Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 116 (2018) 50170-50180

ISSN: 2229-712X

An Inventory Model with a Trade Credit as Demand dependent and Time Variant Parameters

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ARTICLE INFO

Article history:

Received: 15 January 2018;

Received in revised form: 4 March 2018;

Accepted: 16 March 2018;

Keywords

Trade Credit as Demand Dependent Polices,

Optimality,

Deterioration,

Inventory Control,

Time Dependent Parameters,

Inflation,

Time Value of Money.

ABSTRACT

In the classic Economic Order Quantity model the purchasing cost of an order is paid at the time of its receipt. In some cases sellers give their retailers a permissible for payments and some others ak buyers to pay all or a fraction of the purchasing cost in advance and may allow them to divide the prepayment into several equal-sized parts. We propose a generalized supplier–retailer inventory model under a given deterministic planning horizons which consists of n different periods using a trade credit policy that attracts more retailers. The trade credit policy adopted here is a demand dependent policy in which the supplier offers the retailer a permissible delay period as a non-decreasing function of the buyer demand. In addition, the demand rate is assumed to be a function of time and of the related period. During the given permissible period no interest is charged by the supplier on the retailer, but beyond that period an interest, with the conditions agreed upon, is charged on .However, during the permissible credit period, the retailer can accumulate the revenue of salls and earns an interest on that revenue.Hence, determinıng such a trade credit period is recognized as an important strategy to increase seller's profitability and minimize the retailer's costs. Sufficient conditions on the existence and uniqueness of the optimal replenishment policy are provided.

INTRODUCTION

In reality, sellers frequently offer his/her buyers a permissible delay in payment known as a trade credit period. Some suppliers, however, allow a certain period to settle payment accounts, but in most cases such allowed periods are fixed and independent of the demand rate or of any other parameters of the problem such as the parameters which may motivate the supplier to extend hıs allowed permissible period according to the retailers demand rate. For example, the supplier would be encouraged to such extension of the trade credit period given to the retailer if the last increases his demand rate accordingly. Therefore, we propose an economic order quantity model(EOQ) from the seller's prospective to determine his/her optimal trade credit period in which the given trade credit increases not only sales but also minimizes the opportunity cost and default risk. The time horizon is deterministic and is divided into n different periods each of which has its own trade credit and its own demand rate as a non-decreasing function the permissible trade credit of that period. Shortages are not allowed in any period. Moreover in most inventory models it is assumed that the parameters of the model do not vary with time. Here, such restriction shall not be considered. That is, the parameters of our model are to be taken as arbitrary functions of time and as dependent on the related period. The mechanism of our system is working as follows. During the related trade credit given to the retailer , no interest is charged by the supplier on the retailer, but beyond that related period an interest, with the conditions agreed upon, is charged on. Such offers provide the buyers with several advantages. First, the buyers will have enough money to run their jobs. Second, buyers may benefit from the generated sales revenue by depositing it into an interest bearing account Third, allowing the delay in payment may motivate the retailers to order more quantities which in turn lead to a reduction in the purchasing cost, ordering cost, and shortage cost. Further, we shall consider that all model parameters are period dependent. Such consideration gives the flexibility of applying our model on more than one item(commodity) each of which has its own parameters and its own trade credit as a demand rate dependent. However, ordering large quantities will, in general, increase the holding cost, the cost of deteriorated or decaying of the stored items, and the potential cost due to inflation and/or time value of money. Given that an interest can be earned by the retailer from the revenue that he receives during the given credit period, the retailers are likely to balance between the effects of all above cost components so as to minimize the net total relevant cost.

The effect of supplier's trade credit policies on the traditional (EOQ) models has received the attention of many researchers. Chia-ueiHo (2011 and 2013) proposed a generalized, integrated, supplier–retailer inventory model using a two-level trade credit policy from both retailer and supplier. Taleizadeh ,A.A(2014) developed an (EOQ) model for an evaporating item with partial backordering and partial consecutive repayments with a real case study of a gasoline station. Zhang,Yu et al(2014) investigated the buyer's inventory policy under advance payment, including all payment in advance partial-advanced, and partial-delayed payments.

Guria,A. et al (2013) presented an inventory policy for an item with inflation and selling price as demand dependent under deterministic and random planning horizons allowing and not allowing shortages under the existence of a provision for an immediate part payment to the wholesaler and some other conditions.

