



Decision of Intuitionistic Fuzzy Soft Matrix in Weighted Harmonic Mean

S.Subramanian¹ and R. Rathika²

¹Professor, Department of Mathematics, PRIST University, Vallam, Thanjavur-613403, Tamil Nadu, India.

²Research Scholar, Centre for Research & Development, PRIST University, Vallam, Thanjavur-613403, Tamil Nadu, India.

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ABSTRACT

In this study, we explore the fuzzy soft matrix and their operations. In this report, we define intuitionistic fuzzy soft matrices and have entered some new operators with weights. We then prove fuzzy soft algorithm that allows constructing those efficient decision making methods. In the end, we throw an example on the weighted harmonic mean for decision making problems. It indicates that the method can be successfully employed to many problems that contain uncertainties

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Introduction

In the fuzzy set theory [14] there were no scopes thinking about the indisposition in the membership degree, which originate in diverse real life places. To master these situations Atanassov introduced theory of intuitionistic fuzzy set in 1986 as a generalization of fuzzy set. Most of the problems in engineering, medical scientific discipline, economics, environments etc. have various doubts. Molodtsov [12] initiated the concept of soft set theory as a mathematical tool for dealing with doubts. Research works on soft set theory are coming along apace. Combining soft sets with fuzzy sets and intuitionistic fuzzy sets, Feng et al., Maji et al. [8,9] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are annoying capabilities for solving resolution making difficulties. Matrices play an important role in the broad arena of science and engineering. The classical matrix theory cannot work the problems touching on various types of uncertainties. In Yang et al, led up a matrix representation of a fuzzy soft set and presented it in certain decision making problems. The idea of fuzzy soft matrix theory was considered by Borah et al. in [5]. In [5], Chetia et al. And in [13] Rajarajeswari et al. Well-defined intuitionistic fuzzy soft matrix. Once more, it is well recognized that the matrices are important instruments to model/study different mathematical problems specially in linear algebra. And in R. Rathika, S. Subramanian [14] gives an estimate of the weighted geometric mean. Due to huge applications of imprecise information in the above mentioned fields, hence are motivated to take the different matrices containing these data. Soft set is also one of the interesting and popular subject, where different types of decision making problem can be dissolved.

In this broadside, we have presented some operators on intuitionistic fuzzy soft matrix on the basis of weights. We have also deliberated with their examples. In concluding, we have given an elementary application on the decision making, problem on the basis of weighted harmonic mean.

2. Definition and Preliminaries

2.1 Soft set [1]

Let U be a first universe, $P(U)$ be the power set of U , E be the set of all parameters and $A \subseteq E$. A soft set (f_A, E) on the universe U is defined by the set of ordered pairs

$$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$$

where $f_A: E \rightarrow P(U)$ such that $f_A(e) = \phi$ if $e \notin A$.

Here f_A is known as an approximate function of the soft set (f_A, E) . The set $f_A(e)$ is known as e -approximate value set or e -approximate set which consists of related objects of the parameter $e \in E$.

2.2. Fuzzy soft set [6] Let U be an primary universe, E be the set of totally parameters and $A \subseteq E$. A pair (F, A) is known as a fuzzy set over U where $F: A \rightarrow (U)$ is a mapping from A into (U) , where (U) denotes the collection of whole subsets of U . the collection of all subsets of U .

2.3 Fuzzy Soft Matrices (FSM) [5]

Let (f_A, E) be fuzzy soft set over U . And so a subset of $U \times E$ is uniquely determined by

$R_A = \{(u, e) : e \in A, u \in f_A(e)\}$, which is called relation form of (f_A, E) . The characteristic function of R_A is written by $\mu_{R_A}: U \times E \rightarrow [0, 1]$, where $\mu_{R_A}(u, e) \in [0, 1]$ is the membership value of $u \in U$ for each $e \in E$. If $\mu_{ij} = \mu_{R_A}(ui, e.g.)$, we can specify a matrix.

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2n} \\ \dots & \dots & \dots & \dots \\ \mu_{m1} & \mu_{m2} & \dots & \mu_{mn} \end{bmatrix}$$

which is so-called an $m \times n$ soft matrix of the soft set (f_A, E) done U . Thus we can state that a fuzzy soft set (f_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m \times n}$ and both concepts are interchangeable.

