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Some Contributions in Quasi Symmetric 2–Designs With Three Intersection Numbers

Jain Smita and Banerjee Shakti
School of Statistics, Devi Ahilya University,Indore ,India.

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ABSTRACT

Some construction methods of quasi - symmetric 2- designs with no repeated blocks and three intersection numbers $x = 0$, y and z between the blocks are proposed with illustrations.

1. Introduction

In the early 1930's R.A. Fisher and F. Yates gave the concept of design of experiments. A fundamental method of constructing 2- designs (and also the first systematic construction method) is due to Bose [5]. It is known as the method of differences. Suppose an abelian group (G, +). For a subset $B \subseteq G$, $| B | = k$, consider the k(k-1) ordered differences of its elements. We get a collection of m subsets $B_1, B_2, \ldots B_m$ of G, a set of initial blocks if among the mk(k-1) differences arising from these m blocks, each non-zero element of G occurs exactly a constant number λ times. Several series of BIB designs have been constructed by Sprott (1954, 1956) ,using the method of differences. Teirlinck [28] has proved that non – trival t – designs without repeated blocks exist for all t. In the early 1970's Richard Wilson did the most significant improvement in the design theory. He [30]-[32] also gave some useful results on an existence theory for pairwise balanced designs. Hanani [13] has shown that the obvious necessary conditions for the existence of a 2- (v, k, λ) design are λ(v-1) = 0 {mod(k-1)} and λv(v-1)= 0{mod k(k-1)} to be sufficient for the k = 3,4 and 5, and every λ, except v= 15, k = 5 , λ= 2.

A balanced incomplete block design is an arrangement of v treatments in b blocks, of size k where each treatment replicated r times , and every pair of treatment appears together in λ blocks. A BIB design is symmetric iff v = b and r = k . It is also denoted as 2- (v, k, λ) .

Quasi Symmetric Design - Let S be a finite set of v objects (points) , and γ be a finite family of distinct k subsets of S (blocks). Then the pair D = { S , γ } is called a block design (or 2-design) with parameters (v, b, r, k, λ). For $0 \leq x < k$, where an intersection number of D is x , if there exist B , B' ∈ γ such that $| B \cap B' | = x$. A 2-design *D* is **quasi-symmetric design** with two numbers of intersection x and y and $0 \leq x < y < k$ if every two distinct blocks intersect in either x or *y* points.

Ray et al.(see [23]) proved that for a 0- design with t- intersection number b≤ $\binom{v}{t}$. Pawale [17] proved that for a fixed block size k , there exist finitely many parametrically feasible t – designs with t – numbers of intersection and λ > 1. Quasi-symmetric designs and its classification has been important in the study of design theory over the last several years. Sane and Shrikhande gave many important results for quasi-symmetric designs. Goethals et al. [11] gave the concept of graph of a quasi-symmetric design which is strongly regular.

Our main objective in this paper is to study the relations for quasi-symmetric 2- designs with three intersection numbers. So here we consider a multiple solution of symmetrical BIB designs , with no repeated blocks which is also quasi-symmetric 2-designs with three type of intersection numbers $x = 0$, y and z.

Some results which are useful for the development of the paper are discussed below .

Proposition 1.1 [4, 7, 27] If D with standard parameter set (v, b ,r, k, λ ; x, y) is a quasi symmetric design, then the relations between parameters are:

(1) vr = bk and λ (v-1) = r(k-1),

Tele:
E-mail address: smita_313@yahoo.com

(2) k(r-1)(x+y-1) - xy (b-1) = k(k-1)(λ-1)

(3) y- x divides k - x and r- λ

(4) r (-r + kr + λ) = bkλ.

Proposition 1.2 [27] If D with standard parameter set (v, b ,r, k, λ ; x, y) is a quasi symmetric design and (b - 1- α) and α are the number of blocks which intersect given block in x and y number of points , then

$$\alpha = \frac{k^2(\lambda-1)+k(r-rx+x-\lambda)}{y(y-x)}$$

Proposition 1.3 [8] Consider the balanced incomplete block design D_* with parameter set

v = 4t + 1, b_* = 2(4t + 1) , r_* = 4t , k = 2t , λ_* = 2t–1 (*)

where v is a prime and primitive element of GF (v) is x , and the solution of design D_* with above given parameter set is obtained by initial blocks

$(x^0 ,x^2 ,x^4 \ldots\ldots\ldots\ldots\ldots x^{4t-2})$;

$(x ,x^3 ,x^5 \ldots\ldots\ldots\ldots\ldots x^{4t-1})$.

Thus, we get the complete solution of the design D_*, by developing given intial block set.

To illustrate this result, we consider the following example.