JiangWu et al (2014) have dealt with the trade credit period from the seller's perspective in which they try to answer how to determine credit period to increase seller's profitability. Chih-TeYang et al (2015) presented a generalized model to determine the optimal trade credit, in which preservation technology investment and replenishment strategies that maximize the retailer's total profit and to minimize the default risk occurs over a finite planning horizon. XuChen et al (2015) considered a firm facing stochastic demand for two products with a downward, supplier-driven substitution and customer service objectives for a single period problem in which the fundamental challenge is to determine in advance the profit for maximizing inventory levels of both products that will meet given service level objectives. Liang-YuhOuyang and Chun-TaoChang (2013) explored the effects of reworking imperfect quality items and trade credit on the Economic Production Quantity (EPQ) model with imperfect production processes and complete backlogging. Jinn-TsairTeng et al (2014) proposed an (EPQ) model from the seller's prospective to determine his/her optimal trade credit period and production lot size simultaneously in which trade credit increases not only sales but also opportunity cost and default risk, and production cost which declines and obeys a learning curve phenomenon. Goyal (1985) has studied the effects of the permissible delay in payment on the standard (EOQ) model for non-deteriorating items where shortages are not allowed. He showed that such delay in payment leads to an increase in both the order quantity and the replenishment interval and to a sharp decrease in the total annual cost. Aggarwal and Jaggi (1995) presented a model similar to that of Goyal (1985) but with constant deterioration rate where they presented a sensitivity analysis that reveal the effects of such deterioration rate on several factors of the (EOQ) policies. Jamal *et al.* (1994) and Shah *et al.* (1998) extended the models of Aggarwal and Jaggi and Goyal, respectively, by allowing for shortages where it is further shown that the total cost is less than that in the non-shortage case. Jaggi and Aggarwal (1994) and Chung (1989) considered a similar model as that of Aggarwal and Jaggi (1995) but with a discount cash-flow (DCF) approach, and approximately reflected the effect of delay in payment on the optimal inventory policy. Kun et al[2009] considered the optimal ordering policy of the (EOQ) model under trade credit depending on the order quantity from the(DCF) approach. Some useful theoretical results on the subject of permissible delay in payment have been also achieved. Chung (1998) provided conditions under which the total cost introduced by Goyal (1985) is convex. Chu *et al.* (1998) showed that the total cost of the system introduced by Aggarwal and Jaggi is piecewise convex which in turn lead to an improved solution procedure of the considered system. Some other researchers have dealt with the subject from different point of view. Hwang and Shin (1997) studied the case when the demand rate is a function of the retailer's price where it is, then, shown that the retailer's optimal price and the optimal lot size can be determined simultaneously. In Khouja and Mehrez (1996) two main types of supplier credit policies, where the credit policy may be independent or linked to the order quantity, are addressed. Kim *et al.* (1995) showed that the net profit of both the retailer and the supplies can be increased through a wise selection of the credit period. Balkhi (2004 and 2011) introduced a trade credit inventory model that generalizes several of the previously introduced models so that most of these models result as special case of Balkhi (2011) model. Other types of inventory control models with trade credits are proposed by many researchers. Luo (2007) treated a single-vendor, a single-buyer supply chain for a single product, and a model to study and analyze the benefit of coordinating supply chain inventories through the use of credit period is proposed. Jinn-Tsair Teng (2009) established an (EOQ) model for good retailers who receives a full trade credit by his supplier, and offers either a partial or a full trade credit to his bad customers. Sana. And Chaudhuri (2008) modeled the retailer's profit-maximizing strategy when confronted with supplier's trade offer of credit and price-discount on the purchase of merchandise when retailers are facing many scenarios of time dependent demands for different kinds of goods. All above mentioned papers are of infinite time horizon type. A finite horizon inventory model with constant rate of deterioration and without shortages and with equal periods, when a delay in payment is permissible, has been investigated by Liao *et al.* (2000). Balkhi (2011) generalized the model of Liao *et al* by considering that the given time horizon consists of several periods with different lengths each of which has its own demand rate and its own trade credit period. Most of the above mentioned papers considered that the cost parameters of the underlying inventory model as well as the demand and deterioration rates are all known and constant. Also, in most of those papers, neither inflation nor time value of money are taken in to accounts. Balkhi and Tadj (2008) have studied a general EOQ model in which the cost parameters and both demand and deterioration rates are arbitrary functions of time. However, the effect of trade credit policies is not considered in this last model.

As it can be noted from the above literature review, there are several limitations while developing mathematical models in inventory control under trade credit policies. One of these limitations is that the assumption of constant demand and/or deterioration rates may not always be appropriate for many inventory items as, for example, it is the case for products whose demand is affected by seasons or occasions. Another limitation is the case of a given finite horizon divided it into equal inventory cycles. A third one is that the permissible delay in payment (i.e., the trade credit period) is assumed to be the same for all cycles. A fourth limitation is concerned with constant cost parameters which is not realistic for most practical inventory control systems. The fifth limitation is that the trade credit is independent of the demand rate of the retailer as wellas the related period. *Thus, there is a need to drop all above limitations and develop a more general inventory model in which the demand and deterioration rates and all cost parameters are general functions of time, each of the **n** different periods has its own demand rate, its own trade credit, and its own cost parameters so that it can be specified to on or more item. In this case many of the previously introduced models can be obtained as special cases of such a general model.*

The purpose of this paper is to generalize the paper of Balkhi [2011] and most of the above introduced models in the following fronts.

The given time horizon consists of ***n*** different periods each of which has its own trade credit as a non-decreasing function of its demand rate, has its own parameters, and has its own ordered quantity which is equal to the inventory level at the start of that period. Also, both demand and deterioration rates and all cost parameters shall be considered as general and continuous functions of time, and period dependent. These assumptions give the flexibility of specifying one or more of the ***n*** different periods to one or more items. These assumptions lead also to the possibility of choosing the demand not only to be a stock dependent, but also to make a possibility of ordering the required item for one or more period. Therefore, our order quantity is period and time dependent. Further, the effects of both inflation and time value of money shall be incorporated in all cost components. Our main concern will be on the theoretical results. So, the proposed model with the above mentioned general features is developed, solved, and no approximations are used neither in the total net cost nor in any other relations. Then, rigorous mathematical methods are used to prove the existence of a unique vector of the relevant decision variables that solve the underlying inventory system.

MODEL ASSUMPTIONS AND NOTATION.

(1) The time horizon is of ***H*** units long and is divided into ***n*** *different* perıods.

(2) One or more items are ordered at the beginning of each of the n *periods.*

(3) For perıod ***j(j=1,2,….,n)*** we denote by t_{j-1} for the beginning of the perıod , and by t_j for the end of the perıod.

(4) $D_j(t)$ is the demand rate for period ***j*** at time t .

(5) $M_j(D_j(t))$ is the trade credit for period ***j*** ,as a non-decreasing function of the demand rate, offered from the supplier to his retailer. As examples of such trade credit periods which can be considered as period dependent and as a non- decreasing functions of the demand rate are linear and /or exponent functions .Viz; either the case,

$$M_j(D_j(t)) = a_j D_j(t) + b_j \quad \text{, with } a_j \geq 0 \tag{1.1}$$

where $\dfrac{\partial M_j(D_j(t_j))}{\partial D_j(t_j)} = a_j \geq 0$

or

$$M_j(D_j(t)) = c_j e^{a_j D_j(t)+b_j} \text{ , with } a_j, c_j \geq 0 \tag{1.2}$$

where, $M_j'(D_j(t_j)) = \dfrac{\partial M_j(D_j(t_j))}{\partial D_j(t_j)} = c_j a_j e^{a_j D_j(t)+b_j} \geq 0$ when c_j, a_j have the same sign.

But $\dfrac{\partial M_j(D_j(t_j))}{\partial t_j} = \dfrac{\partial M_j(D_j(t_j))}{\partial D_j(t_j)} . \dfrac{\partial (D_j(t_j))}{\partial t_j}$

In (1.1), the assumption $a_j \geq 0$ guarantee that $M_j(.)$ is a non-decreasing function of $D_j(t)$, whereas in (1.2), the assumption a_j, c_j have the same sign guarantee that $M_j(.)$ is a non-decreasing function of $D_j(t)$. The case a_j =0 give us a period dependent trade credit $M_j(D_j(t)) = b_j$, $M_j(D_j(t)) = c_j e^{b_j}$ as the case considered in Balkhi (2011).