2.4 Row- Fuzzy Soft Matrix

A fuzzy soft matrix of order $1 \times n$ i.e., with a only one row is named a row-fuzzy soft Matrix.

2.5 Column -Fuzzy Soft Matrix

A fuzzy soft matrix of order $m \times 1$ i.e., with a single column is termed a column-fuzzy soft matrix.

3. Intuitionistic Fuzzy Soft Matrix Theory

3.1. Intuitionistic Fuzzy Soft Matrix (IFSM) [5]

Let U be an preliminary universe , E be the set of parameters and $A \subseteq E$. Let (f_A, E) be an Intuitionistic fuzzy soft set (IFSS) over U . And so a subset of $U \times E$ is uniquely determined by

$$R_A = \{ (u, e) : e \in A, u \in f_A(e) \},$$

which is called relation form of (f_A, E) . The membership and non-membership functions are written by $\mu_{RA}: U \times E \rightarrow [0, 1]$ and $\nu_{RA}: U \times E \rightarrow [0, 1]$ where $\mu_{RA}: (u, e) \in [0, 1]$ and $\nu_{RA}: (u, e) \in [0, 1]$ are the membership value and non-membership value of $u \in U$ for each $e \in E$.

If $(\mu_{ij}, \nu_{ij}) = (RA(ui, ej), \nu_{RA}(ui, ej))$, we can specify a matrixine a matrix

$$[(\mu_{ij}, \nu_{ij})]_{m \times n} = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \dots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \dots & (\mu_{2n}, \nu_{2n}) \\ \dots & \dots & \dots & \dots \\ (\mu_{m1}, \nu_{m1}) & (\mu_{m2}, \nu_{m2}) & \dots & (\mu_{mn}, \nu_{mn}) \end{bmatrix}$$

which is entitled an $m \times n$ IFSM of the IFSS (f_A, E) done U . Thus, we can say that IFSS (f_A, E) is uniquely characterized by the matrix $[\mu_{ij}, \nu_{ij}]_{m \times n}$ and both concepts are interchangeable. The circle of all $m \times n$ IFS matrices will be denoted by IFSM $m \times n$

Example 3.1

Let $U = \{ u_1, u_2, u_3, u_4, u_5 \}$ is a universal set and $E = \{ e_1, e_2, e_3, e_4, e_5 \}$, is a set of parameters. If $A = \{ e_1, e_3, e_4, e_5 \} \subseteq E$ and $f_A(e_1) = \{ (u_1, .5, .4), (u_2, .6, .1), (u_3, .5, .5), (u_4, .5, .4), (u_5, .2, .1) \}$, $f_A(e_3) = \{ (u_1, .6, .6), (u_3, .2, .2), (u_4, 1, 0), (u_5, .6, .2) \}$, $f_A(e_4) = \{ (u_1, .6, .2), (u_2, 1, 0), (u_3, .8, .2), (u_4, .6, .3), (u_5, .7, .3) \}$, $f_A(e_5) = \{ (u_1, .7, .8), (u_2, 1, 0), (u_3, .6, .5), (u_4, .5, .3), (u_5, .8, .2) \}$

Then the IFS set (F_A, E) is a parameterized family $\{ f_A(e_1), f_A(e_3), f_A(e_4), f_A(e_5) \}$ of all IFS sets over U . Hence IFSM $[(\mu_{ij}, \nu_{ij})]$ can be written every bit

$$[(\mu_{ij}, \nu_{ij})] = \begin{bmatrix} (.5, .4) & (0, 0) & (.6, .6) & (.6, .2) & (.7, .8) \\ (.6, .1) & (0, 0) & (0, 0) & (1, 0) & (1, 0) \\ (.5, .5) & (0, 0) & (.2, .2) & (.8, .2) & (.6, .5) \\ (.5, .4) & (0, 0) & (1, 0) & (.6, .3) & (.5, .3) \\ (.2, .1) & (0, 0) & (.6, .2) & (.7, .3) & (.8, .2) \end{bmatrix}$$

