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Strong semitotal block domination in graphs

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ABSTRACT

For any graph $G=(V,E)$, the semitotal block graph $T_b(G)=H$, whose set of vertices is the union of the set of vertices and blocks of G and in which two vertices are adjacent if and only if the corresponding vertices of G are adjacent or the corresponding members are incident in G. For any two adjacent vertices u and v we say that u strongly dominates v if $\deg(u)\geq\deg(v)$. A dominating set D of a graph H is a strong semitotal block dominating set of G if every vertex in $V[T_b(G)]-D$ is strongly dominated by at least one vertex in D. Strong semitotal block domination number $\gamma_{Stb}(G)$ of G is the minimum cardinality of strong semitotal block dominating set of G. In this paper, we study graph theoretic properties of $\gamma_{Stb}(G)$ and many bounds were obtain in terms of elements of G and its relationship with other domination parameters were found.

1. Introduction

In this paper, all the graphs consider here are simple and finite. For any undefined terms or notation can be found in Harary [5]. In general, we use $<X>$ to denote the subgraph induced by the set of vertices X and $N(v) and N([v])$ denote open (closed) neighborhoods of a vertex v. The minimum distance between any two farthest vertices of a connected G is called the diameter of G and is denoted by $diam(G)$. The notation $\beta_o(G)(\beta_1(G))$ is the maximum cardinality of a vertex (edge) independent set in G. A set $S\subseteq V(G)$ is said to be a dominating set of G, if every vertex in $V-S$ is adjacent to some vertex in S. The minimum cardinality of vertices in such a set is called the domination number of G and is denoted by $\gamma(G)$. The concept of edge dominating sets were also studied by Mitchell and Hedetniemi in [9]. An edge dominating set of G if every edge in $E-F$ is adjacent to at least one edge in F. Equivalently, a set F edges in G is called an edge dominating set of G if for every edge $e\in E-F$, there exists an edge $e_1\in F$ such that e and $e_1$have a vertex in common. The edge domination number $\gamma'(G)$ of graph G is the minimum cardinality of an edge dominating set of G. A dominating set S is called the total dominating set, if for every vertex $v\in V$, there exists a vertex $u\in S$, $u\neq v$ such that u is adjacent to v. The total domination number of G is denoted by $\gamma_t(G)$ is the minimum cardinality of total dominating set of G. A dominating set D of a graph G is a global dominating set if D is also a dominating set of $\overline{G}$. The global domination number $\gamma_g(G)$ in the minimum cardinality of a global dominating set of G. This concept was introduced independently by Brigham and Dutton [2and13]. The concept of Roman domination function (RDF) was introduced by E.J. Cockayne, P.A.Dreyer, S.M.Hedetiniemi and S.T.Hedetiniemi in [3]. A Roman dominating function on a graph $G=(V,E)$ is a function $f:V\to\{0,1,2\}$ satisfying the condition that every vertex u for which $f(u)=0$ is adjacent to at least one vertex of v for which$f(v)=2$. The weight of a Roman dominating function is the value$f(V)=\sum_{v\epsilon V}f(v)$. The Roman domination number of a graph, denoted by $\gamma_R(G)$,equals the minimum weight of a Roman dominating function on G. A semitotal block graph $T_b(G)$ is the graph whose vertices corresponds to the blocks of G and two vertices in $T_b(G)$ are adjacent if and only if the corresponding blocks in$G$ are adjacent.

A dominating set D of a graph G is a split dominating set of G if the induced subgraph <V-D> is disconnected (Kulli, V.R. and Janakiram,B. 1997 (see [4])). The split domination number $\gamma_s(G)$ is the minimum cardinality of the minimal split dominating set of G. If V-D contains a dominating set D' then D' is called the Inverse dominating set of G. Then D' is called an Inverse split dominating set of G if the induced subgraph <V-D'> is disconnected (Ameenal Bibi, K. and Selvakumar,R.2008 (see [1])). The Inverse split domination number $\gamma_s^{-\prime}(G)$ is the minimum cardinality of the minimal Inverse split dominating set of G.