The above non-decreasing functions given in formula (1.1) and (1.2) or any other similar non-decreasing function of the demand rate can be considered so that the parameters $a_j \geq 0$ or a_j, c_j with the same sign can be treated as a decision variables from the supplier.

(4)The ordered items deteriorate while they are effectively in stock and the deterioration rates is an arbitrary and known function of time, and is period dependent denoted as $\theta_j(t)$.

But there is no repair or replacement of deteriorated items.

(5) t_0 =0 and t_n ***=H.***

(6) Q_j is the quantity ordered at the beginning of the period ***j***, and, $I_j(t)$ is the inventory level at time t. *Note also that both* Q_j *and* $I_j(t)$ *are implicitly dependent on* $M_j(D_j(t))$

(7)Lead time is negligible and the replenishment rate is infinite.

(8)Shortages are not allowed in any period. Such assumption can be justified by the fact that the supplier shall provide his retailer by some kind of open permissible trade credit period of demand dependent which in turn shall encourage the retailer to demand more quantities of the required items which in turn does not allow shortages.

(9)A complete order of Q_j units arrives by the beginning of the period ***j***, but there is a demand dependent permissible delay $M_j(D_j(t))$ for payment till the period agreed up on.

(10) The time and period dependent cost structure for period ***j*** of the underlying inventory system is as follows:

(i) $c_j(t)$ is the unit cost in period j at time t .

(ii) $h_j(t)$ is the holding cost in period j per unit per unit time at time t .

(iii) $s_j(t)$ is the unit selling price in period j at time t .

(iv) $k_j(t)$ is the ordering cost in period j per order at time t.

Note that making the above unit cost to be period and time dependent gives the flexibility of specifying each period of the ***n*** different periods for one or more of a specific item(commodity) as well as some or all periods to one item. Note ,also, that the period dependent of the demand rate $D_j(.)$ and the trade credit $M_j(.)$ as well as all cost parameters give us the flexibility to apply this model for each period separately with its separate item(s) or with the same item for one or more of the other periods.

(11)All above time and period dependent costs are free of interest charges.

(12)All costs are affected by inflation rate and time value of money. We shall denote by r_1 the inflation rate and by r_2 the discount rate representing the time value of money so that $r = r_2 - r_1$ is the discount rate net of inflation.

(13) For any the ***n*** different periods, there is an interest charged for the items being held in stock after the given trade credit period $M_j(D_j(t))$ concerned with that period .i.e for those items not being sold during $M_j(D_j(t))$. We denote by i_c the per monetary unit per unit time charge payable at time *t = 0.*

(14) All generated sales revenue can be deposited into an interest bearing account at a rate i_e (at time t= 0) per monetary unit per unit time. We generally have i_c and i_e are the same for all periods and $i_c > i_e$.

The proposed inventory trade credit system operates as follows. A quantity of Q_j units is ordered and stored at the beginning of period ***j***. The supplier (or seller) offers the buyer (or retailer) an agreed up on trade period which is a demand and period dependent $M_j(D_j(t))$ during which there is no charged interest so that the account is to be settled by or after the end of this credit period. Otherwise an interest of rate i_c will be charged for any amount delayed beyond that allowed of the agreed up on period. On the other hand the retailer may deposit all generated sales revenue into an interest bearing account of rate i_e and may pay off his supplier by at most the end of the agreed up on period. Whereas the inventory system operates as follows.

A quantity of Q_j *units* is ordered at the beginning of period ***j***. This quantity is subject to consumption of rate $D_j(t)$ and deterioration rate $\theta_j(t)$ till the inventory level reaches to zero by the time where a new order for the next period, depending on the demand and the period, is ordered. The objective of the retailer is to minimize his net total relevant costs in any or all periods and for all all ordered items. The process is repeated for each period. **Fig 1.** Shows the variation of the underlying inventory system.

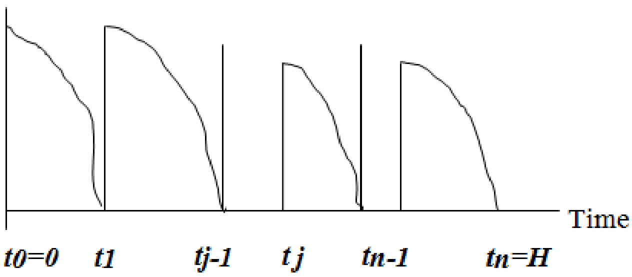

Fig1. The variation of inventory levels in the time horizon [0,H].

MODEL BUILDING.

Following the above assumptions and notations

The changes of $\mathbf{I_j}$ **(t)** in period **j(j=1,2,…,n)** which is $[t_{j-1}, t_j]$ are given by the following differential equation

$$\frac{dI_j(t)}{dt} = -D_j(t) - \theta_j(t).I_j(t) \qquad ; t_{j-1} \le t \le t_j \tag{2.1}$$

With the boundary condition $I_j(t_j) = 0$. The solution of (2.1) is given by:

$$I_j(t) = e^{-\delta_j(t)} \int_t^{t_j} e^{\delta_j(u)} D_j(u)du \qquad ; t_{j-1} \le t \le t_j \tag{2.2}$$

Where

$$\delta_j(t) = \int_0^t \theta_j(u)du \tag{2.3}$$

Next we derive the present worth of all cost components for period j (j=1,2,…,n).