3.2. Complement of Intuitionistic Fuzzy Soft Matrix

Let $\tilde{A} = [(\mu_{ij}^A, \nu_{ij}^A)] \in \text{IFSM}_{m \times n}$. Then Complement of \tilde{A} denoted by \tilde{A}^0 is defined as

$$\tilde{A}^0 = [(\nu_{ij}^A, \mu_{ij}^A)] \text{ for all } i \text{ and } j.$$

3.3. Operators of Intuitionistic Fuzzy Soft Matrices

Let $\tilde{A} = [(\mu_{ij}^A, \nu_{ij}^A)]$, $\tilde{B} = [(\mu_{ij}^B, \nu_{ij}^B)] \in \text{IFSM}_{m \times n}$. Then IFSM $\tilde{C} = [(\mu_{ij}^C, \nu_{ij}^C)]$ is called

a) the “.” (product) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{A} \cdot \tilde{B}$ if $\mu_{ij}^C = \mu_{ij}^A \cdot \mu_{ij}^B$ and $\nu_{ij}^C = \nu_{ij}^A + \nu_{ij}^B - \nu_{ij}^A \cdot \nu_{ij}^B$ For all i and j .

b) the “+” (Probabilistic sum) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{C} = \tilde{A} + \tilde{B}$ if

$$\mu_{ij}^C = \mu_{ij}^A + \mu_{ij}^B - \mu_{ij}^A \cdot \mu_{ij}^B \text{ and } \nu_{ij}^C = \nu_{ij}^A \cdot \nu_{ij}^B \text{ for all } i \text{ and } j.$$

c) the “@” (Arithmetic Mean) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{C} = \tilde{A} @ \tilde{B}$

$$\text{if } \mu_{ij}^C = \frac{\mu_{ij}^A + \mu_{ij}^B}{2} \text{ and } \nu_{ij}^C = \frac{\nu_{ij}^A + \nu_{ij}^B}{2} \text{ for all } i \text{ and } j.$$

d) the “@^w” (Weighted Arithmetic Mean) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{C} = \tilde{A} @^w \tilde{B}$ if

$$\mu_{ij}^C = \frac{(w_1 \mu_{ij}^A + w_2 \mu_{ij}^B)}{w_1 + w_2} \text{ and } \nu_{ij}^C =$$

$$\frac{(w_1 \nu_{ij}^A + w_2 \nu_{ij}^B)}{w_1 + w_2} \text{ for all } i \text{ and } j \text{ } w_1 > 0, w_2 > 0$$

e) the “\$” (Geometric Mean) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{C} = \tilde{A} \$ \tilde{B}$ if

$$\mu_{ij}^C = \sqrt{\mu_{ij}^A \cdot \mu_{ij}^B} \text{ and } \nu_{ij}^C = \sqrt{\nu_{ij}^A \cdot \nu_{ij}^B} \text{ for all } i \text{ and } j.$$

f) the “\$^w” (Weighted Geometric Mean) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{C} = \tilde{A} \$^w \tilde{B}$ if $\mu_{ij}^C = ((\mu_{ij}^A)^{w_1} \cdot (\mu_{ij}^B)^{w_2})^{1/(w_1 + w_2)}$ and $\nu_{ij}^C = ((\nu_{ij}^A)^{w_1} \cdot (\nu_{ij}^B)^{w_2})^{1/(w_1 + w_2)}$ for all i and j $w_1 > 0, w_2 > 0$.

g) the “∞” (Harmonic mean) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{C} = \tilde{A} \infty \tilde{B}$ if $\mu_{ij}^C = 2 \cdot \frac{(\mu_{ij}^A \cdot \mu_{ij}^B)}{\mu_{ij}^A + \mu_{ij}^B}$ and $\nu_{ij}^C = \frac{(\nu_{ij}^A \cdot \nu_{ij}^B)}{\nu_{ij}^A + \nu_{ij}^B}$ for all i and j $w_1 >$

$0, w_2 > 0$.

h) the “ \bowtie^w ” (Weighted Harmonic mean) operation of \tilde{A} and \tilde{B} , denoted by $\tilde{C} = \tilde{A} \bowtie^w \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \frac{w_1 + w_2}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}}$, and $\nu_{ij}^{\tilde{C}} = \frac{w_1 + w_2}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}}$ for all i

and $j \ w_1 > 0, w_2 > 0$.