Example 1.4 Let t = 4 in D_* ,so that the parameters are **v = 17, b_* = 34 , r_* = 16 , k = 8 , λ_* = 7** .Since x = 3 is primitive element of GF (17) ,the initial blocks are ($3^0,3^2, 3^4, 3^6, 3^8 ,3^{10}, 3^{12} ,3^{14}$) = (1, 9, 13, 15, 16, 8, 4, 2) and ($3^1,3^3, 3^5, 3^7, 3^9 , 3^{11}, 3^{13} ,3^{15}$) = (3, 10, 5, 11, 14, 7, 12, 6). By developing the sets of initial block , we get complete solution of the this design.

2. CONSTRUCTION OF 2-MULTIPLE BALANCED INCOMPLETE BLOCK DESIGNS

Theorem 2.1 The existence of the series of symmetrical balanced incomplete block design D with parameters v = 4t - 1= b , r = 2t - 1 = k , λ = t–1 ,where v(=4t-1) = p^n is a prime or prime power and x is a primitive element of GF(p^n) , implies the existence of D* a 2-multiple balanced incomplete block design with no repeated blocks with parameters v= 4t -1 , b*= 2(4t –1) =2b , r*=2(2t –1) =2r , k= 2t -1 , λ*=2(t– 1)= 2λ .

Proof :- Consider as symmetrical BIB designs with r = (v-1)/2 . Then we have

λ(v-1) = r(k-1) = {(v - 1)/2} {(v -3)/2}.

Since v= 4λ + 3 and if we take λ = t -1 ,then the parameters of the design under consideration became

v = 4t - 1 = b , r = 2t - 1 = k , λ = t – 1. (1)

We obtained the solution of the design D with parameters in (1) if v = p^n is a prime or prime power. Since x is a primitive element and all the no zero elements of GF(p^n) can be shown as

$x^0 = 1 , x , x^2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots x^{4t-3}$,

Then ,

$$x^{v-1} = x^{4t-2} = 1 \text{ and } x^{2t-1} = -1. \quad (2)$$

Consider the initial block

$$D_1 = (x^0, x^2, x^4 \ldots\ldots\ldots\ldots\ldots\ldots x^{4t-4}) . \quad (3)$$

The differences from this initial block can be written in this form

$\pm (x^2 - x^0), \pm (x^4 - x^2),\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \pm (x^0 - x^{4t-4})$;

$\pm (x^4 - x^0), \pm (x^6 - x^2),\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \pm (x^2 - x^{4t-4})$;

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$\pm (x^{2t-2} - x^0), \pm (x^{2t} - x^2),\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \pm (x^{2t-4} - x^{4t-4})$.

(4)

Let $x^{2i} - x^0 = x^{qi}$. Then, from (2) ,the differences in the i[th] row of (4) are

$$x^{qi},\ x^{qi+2}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, x^{qi+4t-4}$$
$$x^{qi+2t-1},\ x^{qi+2t+1}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, x^{qi+6t-5} \quad (5)$$

All the differences in (4) can be obtained by setting i = 1,2,………,(t – 1) in (5) .

But, for any given value of i, every non-zero element of GF (p^n) occurs just once in (5) . Therefore, in the differences (5) that arise from the initial block (3), every non-zero element of GF (p^n) occurs exactly (t – 1) times.

Now, multiply (3) by primitive element x of GF (p^n) ,we get another initial block set is

$$\mathbf{D_2} = \quad (x^1, x^3, \ldots\ldots\ldots\ldots\ldots x^{4t-3}). \qquad (6)$$

The differences from initial block set (6) are

$$x^{qi+1}, x^{qi+3}, \ldots\ldots\ldots\ldots\ldots\ldots, x^{qi+4t-3},$$

$$x^{qi+2t}, x^{qi+2t+2}, \ldots\ldots\ldots\ldots\ldots\ldots, x^{qi+6t-4}. \qquad (7)$$

Similarly, all the differences can be obtained by setting **i** = 1,2,…………,(t – 1) in equation (7).

But, for any given value of i, every non-zero elements of GF (p^n) occurs just once in (7). Therefore, in the differences (7) that arise from the initial block (6), every non-zero element of GF (p^n) occurs exactly (t – 1) times.

Again, if we multiply (6) by primitive element x of GF (p^n) ,then we get same initial block set of (3).

Thus, from (3) and (6) we get the initial blocks set in form of **D* = [D_1 : D_2]** . Hence , by developing D* we get the complete solution of 2-multiple BIB designs with no repeated blocks with parameters

v = 4t – 1, b*= 2(4t – 1) = 2b, r*= 2(2t – 1) = 2r , k= 2t – 1 , λ* = 2(t – 1) =2λ. (8)

This complete the proof.