A dominating set D of a graph G is a non split dominating set if the induced subgraph $\langle V(G)-D\rangle$ is complete. The non split domination number $\gamma_{ns}(G)$ of G minimum cardinality of non split dominating set of G. [see (11)].

The concept of strong split block domination in graphs was introduced by M.H.Muddebihal and Nawazoddin U.Patel in [10].

A dominating set D of a graph G is a strong split block dominating set if the induced subgraph $\langle V[B(G)]-D\rangle$ is totally disconnected with at least two vertices. The strong split block domination number $\gamma_{ssb}(G)$of G is the minimum cardinality of strong split block dominating set of G.

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The concept of strong nonsplit block domination in graphs was introduced by M.H.Muddebihal and Nawazoddin U.Patel in [11]. A dominating set D of a graph $B(G)$ is a strong nonsplit block dominating set if the induced subgraph $\langle V[B(G)]-D\rangle$ is complete. The strong nonsplit block domination number $\gamma_{snsb}(G)$ of G is the minimum cardinality of strong nonsplit block dominating set of G. Recently and a variation on the domination theory, which is called strong line domination in graphs, was introduced by M.H.Muddebihal and Nawazoddin U.Patel in [12]. A dominating set D of a graph $L(G)$ is a strong line dominating set if every vertex in $\langle V[L(G)]-D\rangle$ is strongly dominated by at least one vertex in D. Strong line domination number $\gamma_{SL}(G)$ of G is the minimum cardinality of strong line dominating set of G.

The concept of strong domination was introduced by Sampathkumar and Pushpa Latha in [14] and well-studied in [6,7and 8]. Given two adjacent vertices u and v we say that u strongly dominates v if $\deg(u)\geq \deg(v)$. A set $D\subseteq V(G)$ is strong dominating set of G if very vertex in $V-D$ is strongly dominated by at least one vertex in D. The strong domination number $\gamma_s(G)$ is the minimum cardinality of a strong dominating set of G. A dominating set D of a graph $T_b(G)$ is a strong semitotal block dominating set of G if every vertex in $\langle V[T_b(G)]-D\rangle$ is strongly dominated by at least one vertex in D. Strong semitotal block domination number $\gamma_{Stb}(G)$ of G is the minimum cardinality of strong semitotal block dominating set of G. In this paper, many bounds on $\gamma_{Stb}(G)$ were obtained in terms of elements of G but not the elements of $T_b(G)$. Also its relation with other domination parameters were established.

We observed the following results for some standard graphs.

Observation 1: For any path P_n with $n\geq 2$ vertices $\gamma_{Stb}(P_n)=\left\lceil\frac{n}{2}\right\rceil$.

Observation 2: For any non separable graph G, $\gamma_{Stb}(G)=1$.

We needed the following theorem for our later results.

Theorem A [1]: Let T be a any tree such that any two adjacent cutvertices u and v with at least one of u and v is adjacent to an end vertices then $\gamma'(T)=\gamma_s^{-\prime}(T)$. Where $\gamma_s^{-\prime}(T)$ is the inverse split domination number.

2. MAIN RESULTS:

Theorem 1: For any (p,q) graph G with n blocks, then $\gamma_{Stb}(G)\leq n$.

Proof: we consider the following cases:

Case 1: Suppose G is a tree. Then each blocks in an edge. In $T_b(G)$ each block is K_3. Let $A_1=\{v_1,v_2,...,v_n\}$ be the set of all cutvertices of G. Suppose $D=\{v_1,v_2,...,v_k\}$ be the sub set of A_1 such that $\forall v_i\in D$, $\deg(v_i)\geq\deg(v_j), v_j\in V[T_b(G)-D]$, and $N(v_j)\cap D=\{v_i\}$. Then D is a minimal strong semitotal block dominating set. Let $B=\{B_1,B_2,...,B_n\}$ be the set of blocks of $T_b(G)$. Since $\forall v\in A_1$ has at least two blocks which are incident to v. Then $|D|\leq|B|$ gives $\gamma_{Stb}(G)\leq n$.