The present worth of the holding cost, say (PW1), of the amount being held in stock during the period $\lfloor t_{j-1}, t_j \rfloor$ is given by

PW1 = $\displaystyle\int_{t_{j-1}}^{t_j} e^{-rt} e^{-\delta_j(t)} h_j(t).I_j(t)\,dt = \int_{t_{j-1}}^{t_j} e^{-rt-\delta_j(t)} h_j(t).\left(\int_t^{t_j} e^{\delta_j(u)} D_j(u)\,du \right)dt$

Integrating by parts, then PW1 is reduced to:

$$\text{PW1} = \int_{t_{j-1}}^{t_j} e^{\delta_j(t)}\left[H_j(t) - H_j(t_{j-1})\right]D_j(t)\,dt \tag{2.4}$$

Where

$$H_j(t) = \int_0^t e^{-rt-\delta_j(t)} h_j(t)dt \tag{2.5}$$

The number of items received in beginning of the period $[t_{j-1}, t_j]$ is given by

$$Q_j = I_j(t_{j-1}) = e^{-\delta_j(t_{j-1})} \int_{t_{j-1}}^{t_j} e^{\delta_j(u)} D_j(u)\,du \tag{2.6}$$

Since items are received at the beginning of each period (recall that lead time is negligible), the unit item's cost is equal to $c_j(t_{j-1})Q_j$. Hence, the present worth of the items' cost in period j, say PW2, is equal to

$$PW2 = c_j(t_{j-1})Q_j = c_j(t_{j-1})e^{-rt_{j-1}-\delta_j(t_{j-1})} \int_{t_{j-1}}^{t_j} e^{\delta_j(u)} D_j(u)du \tag{2.7}$$

Note that the items' cost include the cost of deteriorated and consumed (none deteriorated) items.
The present worth of ordering cost in period **j**, say PW3, is equal to

$$PW3 = e^{-rt_{j-1}} k_j(t_{j-1}) \tag{2.8}$$

To find the present worth for each of the interest charged and the interest earned. We distinguish two cases.

Case (1). Cycles with $M_j(D_j(t_j)) \le t_j$.

In this case we need to find the present worth of the cost which will result from interest charged for items in inventory not being sold after $M_j(D_j(t))$. But the number of items not being sold in a small time interval dt after $M_j(D_j(t_j))$ is equal $I_j(t)d(t)$. Thus the present worth of interest charged, say PW4, is equal to

$$PW4 = i_c \int_{M_j(D_j(t_j))}^{t_j} e^{-rt} c_j(t) I_j(t)dt = i_c \int_{M_j(D_j(t_j))}^{t_j} e^{-rt} c_j(t) e^{-\delta_j(t)} \left(\int_t^{t_j} e^{\delta_j(u)} D_j(u)du \right)dt$$

Integrating by parts, we find

$$PW4 = i_c \int_{M_j(D_j(t_j))}^{t_j} \left[G_j(t) - G_j(M_j(D_j(t_j))\right] e^{\delta_j(t)} D_j(t)dt \tag{2.9}$$

Where

$$G_j(t) = \int_0^{t_j} e^{-rt-\delta_j(t)} c_j(t)dt \tag{2.10}$$

Similarly, the present worth of the interest earned, say PW5, in the permissible period [$t_{j-1}, M_j(D_j(t_j))$], which has positive stock, and in the rest of the period [$M_j(D_j(t_j))$, t_j] from the remaining cash, is given by:

$$PW5 = i_e \int_{t_{j-1}}^{M_j(D_j(t_j))} e^{-rt} s_j(t) D_j(t)dt + i_e \int_{M_j(D_j(t_j))}^{t_j} e^{-rt} s_j(t) D_j(t)dt = i_e \int_{t_{j-1}}^{t_j} e^{-rt} s_j(t) D_j(t)dt \tag{2.11}$$

Hence, the net total variable cost in period j in Case (1) as a function of t_{j-1}, t_j, and $M_j(D_j(t))$ say W_{1j} (t_{j-1}, t_j, $M_j(D_j(t_j))$), is given by

$$\boldsymbol{W_{1j}\left(t_{j1}, t_j, {}_{M_t(D_t(t_t)}\right) = \int_{t_{t-1}}^{t_t} e^{\delta_t(t)} \left[H_j(t) - H_j\left(t_{j-1}\right)\right] D_j(t)dt + c_j\left(t_{j-1}\right) e^{-rt_{t-1}-\delta_t(t_{t-1})} \int_{t_{t-1}}^{t_t} e^{\delta_t(t)} D_j(u)du + e^{-rt_{t-1}} K_j\left(t_{j-1}\right) + i_c \int_{M_t(D_t(t_t))}^{t_t} \left[G_j(t) - G_j\left(M_j\left(D_j(t)\right)\right] e^{\delta_t(t)} D_j(t)dt - i_e \int_{t_{t-1}}^{t_t} e^{-rt} s_j(t) D_j(t)dt} \tag{2.12}$$

Case (2).Cycles with $M_j(D_j(t_j)) \ge t_j$.

In this case we do not have an interest charge for inventory not being sold after $M_j(D_j(t_j))$ since, then, we have $M_j(D_j(t_j)) \ge$ **tj .** But the interest earned per period j consists, here, from two parts. The first part is the interest earned during the positive inventory in period [t_{j-1}, t_j,] which is given by

$$i_e \int_{t_{j-1}}^{t_j} e^{-rt} s_j(t) D_j(t) dt \qquad (2.13)$$

The second part is the interest earned from the remaining cash during the time period [t_j, $M_j(D_j(t_j))$] after the depletion of inventory. The present worth of this last part is equal to

$$i_e \ [M_j(D_j(t_j)) - \text{tj}] \int_{t_j}^{M_j(D_j(t_j))} e^{-rt} s_j(t) D_j(t) dt \qquad (2.14)$$

Thus the net total variable cost in period j in Case (2) as a function of t_{j-1}, t_j, and $M_j(D_j(t))$. say W_{2j} (t_{j-1}, t_j, , $M_j(D_j(t_j)$) , is given by

$$\begin{aligned} \boldsymbol{W_{2j}}\left(\boldsymbol{t_{j-1}}, \boldsymbol{t_j}, {}_{M_t(D_t(t_t)}\right) &= \int_{t_{t-1}}^{t_t} e^{\delta_t(t)} [H_j(t) - H_j(t_{j-1})] D_j(t) dt \\ &+ c_j(t_{j-1}) e^{-rt_{t-1} - \delta_t(t_{t-1})} \int_{t_{t-1}}^{t_t} e^{\delta_t(t)} D_j(u) du + e^{-rt_{t-1}} K_j(t_{j-1}) \\ &- i_e \int_{t_{t-1}}^{t_t} e^{-rt} s_j(t) D_j(t) dt \, i_e \left[\left(M_j\left(D_j(t)\right) - tj\right] \int_{t_t}^{M_t(D_t(t_t))} e^{-rt} s_j(t) D_j(t) dt \end{aligned}$$