Example 2.2 We illustrate the theorem 2.1, if t = 3 then BIB design with the parameters **v = b = 11 , r = k = 5 , λ = 2 .**

Since primitive element of GF (11) is x **= 2** and the solution of the design is given by the initial set (2^0,2^2, 2^4, 2^6, 2^8), that is (1, 3, 4, 5, 9) mod 11 provides

	(1, 3, 4, 5, 9) ;	(2, 4, 5, 6, 10) ;	(3, 5, 6, 7, 0) ;
D_1 :	(4, 6, 7, 8, 1) ;	(5, 7, 8, 9, 2) ;	(6, 8, 9, 10, 3) ;
	(7, 9, 10, 0, 4);	(8, 10, 0, 1, 5) ;	(9, 0, 1, 2, 6) ;
	(10, 1, 2, 3, 7);	(0, 2, 3, 4, 8) .	

And ,the another initial block set is (2^1,2^3, 2^5, 2^7, 2^9), that is (2, 6, 7, 8, 10) mod 11 provides

	(2, 6, 7, 8, 10) ;	(3, 7, 8, 9, 0) ;	(4, 8, 9, 10, 1) ;
D_2 :	(5, 9, 10, 0, 2) ;	(6, 10, 0, 1, 3);	(7, 0, 1, 2, 4) ;
	(8, 1, 2, 3, 5) ;	(9, 2, 3, 4, 6) ;	(10, 3, 4, 5, 7) ;
	(0, 4, 5, 6, 8) ;	(1, 5, 6, 7, 9) .	

Thus, by developing these initial block sets in form of **D* = [D_1 : D_2]** we get the 2 multiple BIB design with no repeated blocks and with parameters **v = 11, b* = 22 , r* = 10, k = 5 , λ* = 4.**

3. QUASI SYMMETRIC DESIGNS WITH THREE INTERSECTION NUMBERS

Theorem 3.1 : Let D* be a quasi-symmetric design with parameter set (v, b*, r*, k, λ*; x, y, z) have three intersection numbers x = 0 , y = λ and z = λ +1. Then corresponding to a block there are

$$\alpha_1 = \frac{k^2(\lambda^* - 1) + k\{(x+z)(1-r^*) - \lambda^* + r^*\} + xz(b^* - 1)}{(y-x)(y-z)}$$

with one type of intersection number ,

$$\alpha_2 = \frac{k^2(\lambda^* - 1) + k\{(y+x)(1-r^*) - \lambda^* + r^*\} + xy(b^* - 1)}{(z-x)(z-y)}$$

with second type of intersection number , and

$$b^* - 1 - \alpha_1 - \alpha_2 = \frac{k^2(\lambda^* - 1) + k\{(y+z)(1-r^*) - \lambda^* + r^*\} + yz(b^* - 1)}{(y-x)(z-x)}$$

with third type of intersection number and where α_1 , α_2 and ($b^*-1-\alpha_1-\alpha_2$) be the number of blocks intersecting B_* in y , z and x number of points.

Proof :- Let us consider a block B_* of D* and make it fixed. Then α_1 and α_2 be the number of blocks intersecting B_* in y and z points, the remaining ($b^*-1-\alpha_1-\alpha_2$) blocks intersecting with B_* in x points. Then count in two ways process, the number of pairs {(u_1, u_2), B} , where B is a block of D* other than B_* and where u_1, u_2 are distinct points of D* contained in $B \cap B_*$, is obtained by the following equations,

$$\alpha_1 y + \alpha_2 z + (b^*-1-\alpha_1-\alpha_2)\, x = k(r^*-1) \quad \text{and}$$

$$\alpha_1 y(y-1) + \alpha_2 z(z-1) + (b^*-1-\alpha_1-\alpha_2)\, x(x-1) = k(k-1)(\lambda^*-1)$$

By simplify above two equations , we get

$$\alpha_1 (y-x) + \alpha_2 (z-x) + (b^*-1)\, x = k(r^*-1) \quad \text{and} \qquad (9)$$

$$\alpha_1\{(y-x)(y+x-1)\} + \alpha_2\{(z-x)(z+x-1)\} + (b^*-1)\, x(x-1) = k(k-1)(\lambda^*-1) \qquad (10)$$

By solving equations (9) and (10) ,we eliminate α_2 and get

$$\alpha_1 = \frac{k^2(\lambda^* - 1) + k\{(x+z)(1-r^*) - \lambda^* + r^*\} + xz(b^*-1)}{(y-x)(y-z)}$$

Similarly, we eliminate α_1 from equations (9) and (10) , and get

$$\alpha_2 = \frac{k^2(\lambda^* - 1) + k\{(y+x)(1-r^*) - \lambda^* + r^*\} + xy(b^*-1)}{(z-x)(z-y)}$$

Substitute the value of α_1 and α_2 in equation (9) , and get

$$b^* - 1 - \alpha_1 - \alpha_2 = \frac{k^2(\lambda^* - 1) + k\{(y+z)(1-r^*) - \lambda^* + r^*\} + yz(b^*-1)}{(y-x)(z-x)}$$

Hence, the theorem proved.