Case 2: Suppose G is not a tree. Then there exists at least one block which is not an edge. Since v_i be the number of block vertices corresponding to blocks of G which are not edges. Then $\{v_i\}\cup\{D\}$ forms a strong semitotal block dominating set. Hence from the case 1, $|\{v_i\}\cup\{D\}|\leq|B|$ gives $\gamma_{Stb}(G)\leq n$. One can see for the equality, if G is a block.

Theorem 2: For any tree T, $\gamma_{Stb}(T)\geq\gamma'(T)$.

Proof: Let $E=\{e_1,e_2,...,e_n\}$ be the set of edges which are adjacent to the end edges and $E_1=\{e_1,e_2,...,e_k\}$ be the set of end edges, $E_2=E(T)-E\cup E_1$. Suppose $E_2'\subseteq E_2$. Then $\forall e_i\in\{E\cup E_2'\}$ is adjacent to at least one element of $E(T)-\{E\cup E_2'\}$. Hence $\{E\cup E_2'\}$ form $\gamma'-set$. In $T_b(T)$ each block is K_3 and each cutvertex of $T_b(T)$ lie on at least two blocks. Hence degree of each cutvertex is at least 4. Let $D=\{v_1,v_2,...,v_n\}$ be set of cutvertices in $T_b(T)$ with $\deg(v_i)\geq 4$. Suppose $D_1=\{v_1,v_2,...,v_k\}\subseteq D$ in which $\deg(v_j)=4\,\forall v_j\in D_1, 1\leq j\leq k$. Then consider $D_1'\subseteq D_1$ and if at least one $v\in D_1'$ in $\gamma_{Stb}-set$. Then $\{D\cup D_1'\}$ is a minimal $\gamma_{Stb}-set$. Otherwise, if $D_1'=\phi$, then $\{D\}$ is $\gamma_{Stb}-set$. Hence $|\{E\cup E_2'\}|\leq|\{D\cup D_1'\}|$ or $|\{D\}|$, which gives $\gamma_{Stb}(T)\geq\gamma'(T)$.

Theorem 3: For any (p,q) tree T, $\gamma_{Stb}(T) \geq \gamma_s^{-'}(T)$.

Proof: From Theorem A, $\gamma^{'}(T) = \gamma_s^{-'}(T)$ ----------- (1)

From Theorem 2, $\gamma_{Stb}(T) \geq \gamma^{'}(T)$ ------------------- (2)

From (1) and (2) we get the required result.

Theorem 4: For any (p,q) graph G, $\gamma_{Stb}(G) \leq \beta_0(G)$. Where $\beta_0(G)$ is the maximum vertex independent number of G.

Proof: Suppose $G = K_2$. Then $T_b(G) = K_3$. Hence $\gamma_{Stb}(G) = \beta_0(G)$. Now we consider G with $p \geq 3$ vertices. Let $A = \{v_1, v_2, ..., v_n\}$ be the set of all end vertices and $B = \{v_1, v_2, ..., v_k\} \subseteq \{V(G) - A\}$. $\forall v_i \in B, 1 \leq i \leq k$ Which are at a distance two. Since $N(A) \cap N(B) = \{v_j\} \in V(G) - \{A \cup B\}$, then $\{A \cup B\}$ is a independent set of G with $|A \cup B| = \beta_0(G)$. Suppose $C = \{v_1, v_2, ..., v_m\} \subseteq V[T_b(G)]$ be the set of vertices with maximum degree and $\forall v_l \in C, 1 \leq l \leq m$ is adjacent to at least one vertex $v_p \in V[T_b(G)] - C$, such that $N[C] = V[T_b(G)]$. Furthermore, $\deg(v_l) \geq \deg(v_p)$, since G has at least 3-vertices, then $\{C\} \subset \{A \cup B\}$. Hence $|\{A \cup B\}| \geq |C|$, which gives $\gamma_{Stb}(G) \leq \beta_0(G)$.