Proposition 1:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{m \times n}$. Then (i) $\tilde{A}^0 @^w \tilde{B}^0 = \tilde{A} @^w \tilde{B}$, (ii) $\tilde{A}^0 \$^w \tilde{B}^0 = \tilde{A} \$^w \tilde{B}$, (iii) $\tilde{A}^0 \bowtie^w \tilde{B}^0 = \tilde{A} \bowtie^w \tilde{B}$.

Proof:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{m \times n}$ and $w_1 > 0, w_2 > 0$.

$$\begin{aligned} \text{(i)} \tilde{A}^0 @^w \tilde{B}^0 &= ([\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}}] @^w [\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{B}}])^0 = \left[\left(\frac{w_1 \nu_{ij}^{\tilde{A}} + w_2 \nu_{ij}^{\tilde{B}}}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}}, \frac{w_1 \mu_{ij}^{\tilde{A}} + w_2 \mu_{ij}^{\tilde{B}}}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}} \right) \right]^0 = \left[\left(\frac{w_1 \nu_{ij}^{\tilde{A}} + w_2 \nu_{ij}^{\tilde{B}}}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}}, \frac{w_1 \mu_{ij}^{\tilde{A}} + w_2 \mu_{ij}^{\tilde{B}}}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}} \right) \right] = \tilde{A} @^w \tilde{B} \\ \text{(ii)} \tilde{A}^0 \$^w \tilde{B}^0 &= ([\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}}] \$^w [\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{B}}])^0 = \left[\left((\nu_{ij}^{\tilde{A}})^{w_1} \cdot (\nu_{ij}^{\tilde{B}})^{w_2} \right)^{1/(w_1+w_2)}, \left((\mu_{ij}^{\tilde{A}})^{w_1} \cdot (\mu_{ij}^{\tilde{B}})^{w_2} \right)^{1/(w_1+w_2)} \right]^0 \\ &= \left[\left((\nu_{ij}^{\tilde{A}})^{w_1} \cdot (\mu_{ij}^{\tilde{B}})^{w_2} \right)^{1/(w_1+w_2)}, \left((\nu_{ij}^{\tilde{A}})^{w_1} \cdot (\nu_{ij}^{\tilde{B}})^{w_2} \right)^{1/(w_1+w_2)} \right] = \tilde{A} \$^w \tilde{B} \\ \text{(iii)} \tilde{A}^0 \bowtie^w \tilde{B}^0 &= ([\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}}] \bowtie^w [\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{B}}])^0 = \left[\left(\frac{w_1 + w_2}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}}, \frac{w_1 + w_2}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}} \right) \right]^0 = \left(\frac{w_1 + w_2}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}}, \frac{w_1 + w_2}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}} \right) = \tilde{A} \bowtie^w \tilde{B}. \end{aligned}$$

Proposition 2:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{m \times n}$. Then (i) $\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$ (ii) $\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$ (iii) $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$ (iv) $\tilde{A} \cdot \tilde{B} = \tilde{B} \cdot \tilde{A}$

Proof:

$$\begin{aligned} \text{(i)} \tilde{A} \cup \tilde{B} &= [(\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\})] = [(\max\{\mu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{A}}\}, \min\{\nu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{A}}\})] = \tilde{B} \cup \tilde{A} \\ \text{(ii)} \tilde{A} \cap \tilde{B} &= [(\min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \max\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\})] = [(\min\{\mu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{A}}\}, \max\{\nu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{A}}\})] = \tilde{B} \cap \tilde{A} \\ \text{(iii)} \tilde{A} + \tilde{B} &= [(\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\})] = [(\max\{\mu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{A}}\}, \min\{\nu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{A}}\})] = \tilde{B} + \tilde{A} \end{aligned}$$

Validity: Since $0 \leq \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \leq 1, 0 \leq \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \leq 1, \min\{\mu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{A}}\} \leq 1, \max\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\} \leq 1$

Proposition 3:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{m \times n}$. Then (i) $\tilde{A} @^w \tilde{B} = \tilde{B} @^w \tilde{A}$ (ii) $\tilde{A} \$^w \tilde{B} = \tilde{B} \$^w \tilde{A}$ (iii) $\tilde{A} \bowtie^w \tilde{B} = \tilde{B} \bowtie^w \tilde{A}$.