(2.15)

Now we define

$$\delta_j = \begin{cases} 1 & if \quad M_j(D_j(t_j)) \le \text{tj} \\ 0 & otherwise \end{cases} \qquad (2.16)$$

Then by (2.16), we can unify (2.12) and (2.13) by the following formula

$$\begin{aligned} \boldsymbol{W_{2j}}\left(\boldsymbol{t_{j-1}}, \boldsymbol{t_j}, {}_{M_t(D_t(t_t)}\right) &= \int_{t_{t-1}}^{t_t} e^{\delta_t(u)} [H_j(u) - H_j(t_{j-1})] D_j(u) du \\ &+ c(t_{j-1}) e^{-rt_{t-1} - \delta_t(t_{t-1})} \int_{t_{t-1}}^{t_t} e^{\delta_t(t)} D(u) du + e^{-rt_{t-1}} K_j(t_{j-1}) \\ &+ \alpha_j i_c \int_{M_t(D_t(t_t))}^{t_t} \left[G_j(u) - G_j\left(M_j\left(D_j(tj)\right)\right)\right] e^{\delta_t(u)} D_j(u) du \\ &- i_e \int_{t_{t-1}}^{t_t} e^{-ru} s_j(u) D_j(u) du - i_e(1 - \alpha_j)[M_j\left(D_j(tj)\right) - tj \int_{t_t}^{M_t(D_t(t_t))} e^{-ru} s_j(u) D_j(u) du \end{aligned}$$

(2.17)

Note that

$$\text{Wj}(\text{t}_{\text{j-1}}, \text{t}_{\text{j}}, M_j(D_j(t_j))) = \begin{cases} \text{W1j}(\text{t}_{\text{j-1}}, \text{t}_{\text{j}}, M_j(D_j(t_j))) & if \quad \alpha_j = 1 \\ \text{W2j}(\text{t}_{\text{j-1}}, \text{t}_{\text{j}}, M_j(D_j(t_j))) & if \quad \alpha_j = 0 \end{cases} \qquad (2.18)$$

Hence, the net total relevant cost in the whole time horizon [0, H], where $\mathbf{t_0=0}$ and $\mathbf{t_n=H}$**, is given by**

W = $\sum_{j=1}^{j=n}$ W_j (t_{j-1}, t_j, $M_j(D_j(t)$)

$$= \sum_{j-1}^{j-n} \{ \int_{t_{t-1}}^{t_t} e^{\delta_t(u)} [H_j(u) - H_j(t_{j-1})] D_j(u) du$$
$$+ c(t_{j-1}) e^{-rt_{t-1}-\delta_t(t_{t-1})} \int_{t_{t-1}}^{t_t} e^{\delta_t(t)} D(u) du + e^{-rt_{t-1}} K_j(t_{j-1})$$
$$+ \alpha_j i_c \int_{M_t(D_t(t_t))}^{t_t} [G_j(u) - G_j(M_j(D_j(tj)))] e^{\delta_t(u)} D_j(u) du$$
$$- i_e \int_{t_{t-1}}^{t_t} e^{-ru} s_j(u) D_j(u) du - i_e(1-\alpha_j)[M_j(D_j(tj)) - tj \int_{t_t}^{M_t(D_t(t_t))} e^{-ru} s_j(u) D_j(u) du$$

(2.19)

Thus our problem, which we shall refer to as **(P)**, is

Minimize **W** given by (2.19) subject to:

$\mathbf{0 = t_0 \le t_1 \le t_2 \le \ldots\ldots\ldots\ldots\ldots \le t_n = H}$ (2.20)

Note that constraints (2.20) must be satisfied for any feasible solution of the underlying inventory system since otherwise the problem would have no meaning.

PROBLEMS' SOLUTION

To solve problem (P) *we shall first ignore constraints (2.20) and consider* ***n*** *to be fixed* .We refer to the resulting unconstrained problem as ***(P₁)***. Then, the necessary conditions for having a minimizing solution for ***(P₁)*** are:

$$\frac{\partial W}{\partial t_j} = \sum_{j=1}^{j=n} \partial \mathrm{W_j}(\mathrm{t_{j-1}}, \mathrm{t_j}, M_j(D_j(t_j)))/\partial \mathrm{t_j} = 0, \quad j=1, 2, \ldots\ldots, n-1 \qquad (2.21)$$

Note that $\partial \mathrm{W_j}(\mathrm{t_{j-1}}, \mathrm{t_j}, M_j(D_j(t_j)))/\partial \mathrm{t_j} = 0$ for all j $\Rightarrow \frac{\partial W}{\partial t_j} = 0$. Hence, (2.21) give the critical points for both $\mathrm{W_j}$ *and W*.

To find $\partial \mathrm{W_j}(\mathrm{t_{j-1}}, \mathrm{t_j}, M_j(D_j(t_j)))/\partial \mathrm{t_j}$ we note that t_j appears in the following Terms

$\int_{t_{t-1}}^{t_t} e^{\delta_t(u)} [H_j(u) - H_j(t_{j-1})] D_j(u) du + c(t_{j-1}) e^{-rt_{t-1}-\delta_t(t_{t-1})} \int_{t_{t-1}}^{t_t} e^{\delta_t(u)} D_j(u) du - i_e \int_{t_{t-1}}^{t_t} e^{-ru} s_j(u) D_j(u) du + \int_{t_t}^{t_{t+1}} e^{\delta_{t+1(u)}} [H_{j+1}(u) - H_j(t_j)] D_{j+1}(u) du + c(t_j) e^{-rt_t - \delta_{t+1}(t_t)} \int_{t_t}^{\delta_{t+1(u)}} D_j(u) du + \alpha_j i_c \int_{M_t(D_t(t_t))}^{t_t} [G_j(u) - G_j(M_j(D_j(tj)))] e^{\delta_t(u)} D_j(u) du + e^{-rt_t} K_j t_j - i_e \int_{t_t}^{t_{t+1}} e^{-ru} s_j(u) D_{j+1}(u) du - i_e(1-\alpha_j)[M_j(D_j(tj)) - tj] \int_{t_t}^{M_t(D_t(t_t))} e^{-ru} s_j(u) D_j(u) du.$ Let $Y'(x) = \partial Y / \partial x$, then