Lemma 1: If $\gamma^{-'}(G) \leq \gamma_{Stb}(G)$,then $\gamma^{-'}(G) \leq n$. Where n is the number of blocks of G.

Proof: Suppose $\gamma^{-'}(G) \leq \gamma_{Stb}(G)$. Then by Theorem [1], $\gamma_{Stb}(G) \leq n$. It follows that $\gamma^{-'}(G) \leq \gamma_{Stb}(G) \leq n$, thus $\gamma^{-'}(G) \leq n$.

Theorem 5: For any (p,q) graph G, $\gamma_{Stb}(G) \leq diam(G)$.

Proof: Let $A = \{e_1, e_2, ..., e_k\}$ be the set of edges which constitutes the largest path between any two vertices of G such that $|A| = diam(G)$.

We consider the following cases.

Case 1: If all the elements of A belongs to a single block, then $\gamma_{Stb}(G) = 1 \leq |A|$.

Case 2: If all the elements of A belongs to different blocks and G is without end vertices, then $n \leq |A|$. Then by the Theorem [1], $\gamma_{Stb}(G) \leq n \leq |A|$. Hence $\gamma_{Stb}(G) \leq diam(G)$.

Theorem 6: For any (p,q) graph G, $\gamma_{Stb}(G) \leq \gamma(G) + \gamma^{'}(G) - 1$. Equality holds for P_p.

Proof: Let $D = \{v_1, v_2, ..., v_i\}$ be the minimal set of vertices such that for each $v_i \in D$ and $N[\{v_i\}] = V[G]$. Then D is minimal dominating set of G, $|D| = \gamma(G)$. Further, let $F^{'} = \{e_1, e_2, ..., e_i\}$ be a minimal edge set of G, $\forall e_i \in F^{'}, N(e_i) \cap F^{'} = \phi$. Thus $|F^{'}| = \gamma^{'}(G)$. Since $V(G) \subset V[T_b(G)]$ and let $S = \{v_1, v_2, ..., v_n\} \subseteq V[T_b(G)] - D$, then there exists $S^{'} \subseteq S$ such that the closed neighborhood of $\{D\} \cup \{S^{'}\} = V[T_b(G)]$. Hence $\{D\} \cup \{S^{'}\}$ is a minimal dominating set of $T_b(G)$. Suppose $\deg(v_i) \geq \deg(v_j), \forall v_i \in \{D\} \cup \{S^{'}\}$ and $v_j \in V[T_b(G)] - [\{D\} \cup \{S^{'}\}]$. Then $\{D\} \cup \{S^{'}\}$ is strong dominating set $T_b(G)$. Since $|D| > |\{D\} \cup \{S^{'}\}| \leq |D| + |F| - 1$, which gives $\gamma_{Stb}(G) \leq \gamma(G) + \gamma^{'}(G) - 1$.

Theorem 7: For nay non-trivial tree T, with $p \geq 4$ vertices, then $\gamma_{Stb}(T) \leq \gamma_{ns}(T)$.

Proof: Let $C=\{v_1,v_2,...,v_n\}\subseteq V[T]$ be the set of all cutvertices in T. Then $C^{'}\subseteq C$ forms a $\gamma-set$ of T. Since each edge of T is K_2 and each block in $T_b(T)$ is K_3 which are incident with each $v_i,\forall v_i\in C, 1\le i\le n$, then there exists $C^{"}\subseteq V[T_b(T)]-C$, such that $C^{'}\cup C^{"}$ is a minimal dominating set of $T_b(T)$. Suppose $\deg(v_i)\ge\deg(v_j),\forall v_i\in\{C^{'}\cup C^{"}\}$ and $\forall v_j\in V[T_b(T)]-C$. Then $\{C^{'}\cup C^{"}\}$ is a minimal $\gamma_{Stb}(T)-set$ of a tree T. Now nonsplite dominating set for a tree is $V(T)-[V(T)-2]=K_2$. Hence $|V(T)-[V(T)-2]|\le|C^{'}\cup C^{"}|$ gives $\gamma_{Stb}(T)\le\gamma_{ns}(T)$.