Proof: For all I and j $w_1 > 0, w_2 > 0$

$$\begin{aligned} \text{(i)} \tilde{A} @^w \tilde{B} &= \left[\left(\frac{w_1 \mu_{ij}^{\tilde{A}} + w_2 \mu_{ij}^{\tilde{B}}}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}}, \frac{w_1 \nu_{ij}^{\tilde{A}} + w_2 \nu_{ij}^{\tilde{B}}}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}} \right) \right] = \left[\left(\frac{w_2 \mu_{ij}^{\tilde{B}} + w_1 \mu_{ij}^{\tilde{A}}}{\frac{w_2}{\mu_{ij}^{\tilde{B}}} + \frac{w_1}{\mu_{ij}^{\tilde{A}}}}, \frac{w_2 \nu_{ij}^{\tilde{B}} + w_1 \nu_{ij}^{\tilde{A}}}{\frac{w_2}{\nu_{ij}^{\tilde{B}}} + \frac{w_1}{\nu_{ij}^{\tilde{A}}}} \right) \right] = \tilde{B} @^w \tilde{A} \\ \text{(ii)} \tilde{A} \$^w \tilde{B} &= \left[\left((\mu_{ij}^{\tilde{A}})^{w_1} \cdot (\mu_{ij}^{\tilde{B}})^{w_2} \right)^{1/(w_1+w_2)}, \left((\nu_{ij}^{\tilde{A}})^{w_1} \cdot (\nu_{ij}^{\tilde{B}})^{w_2} \right)^{1/(w_1+w_2)} \right] = \left[\left((\mu_{ij}^{\tilde{B}})^{w_2} \cdot (\mu_{ij}^{\tilde{A}})^{w_1} \right)^{1/(w_1+w_2)}, \left((\nu_{ij}^{\tilde{B}})^{w_2} \cdot (\nu_{ij}^{\tilde{A}})^{w_1} \right)^{1/(w_1+w_2)} \right] = \tilde{B} \$^w \tilde{A} \\ \text{(iii)} \tilde{A} \bowtie^w \tilde{B} &= \left(\frac{w_1 + w_2}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}}, \frac{w_1 + w_2}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}} \right) = \left(\frac{w_1 + w_2}{\frac{w_1}{\mu_{ij}^{\tilde{B}}} + \frac{w_2}{\mu_{ij}^{\tilde{A}}}}, \frac{w_1 + w_2}{\frac{w_1}{\nu_{ij}^{\tilde{B}}} + \frac{w_2}{\nu_{ij}^{\tilde{A}}}} \right) = \tilde{B} \bowtie^w \tilde{A}. \end{aligned}$$

Proposition 4:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m \times n}$. Then (i) $\tilde{A} @^w \tilde{A} = \tilde{A}$, (ii) $\tilde{A} \$^w \tilde{A} = \tilde{A}$, (iii) $\tilde{A} \bowtie^w \tilde{A} = \tilde{A}$

Proof: For all I and j $w_1 > 0, w_2 > 0$

$$\begin{aligned} \text{(i)} \tilde{A} @^w \tilde{A} &= \left[\left(\frac{w_1 \mu_{ij}^{\tilde{A}} + w_2 \mu_{ij}^{\tilde{A}}}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{A}}}}, \frac{w_1 \nu_{ij}^{\tilde{A}} + w_2 \nu_{ij}^{\tilde{A}}}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{A}}}} \right) \right] = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] = \tilde{A} \\ \text{(ii)} \tilde{A} \$^w \tilde{A} &= \left[\left((\mu_{ij}^{\tilde{A}})^{w_1} \cdot (\mu_{ij}^{\tilde{A}})^{w_2} \right)^{1/(w_1+w_2)}, \left((\nu_{ij}^{\tilde{A}})^{w_1} \cdot (\nu_{ij}^{\tilde{A}})^{w_2} \right)^{1/(w_1+w_2)} \right] = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] = \tilde{A} \\ \text{(iii)} \tilde{A} \bowtie^w \tilde{A} &= \left(\frac{w_1 + w_2}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{A}}}}, \frac{w_1 + w_2}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{A}}}} \right) = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] = \tilde{A} \end{aligned}$$