$$\frac{\partial W_j(t_{j-1}, t_j, M_j(D_j(t_j)))}{\partial t_j} = 0 \Leftrightarrow e^{\delta_t(t_t)}[H_j(t_j) - H_j(t_{j-1})] D_j(t_j) + c(t_{j-1}) e^{-rt_{t-1}-\delta_t(t_{t-1})} e^{\delta_t(t_t)} D_j(t_j) - i_e D_j(t_j) e^{-rt_t} s_j(t_j) + H'_j(t_j) e^{\delta_{t+1}(t_t)} [H_{j+1}(t_j) - H_j(t_j)] D_{j+1}(t_j) + \left[c'(t_j) e^{-rt_t - \delta_{t+1}(t_t)} - c(t_j)\left(r + \delta'_{j+1}(t_j)\right) e^{-rt_t - \delta_{t+1}(t_t)}\right] \int_{t_t}^{t_{t+1}} e^{\delta_{t+1}(u)} D_j(u) du - c(t_j) e^{-rt_t - \delta_{t+1}(t_t)} e^{\delta_{t+1}(t_t)} D_j(t_j) - \alpha_j i_c D'_j(t_j) G'_j(M_j(D_j(t_j)) M'_t(D_j(t_j)) \int_{M_t(D_t(t_t))}^{t_t} [G_j(u) - G_j(M_j(D_j(t_j)))] e^{\delta_t(u)} D_j(u) du + \alpha_j i_c [G_j(t_j) - G_j(M_j(D_j(t_j)))] e^{\delta_t(t_t)} D_j(t_j) + i_e e^{-rt_t} s_{j+1}(t_j) D_{j+1}(t_j) - r e^{-rt_t} k_j(t_j) + e^{-rt_t} k'_j(t_j) - [i_e(1-\alpha_j)[M_j(D_j(t_j)) - t_j] \times [-e^{-rt_t} s_j(t_j) D_j(t_j) + e^{-rM_t(D_t(t_t))} s_j M_j(D_j(t_j)) D_j(M_j(D_j(t_j)) \times D'_j(t_j)[M'_j(D_j(t_j)) + [D'_j(t_j)[M'_j(D_j(t_j)) - 1 \int_{t_t}^{M_t(D_t(t_t))} e^{-ru} s_j(u) D_j(u) du = 0$$
$$; j = 1 \ldots\ldots\ldots n-1 \qquad (2.22)$$

Note that (2.22) are *(n-1)* equations with *(n-1)* decision variable, namely $t_1, t_2, \ldots, t_{n-1}$.*(recall that* $0 = t_0$, $t_n = H$*)*.The solution of these equations (if it exists) gives the critical points $\mathrm{W_j}(\mathrm{t_{j-1}}, \mathrm{t_j}, M_j(D_j(t_j)))$, hence of *W*.

Next we deliver sufficient conditions for which any existing solution of ***(P1)*** is a minimizing solution to ***(P1).*** For this purpose ,let $\boldsymbol{T} = (0 = t_0, t_1, t_2 \ldots, t_n = H$ *.)* be a solution of equations (2.21) and let ***HM(T)*** be the value of the **Hessian Matrix at *T*** , then , by Bazara et al (1993), Stewart (1973, page 143 Chapter 3) and Theorem 3 of Balkhi and Benkhrouf (2004), ***HM(T)*** is positive definite if

$W_{jj} > \left| W_{(j+1)j} \right| + \left| W_{(j-1)j} \right|$ for $j = 2, \ldots\ldots, n-2$, $W_{jj} > \left| W_{j(j+1)} \right|$ *for* $j=1$, *and*

$$W_{jj} > \left| W_{(j-1)j} \right| \quad for \;\; j = n-1 \tag{2.23}$$

Where

$$W_{jj} = \partial^2 \mathrm{W} \Big/ \partial \mathrm{t_j}^2 , \quad W_{jk} = \partial^2 \mathrm{W} \Big/ \partial t_j \partial t_k$$

The above arguments lead to the following theorem.

Theorem 1. *For fixed **n,** any existing solution of (P1) is a minimizing solution to (P1) if this solution satisfies conditions (2.23).*

UNIQUENESS AND GLOBAL OPTIMALITY OF THE SOLUTION.

In this section we shall show that any existing and minimizing solution of ***(P1)*** is unique (hence global optimal) of both ***(P1)*** and ***(P)***. We shall show this in three steps:

First we show that ***(P1)*** depend only on one of the variables $t_1, t_2 \ldots\ldots, t_{n-1}$. Second, we show that under the hypothesis of Theorem **1.** , then any existing and minimizing solution of ***(P1)*** is unique.

Third, we show that under the result of Theorem **1**. then, the total net relevant cost ***W*** is convex with respect to ***n.***

Now, from equations (2.22) and for ***j=1*** we have

$$\boldsymbol{e^{\delta_t(t_1)}[H_1(t_1) - H_1(0)]D_1(t_1) + c(0)e^{\delta_t(0)}e^{\delta_t(t_1)}D_1(t_1) - i_e D_1(t_1)e^{-rt}s_1(t_1) + H_1'(t_1)e^{\delta_2(t_t)}[H_2(t_1) - H_1(t_1)]D_2(t_1) +}$$
$$\boldsymbol{\left[c'(t_1)e^{-rt-\delta_2(t_1)} - c(t_1)\big(r + \delta_2(t_1)\big)e^{-rt-\delta_2(t_1)}\right] \int_{t_1}^{t_2} e^{\delta_{2(u)}}\, D_1(u)du - c(t_1)e^{-rt_1}D_j(t_j) -}$$
$$\boldsymbol{\alpha_j i_c D_1'(t_1)G_1'(M_1\big(D_1(t_1)\big)M_1'(D_1(t_1)) \times \int_{M_t(D_t(t_t))}^{t_t} [G_1(u) - \; G_1\big(M_1\big(D_1(t_1)\big)\big)]e^{\delta_1(u)}D_1(u)du +}$$
$$\boldsymbol{\alpha_j i_c\big[-G_1 M_1\big(D_1(t_1)\big)\big]e^{\delta_1 t_1}D_1(t_1)\big)\, e^{\delta_1(t_1)}D_1(t_1) + i_e e^{-rt_1}s_2(t_1)D_2(t_1) + i_e(1 - \alpha_1)[M_1\big(D_1(t1)\big) - t_1] \times}$$
$$\boldsymbol{e^{-rt_1}s_1(t_1)D_1(t_1) - e^{-rM_t(D_t(t_t))}s\big(M_1\big(D_1(t_1)\big)D_1(M_1(D_1(t_1)) \times D_1'(t_1)[M_1'(D_1(t_1))\big] + [D_1'(t_1)[M_1'\big(D_1(t_1)\big) -}$$
$$\boldsymbol{1] \int_{t_t}^{M_t(D_t(t_t))} e^{-ru}s_1(u)D_1(u)du - r^{e^{-rt_t}}k_1'(t_1) = 0}$$