Theorem 8: For any (p,q) graph G, $\gamma_{Stb}(G)\le\gamma_s(G)$. Further, equality holds for $G=K_p; P\ge 2$.

Proof: Let $S=\{v_1,v_2,...,v_n\}\subseteq V[G]$ such that $\forall v_i\in S, N[S]=V[G]$. Then S is a minimal dominating set. Suppose $\langle V(G)-S\rangle$ is disconnected. Then S is a minimal split dominating set of G. Now assume if $\forall v_i\in S$ and $\forall v_j\in V[T_b(G)]-S$ is adjacent to at least an vertex of S and $\deg(v_i)\ge\deg(v_j)$. Then clearly S is a $\gamma_{Stb}-set$ of G. Otherwise there exists a vertex $v_k\in V[T_b(G)]-S$ such that $S\cup\{v_k\}$ dominates all vertices of $V[T_b(G)-S]$ .Hence in any one, $|S\cup\{v_k\}|=|S|$ gives $\gamma_{Stb}(G)=\gamma_s(G)$.

On the other hand suppose G has a block B with maximum number of vertices which is B not a complete graph. Then this block has at least two vertices $v_1,v_2\in\{S\}$, where as in $T_b(G)$, u be a block vertex adjacent to all vertices of B and $u\in\gamma_{Stb}-set$. Hence $|S\cup\{v_k\}|\le|S|$ gives $\gamma_{Stb}(G)\le\gamma_s(G)$.

Lemma 2: For any star $K_{1,p}; p\ge 1,\ \gamma_{Stb}(K_{1,p})=1$.

The following theorem gives the result on strong semitotal block domination number of a graph G.

Theorem 9: For any connected (p,q) graph G, $\gamma_{Stb}(G)\le p-\gamma_t(G)$.

Proof: Let $H_1=\{v_1,v_2,...,v_n\}$ be the minimum set of vertices which covers all the vertices in G. Suppose $\deg(v_j)\ge 1,\forall v_j\in H_1, 1\le j\le m$ in the subgraph $\langle H_1\rangle$ then H_1 forms a $\gamma_t(G)-set$ of G. Otherwise if $\deg(v_j)<1,$ then attaché the vertices $w_i\in N(v_i)$ to make $\deg\ge 1$ such that $\langle H_1\cup\{w_i\}\rangle$ does not contains any isolated vertex. Clearly $H_1\cup\{w_i\}$ forms a minimal total dominating set of G.

Now in $T_b(G)$, let $A\le V[T_b(G)]$, let there exists a subset $D=\{u_1,u_2,...,u_k\}\subseteq A$ of vertices with $\deg(u_i)\ge 3, 1\le i\le k$ and $N[\{u_i\}]=V[T_b(G)]$ .Further, $|\deg(u)-\deg(w)|\le 2,\forall u\in D$ and $w\in V[T_b(G)]-D$ has at least one vertex in D. Clearly D forms a minimal strong dominating set in $T_b(G)$. Therefore it follows that $|D|\le|V(G)|-|H_1\cup\{w_i\}|$ and hence $\gamma_{Stb}(G)\le p-\gamma_t(G)$.

Theorem 10: For any connected (p,q) graph G with $p\ge 3,$ then $\gamma_{Stb}(G)\le\gamma_R(G)-1$.

Proof: Let $f:V(G)\to\{0,1,2\}$ and partition the vertex set $V(G)$ into (V_0,V_1,V_2) induced by f with $|V_i|=n_i$ for $i=0,1,2$. Suppose the set V_2 dominates V_0. Then $S=V_1\cup V_2$ forms a minimal Roman dominating set of G. Further, let $A=\{v_1,v_2,...,v_i\}\subseteq V[T_b(G)]$ be the set of vertices with $\deg(v_j)\ge 3$. Suppose there exists a vertex set $D\subseteq A$ with $N[D]=V[T_b(G)]$ and if $|\deg(x)-\deg(y)|\le 2$, $\forall x\in D$, $y\in V[T_b(G)]-D$. Then D forms a Strong dominating set in $T_b(G)$. Otherwise there exists at least one vertex $\{w\}\subseteq A$ where $\{w\}\notin D$ such that $D\cup\{w\}$ forms a minimal $\gamma_{Stb}-set$ in $T_b(G)$ which gives $|D\cup\{w\}|\le|S|$. Clearly, $\gamma_{Stb}(G)\le\gamma_R(G)-1$.