4. Presentation of Weighted Harmonic Mean (H_{WHM}) of Intuitionistic Fuzzy Soft Matrix in Decision Making :

In this segment, we express harmonic mean and weighted harmonic mean of intuitionistic fuzzy soft matrix.

4.1. Weighted Harmonic mean of Intuitionistic Fuzzy Soft Matrix:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m \times n}$. Then weighted harmonic Mean (H_{WHM}) of Intuitionistic Fuzzy Soft Matrix \tilde{H} denoted by $\tilde{H}_{WHM} = [(\sum_{j=1}^n \frac{w_j + w_j}{\frac{w_j}{\mu_{ij}^{\tilde{A}}} + \frac{w_j}{\nu_{ij}^{\tilde{A}}}}, \sum_{j=1}^n \frac{w_j + w_j}{\frac{w_j}{\mu_{ij}^{\tilde{B}}} + \frac{w_j}{\nu_{ij}^{\tilde{B}}})]$, w_j for $j = 1, 2, \dots, n$ are corresponding weights for membership and non-membership

value .

Input: Intuitionistic fuzzy soft sets by m objects , each of which has n parameters.

Output: An optimal solution.

Algorithm

Step- 1: Choose the set of parameters.

Step -2: Make the intuitionistic fuzzy soft matrix for the set of parameters.

Step -3: Compute the harmonic mean for the intuitionistic fuzzy soft matrix for the set of parameters.

Step- 4: Compute the weighted harmonic mean of membership and non-membership value of intuitionistic fuzzy soft matrix as

G_{WAM} .

Step-5: Choose the object with highest membership value. In event of tie, i.e. when more than one object with same highest membership value, select the object with highest membership value as well as the lowest non- membership value.

Example:

Let $X = \{h_1, h_2, h_3, h_4, h_5\}$ be the set of places under consideration and $U = \{\text{expensive (e1), nearby city (e1), cheap (e1)}\}$ be the set of parameters. Suppose Mr.Z is going to buy a household and an intuitionistic fuzzy soft matrix

$$X = \begin{bmatrix} (0.5, 0.3) & (0.6, 0.5) & (0.7, 0.2) \\ (0.7, 0.2) & (0.5, 0.3) & (0.5, 0.3) \\ (0.6, 0.2) & (0.7, 0.2) & (0.7, 0.2) \\ (0.7, 0.3) & (0.6, 0.3) & (0.5, 0.1) \\ (0.8, 0.1) & (0.3, 0.2) & (0.3, 0.1) \end{bmatrix}$$

$$X_{HM} = \begin{bmatrix} 0.350 & 0.0900 \\ 0.308 & 0.0675 \\ 0.441 & 0.0400 \\ 0.350 & 0.0385 \\ 0.154 & 0.015 \end{bmatrix} \dots\dots\dots (1)$$

If we choose to 'near by city' of the homes and weights. 3, .5, .2 are given in the parameters expensive, near by city, and cheap respectively then

$$X_{WGM} = \begin{bmatrix} 0.5330 & 0.309 \\ 0.625 & 0.222 \\ 0.626 & 0.200 \\ 0.652 & 0.250 \\ 0.533 & 0.110 \end{bmatrix} \dots\dots\dots (2)$$

From the above results (1) and (2), it is clear that if we give equal preference, we have .441 and .652 is the highest membership value in both (1) and (2). So h3 home is virtually suited for Mr. Z. But if we give more preference on 'near by city' than other parameters, then h4 home is the most suitable for Mr.Z.

Conclusion

In this report, we have utilized the properties of intuitionistic fuzzy soft matrix. we have given one simple application for decision making problem on the basis weighted harmonic mean. This method can be applied to other decision making problems with uncertain parameters.

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