(3.1)

From the second term of equation (3.1) we can deduce that t_2 is a function of t_1. Also, from (2.22) and for *j=2* , we can see that t_3 is a function of t_1 and t_2 hence it is a function of t_1 and so forth we can see that each of the variables $t_1, t_2 \ldots\ldots, t_{n-1}$ is a function of t_1 , say

$$t_j = f_j\,(t_1), \qquad j=1,..,n-1 \text{ with } t_1 = f_1\,(t_1) = t_1 \tag{3.2}$$

Next we state some important results

Lemma1. Under the hypothesis of Theorem **1**. we have

$$t'_{j+1} > t'_j > 0 \qquad\qquad j = 1,2,\ldots\ldots\ldots, n-1 \tag{3.3}$$

where $t'_j = f'(t_1)$

Proof. The proof of this theorem is similar to the proof of Lemma **1.** of Balkhi (2011), so it will not be repeated again.

Now relations (3.3) and (3.2) lead to the following corollary

Corollary 1. *Under the hypothesis of* **Theorem 1**.,*then*

(i)All variables $t_1, t_2 \ldots\ldots, t_{n-1}$ are increasing functions of t_1 and of each other's.

*(ii) For fixed **n** , any existing and minimizing solution to problem **(P1)** is an existing and minimizing solution to problem **(P)***

Proof. The proof of part (i) is clear from relations (3.2) and(3.3). Again, from (3.3) and by the theory of Real Analysis we have

$$t_{j+1} \geq t_j \geq 0 \tag{3.4}$$

Thus constraints (2.20) hold for any existing and minimizing solution of (***P1)*** if $t_1 \geq 0$. But, as an implication of Kohn-Tucker necessary conditions, the last inequality $t_1 \geq 0$ needs not to be considered. Hence, constraints (2.20) are satisfied for any feasible solution of (2.22) .Hence, such a solution is an existing and minimizing solution to ***(P).***

Some other main results follow.

Theorem 2. Under the hypothesis of Theorem **1**., and Lemma**1**., any existing and minimizing solution of ***(P)*** is the unique solution of ***(P).***

Proof. Let ***T = (0 =t₀, t₁,t₂ ……,tₙ₋₁,tₙ=H)*** be an existing and minimizing solution of ***(P) .*** By relations (3.2), the amount $\sum_{j=1}^{j=n}(t_j - t_{j-1}) - H$ is a function of, t_1. Our idea in is to show that the equation

$$\sum_{j=1}^{j=n} (t_j - t_{j-1}) - H = 0 \tag{3.5}$$

as an equation of t_1 ,either has a unique solution or it does not have any solution. To see this, let us denote by **Z(*t₁*)** the left hand side of (3.5). Then $Z'(t_1) = \sum_{j=1}^{j=n} (t'_j - t'_{j-1}) > 0$ by(3.3) , which means that **Z(*t₁*)** is an increasing function of t_1 . Now, if $Z(0) \leq 0$, then (3.5) has a unique solution. If however Z(0) >0 then (3.5) does not have any solution (see **Fig. 2**) . This leads to the desired result.

Theorem 3. Consider the following two different existing schedules with ***n*** and (***n+1)*** entries, say *T = (0 =t0 , t1 , t2, …,tn=H), S=(T,n) and* $\hat{T} = (0 = \hat{t}_0, \hat{t}_1, \hat{t}_2, \ldots\ldots, \hat{t}_{n+1} = H)$, $\hat{S} = (\hat{T}, n+1)$ .If the hypothesis of Theorem **1.,**and

Lemma **1.** hold, then the entries of $\hat{S}$ lies between the entries of S, that is;

$$0=\hat{t}_0 \le, \hat{t}_1 \le t_1 \le, \hat{t}_2 \le t_2 \le, \ldots\ldots\ldots\ldots \hat{t}_n \le t_n = \hat{t}_{n+1} = H \tag{3.6}$$

Proof. The proof of this theorem mimics the proof of Theorem 3 of Balkhi (2001) so it will not be repeated again.

Theorem 4. Suppose that the hypothesis of Theorem **1.** and Lemma **1.** hold and ***MS*** is the set of all minimizing solutions of the form ***S=(T,n)*** for the underlying inventory system ,then under some seemingly possible conditions , there exists a unique vector $S^* = (T^*, n^*)$ from ***MS*** for which the net total relevant cost of this system is minimum .

Proof. From (2.17) and (2.19) we can rewrite *W* as the sum of set up costs,say *W0* ,and the rest of the cost components ,say *WR*. That is