Next,we obtained the upper bound for $\gamma_{Stb}(T)$ in forms of $\gamma_{Ssb}(T)$.

Theorem 11: For nay non-trivial tree T, $T \neq K_{1,p}$, then $\gamma_{Stb}(T) \leq \gamma_{Ssb}(T)$.

Proof: Suppose $T = K_{1,p}$. Then block graph of T, $B(T) = K_p$ and by the definition of strong split domination $\gamma_{Ssb} - set$ does not exists. Hence $T \neq K_{1,p}$. Let $H = \{v_1, v_2, ..., v_n\} \subseteq V(T)$ be the set of non end vertices. Since $H \subset V[T_b(T)]$ and let $H' \subseteq H$ such that $\forall v_i \in H'$, $\deg(v_i) \geq \deg(v_j)$, $\forall v_j \in V[T_b(T)] - H'$, then H' is a minimal $\gamma_{Stb} - set$. Suppose the edge set of T, $E(T) = V[B(T)]$. Then in $B(T)$ each block is complete. Let $B_1, B_2, ..., B_k$ be the number of blocks in $T_b(T)$, if each block $B_i, 1 \leq i \leq k$ contains p vertices. Then $p-1$ vertices from each block form a set $S = [\{p-1\}_1, \{p-2\}_2, ..., \{p-1\}_k] \subseteq T_b(T)$ such that $M = V[T_b(T)] - S$ in which $\langle H \rangle$ is a null graph with at least two vertices. Hence H is a $\gamma_{Ssb} - set$. Clearly $|H'| \leq |M|$, which gives $\gamma_{Stb}(T) \leq \gamma_{Ssb}(T)$.

Next, we obtain the relationship between $\gamma_{Snsb}(G)$ and $\gamma_{Stb}(G)$.

Theorem 12: For any connected (p,q) graph G, $\gamma_{Stb}(G) \leq \gamma_{Snsb}(G) + \gamma(G) - 1$.

Proof: Suppose G is a block. Then $\gamma_{Snsb}(G) = 1$, $\gamma(G) \geq \gamma_{Stb}(G)$. Hence we have required result. Now assume G has at least two blocks. Then $\gamma_{Stb}(G) \geq \gamma(G)$ and hence $\gamma(G) + \gamma_{Snsb}(G) - 1 \geq \gamma_{Stb}(G)$, as required.

We conclude this section by giving the following result that is relation between $\gamma_{SL}(T)$ and $\gamma_{Stb}(T)$.

Theorem 13: For nay non-trivial tree T, then $\gamma_{SL}(T) \leq \gamma_{Stb}(T)$.

Proof: Suppose $G = K_{1,p}, p \geq 2$. Then $\gamma_{SL}(T) = \gamma_{Stb}(T)$. Now assume $G \neq K_{1,p}, p \geq 2$, then every block in $L(T)$ is complete and every block in $T_b(T)$ is a triangle. Let $S = \{v_1, v_2, ..., v_n\} \subseteq V(T)$ be the set of all non-end vertices, $H = \{e_1, e_2, ..., e_n\} \subseteq E(T)$ be the set of edges which are incident to the vertices of S. In $L(T)$, $H \subseteq V[L(T)]$ and $\forall e_i \in H$ can be denoted as $H = \{v_1, v_2, ..., v_m\}$ in $L(T)$. Now consider a set $H' = \{v_1, v_2, ..., v_k\} \subseteq H$ in which $\forall v_j, 1 \leq j \leq k$ $\deg(v_j) \geq \deg(v_p)$, $\forall v_p \in V[L(T)] - H'$. Since $S \subseteq V[T_b(T)]$, then $S' \subseteq S$ such that $\forall v \in S'$ $\deg(v) \geq \deg(u)$, $\forall u \in V[T_b(T)] - S'$. Also $|S'| \geq |H'|$ which gives $\gamma_{SL}(T) \leq \gamma_{Stb}(T)$.

