions , and then we have:

$$v^2K(u,v)-K(u,0)-\frac{1}{v}\frac{[\partial K(u,0)]}{\partial t}+avK(u,v)-\frac{a}{v}K(u,0)+bK(u,v)=$$

$$c^2u^2K(u,v)-c^2K(0,v)-\frac{c^2}{u}\frac{\partial[K(0,v)]}{\partial x}\qquad(2-18)$$

And

$$K(0,v)=F_1(v)\ ,\ \frac{\partial[K(0,v)]}{\partial x}=G_1(v)\ \ (2-19)$$

$$K(u,0)=F_2(u)\ ,\ \frac{\partial[K(u,0)]}{\partial t}=G_2(u)\ \ (2-20)$$

Substituting $(2-19)$ and $(2-20)$ in $(2-18)$ we obtain:

$$v^2K(u,v)-F_2(u)-\frac{1}{v}G_2(u)+avK(u,v)-\frac{a}{v}F_2(u)+bK(u,v)$$
$$-c^2u^2K(u,v)+c^2F_1(v)+\frac{c^2}{u}G_1(v)=0$$

$$(v^2+av+b-c^2u^2)K(u,v)$$
$$=F_2(u)+\frac{1}{v}G_2(u)+\frac{a}{v}F_2(u)-c^2F_1(v)-\frac{c^2}{u}G_1(v)$$

$$K(u,v)=\frac{F_2(u)+\frac{1}{v}G_2(u)+\frac{a}{v}F_2(u)-c^2F_1(v)-\frac{c^2}{u}G_1(v)}{v^2+av+b-c^2u^2}\equiv H(u,v)$$

Take double inverse Aboodh transform to obtain the solution of general linear telegraph equation $(2-15)$ in the form:

$$U(x,t)=A_2^{-1}[H(u,v)]=K(x,t)$$

Assumed that the double inverse Aboodh transform is exists.

3. Conclusion

In this work, double Aboodh transform is applied to obtain the solution of general linear telegraph. It may be concluded that double Aboodh transform is very powerful and efficient in finding the analytical solution for a wide class of partial differential equations.

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