W= W0 +WR where *W0* = $\sum_{j=1}^{j=n} e^{-rt_{j-1}1} k(t_{j-1})$ and *WR=W- W0,* .If we ignor the terms $-re^{-rt_1}k_1(t_1)+e^{-rt_1}k_1'(t_1)$, then from (3.1) we have

$$\frac{\partial W_R}{\partial t_1} = e^{\delta_1(t_1)}\left[H_1(t_1)-H_1(0)\right]D_1(t_1) + c(0)e^{\delta_j(0)} e^{\delta_1(t_1)} D_1(t_1) + H_1'(t_1)e^{\delta_2(t_1)}\left[H_2(t_1)-H_1(t_1)\right]D_2(t_1)$$

$$\boldsymbol{\frac{\partial W_R}{\partial t_1} = e^{\delta_t(t_1)}[H_1(t_1)-H_1(0)]c + c(0)e^{\delta_t(0)}e^{\delta_t(t_t)}D_1(t_1) + H_1'(t_1)e^{\delta_2(t_t)}[H_2(t_1)-H_1(t_1)]D_2(t_1) +}$$
$$\boldsymbol{\Big[c'(t_1)e^{-rt-\delta_2(t_1)}\int_{t_1}^{t_2} e^{\delta_{2(u)}} D_1(u)du + \alpha_j i_c\big[G_1(t_1) - G_1M_1\big(D_1(t_1)\big)\big]e^{\delta_1 t_1}D_1(t_1)\Big) + i_e e^{-rt_1}s_2(t_1(t_1)D_2(t_1) +}$$
$$\boldsymbol{e^{-rM_t(D_t(t_t))}s\big(M_1\big(D_1(t_1)\big)D_1(M_1(D_1(t_1))D_1'(t_1)[M_1'(D_1(t_1))]\big] + [D_1'(t_1)[M_1'\big(D_1(t_1)\big) -}$$
$$\boldsymbol{1]\int_{t_t}^{M_t(D_t(t_t))} e^{-ru}s_1(u)D_1(u)du - i_eD_1(t_1)e^{-rt}s_1(t_1) - c(t_1)(r+\delta_2'(t_1))e^{-rt_1-\delta_2(t_1)}]\int_{t_1}^{t_2} e^{\delta_{2(u)}} D_1(u)du -}$$
$$\boldsymbol{c(t_1)e^{-rt_1}D_1(t_1) - i_e(1-\alpha_1)[M_1\big(D_1(t1)\big) - t_1]e^{-rt_1}s_1(t_1)D_1(t_1) -}$$
$$\boldsymbol{\alpha_j i_c D_1'(t_1)G_1'M_1\big(D_1(t_1)\big)M_{11}'\big(D_1(t_1)\big) \times \int_{M_t(D_t(t_t))}^{t_t}[G_1(u) - G_1\big(M_1\big(D_1(t_1)\big)\big)]e^{\delta_1(u)}D_1(u)du} \tag{3.7}$$

Suppose that in (3.7) the following condition hold:

sum of all positive terms in (3.7) $\geq$ *than the sum of negative terms* (3.8)

Recalling that

$$\boldsymbol{\delta_j(t) = \int_0^t \theta_j(u)du\,, H_j(t) = \int_0^t e^{-rt-\delta_t(t)} h_j(t)dt, and\ G_j(t) = \int_0^{t_t} e^{-rt-\delta_t(t)} c_j(t)dt, then\ \frac{\partial W_W}{\partial t_1} \geq 0} \tag{3.8}$$

Holds .In this case, *WR* is an increasing function of t_1. Now, consider the two replenishment schedules *S=(T,n) and* $\hat{S} = (\hat{T}, n+1)$, then we have that $\hat{t}_1 \le t_1$ by (3.6). , hence $W_R(\hat{S}) \le W_R(S)$. This means that *WR* is an increasing function of t_1 but a non-increasing function of ***n.*** On the other hand we have *W0* = $\sum_{j=1}^{j=n} e^{-rt_{j-1}} k(t_{j-1})$ > k(0) >0 . As a set up cost it is clear that *W0* is an increasing function of of the period number ***n***. Now, combining the above results we can reach to the following conclusion. While *WR* decreases with ***n*** , *W0* increases with ***n*** so that *W0* will eventually dominate *WR* after certain value of ***n*** say ***n**** where *W= W0 +WR* starts to increase with ***n*** . Hence, there is a unique vector say $S^* = (T^*, n^*)$ that solves the underlying inventory problem. This completes the proof of the theorem.

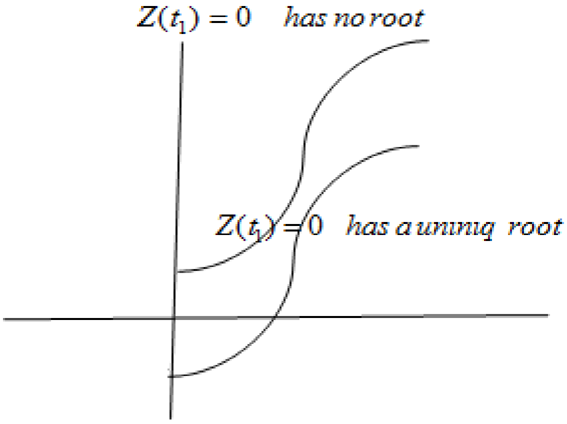

Figure.2

CONCLUSION

In this paper we have presented a general economic order quantity (EOQ) inventory model for deteriorating item for a given time horizon under a trade credıt polıcy which may motıvate the buyer to demand more quantities.The given time horizon consists of n different periods each of which has its own trade credit as a non decreasing function of its demand rate, has its own parameters and has its own cost structure . Both demand and deterioration rates as well as all cost parameters are time and period dependent, and general continuous functions of time. As examples of such trade credit periods introduced linear and /or exponent functions.These assumptions leads to the possibility of making the demand not only to be a stock dependent, but also to make a possibility of ordering the required item for one or more of the n periods.The two possibilities, that the trade credit of any period may be less or greater than the period length, are conventionally incorporated for each period so that the proposed model can suite more practical and possible cases, including the cases of prepayments, immediate payments, and late payments.The effects of both inflation and time value of money are also incorporated in all cost components.The period dependent trade credıt of the demand rate as well as all cost parameters via the index j of the model give us the flexibility of applying our model for each period seperatly with a separate item (commodity) or with the same item for one or more of the other periods. Each of the demand, deterioration rates as well as all cost parameters are known and arbitrary functions of time and are dependent on the related period. The two possibilities, that the credit period may be less or greater than the period length, are conventionally incorporated for each period so that the proposed model can suite more practical and possible cases, including the cases of prepayments, immediate payments, and late payments. Both inflation and time value of money are, also, incorporated in all cost components. Our main concern were on the theoretical results. The proposed model with the above mentioned general features is developed, and no approximations were used neither in the total net cost nor in any other relations .A closed form of the net total cost is derived and the resulting model is solved. Then, a rigorous mathematical methods are used to show that, under some seemingly Possible conditions, there exists a unique vector of the relevant decision variables that solve the underlying inventory system.

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