Theorem 14: For any connected $(\mathbf{p}, \mathbf{q})$ graph $\mathbf{G}, \boldsymbol{\gamma_{Stb}(G) \leq \gamma_g(G)}$. Where $\boldsymbol{\gamma_g(G)}$ is a global domination number of $\boldsymbol{G}$.

Proof: Let $\boldsymbol{S = \{v_1, v_2, v_3, \dots\dots\dots, v_i\} \subseteq V(G)}$ be an independent set of $\boldsymbol{G}$. Since $\boldsymbol{G}$ has no isolated vertices, $\boldsymbol{V - S}$ is dominating set of $\boldsymbol{G}$. Clearly for very vertex $\in \boldsymbol{S}$, $(\boldsymbol{V - S}) \cup \{v\}$ is a global dominating set of $\boldsymbol{G}$. Since $|(V-S) \cup \{v\}| = \boldsymbol{\gamma_g(G)}$. Let $\boldsymbol{D' = \{v_1, v_2, v_3, \dots\dots\dots, v_i\} \subseteq V[T_b(G)]}$ be the minimal dominating set of $\boldsymbol{T_b(G)}$ and if $\mathbf{deg}(\boldsymbol{v_i}) \geq \mathbf{2}\ \forall \boldsymbol{v_i} \in \boldsymbol{D'}$ with $\mathbf{deg}(\boldsymbol{v_k}) \leq \mathbf{2}, \forall \boldsymbol{v_k} \in \boldsymbol{V[T_b(G)] - D'}$. Then $\boldsymbol{D'}$ is a Strong dominating set of $\boldsymbol{T_b(G)}$. It follows that $|D'| \leq |(V-S) \cup \{v\}|$ and hence $\boldsymbol{\gamma_{Stb}(G) = \gamma_g(G)}$.

Theorem 15: For any acyclic (p,q) graph G, with $p \geq 3$ vertices, then $\gamma_{Stb}(G) \leq c_0 + e - 2$. Where c_0 is the number of cutvertices and e be the number of end edges of G.

Proof: Suppose acyclic graph G, with $p < 3$ vertices. Then result does not hold. Hence acyclic graph G has $p \geq 3$ vertices. Consider the following cases.

Case 1: Suppose acyclic graph G is a path with $p \geq 3$ vertices. Then path with $3-$ vertices, $\gamma_{Stb}(T) = 1$, $c_0 = 1, e = 2$. Hence equality holds. Further if path has $p > 3$ vertices then $\gamma_{Stb} = \left\lceil \frac{p}{2} \right\rceil$, $c_0 = p-2, e = 2$. Thus $\left\lceil \frac{p}{2} \right\rceil \leq (p-2) + 2 - 2$ which gives $\gamma_{Stb}(G) \leq c_0 + e - 2$.

Case 2: Suppose acyclic graph G is not a path. Then there exists at least two vertices of degree at least 3. Then $E=\{e_1,e_2,...,e_n\}$ be the set of end edges incident to the cutvertices $C_0=\{c_1,c_2,...,c_n\}$. Since $|E|>|C_0|$ and $C_0 \in V[T_b(G)]$, then there exits $C_0^{'} \subseteq C_0$ such that $\forall v_i \in C_0^{'}$ is adjacent to at least one vertex of $V[T_b(G)]-C_0^{'}$. Also $\deg(v_i) \geq \deg(v_j)$ $\forall v_i \in C_0^{'}$ and, $\forall v_j \in V[T_b(G)-C_0^{'}]$. Clearly it is known that $|C_0^{'}| \leq |C_0|$. Obviously $|C_0^{'}| \leq |C_0|+|E|-2$, which gives $\gamma_{Stb}(G) \leq c_0+e-2$.

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