Example 3.2 If t= 8, in equation (8), then the quasi-symmetric design D with parameter set (31, 31, 15 , 15 ,7) and D* is also a quasi-symmetric design with parameters v = 31, b* = 62 , r* = 30, k =15 , λ* =14 , χ=0, y = 7 and z = 8 and for D* from theorem 3.1 , α_1 = 45 , α_2 = 15 and (b* -1 – α_1 – α_2) = 1.

Theorem 3.3 Let D_* with parameters **v = 4t + 1, b_* = 2(4t + 1) , r_* = 4t , k = 2t , λ_* = 2t–1** , is quasi-symmetric design have three intersection numbers χ = 0 , y = t - 1 and z = t. Then corresponding to a block there are

$$\alpha_1 = \frac{k^2(\lambda_* - 1) + k\{(x+z)(1-r_*) - \lambda_* + r_*\} + xz((b_*-1)}{(y-x)(y-z)}$$

with one type of intersection number ,

$$\alpha_2 = \frac{k^2(\lambda_* - 1) + k\{(y+x)(1-r_*) - \lambda_* + r_*\} + xy(b_*-1)}{(z-x)(z-y)}$$

with second type of intersection number , and

$$b_* - 1 - \alpha_1 - \alpha_2 = \frac{k^2(\lambda_* - 1) + k\{(y+z)(1-r_*) - \lambda_* + r_*\} + yz(b_*-1)}{(y-x)(z-x)}$$

with third type of intersection number and where α_1 , α_2 and (b_*-1 - α_1 - α_2) be the number of blocks intersecting B_* in y , z and χ points.

Example 3.4 D_* with parameters v = 13 , b_* = 26 , r_* =12 , k = 6 , λ_* = 5 is a quasi-symmetric design have three intersection numbers χ=0, y = 2 and z = 3 and for D_* from theorem 3.3 , α_1 = 6 , α_2 = 18 and b_* -1 – α_1 – α_2 = 1.

Theorem 3.5 D* with parameter set (v, b*, r*, k, λ*; χ, y, z) is a quasi-symmetric design have three intersection numbers χ = 0 , y = λ and z = λ +1. Then D* holds the following relations:

1. vr* = b*k ,
2. λ* (v-1) = r*(k-1),
3. r^{*2}(k-1)+r*λ* = b*k λ*,
4. α_1y + α_2 z + (b* -1- α_1 – α_2) χ = k (r*- 1) ,
5. α_1y(y-1) + α_2 z(z-1) + (b* -1- α_1 – α_2) $\chi(\chi$-1) = k (k- 1)(λ*-1) ,
6. k(r*-1)(y + z -1) – yz (b*- 2) – χ (y + z – χ) = k(k-1)(λ*-1) ,
7. z – y divides k – y and r* - λ* .

Proof :-

(1) and (2) From proposition 1.1-(1) ,vr = bk. Since r*= 2r ,b* = 2b and λ*= 2λ. Hence vr* = b*k and λ*(v- 1) = r*(k-1).

(3) From above proved results (1) and (2), r*(-r*+ kr* + λ*) = r^{*2}(k-1)+ λ*r* = r*λ*(v-1) + λ*r* = b*k λ*.

(4) and (5) Let α_1 and α_2, be the number of blocks intersecting B_* in y and z points, the remaining (b*-1- α_1- α_2) blocks intersecting with B_* in χ points. Fix a block B_* and count in two ways process the number of pairs {(u_1, u_2), B} , where B is a block of D* other than B_* and where u_1, and u_2 are distinct points of D* contained in B∩ B_* , is obtained by the following equations,

α_1y + α_2z + (b*-1- α_1 -α_2) χ = k (r*-1) and (11)

α_1y(y-1) + α_2z(z-1) + (b* -1- α_1 -α_2) $\chi(\chi$-1) = k(k-1)(λ*-1) (12)

(6) By solving equations (11) and (12), we get the solution of relation

k(k-1)(λ*-1) = k(r*-1)(y+z-1) – yz(b* -1) + (y-χ)(z-χ)

Hence, k(r*-1)(y+z-1) – (b*-2)yz – χ(y+z-χ) = k(k-1)(λ*-1).

(7) From proposition 1.1- (3) y-x divides k – y and r – λ . Since z-y = λ+1-λ=1 and r*=2r and λ* =2λ. Hence, z– y divides k -y and r* - λ*.

Example 3.6 D* is a quasi-symmetric design with parameters v = 11, b* = 22 , r* = 10, k = 5 , λ* = 4 with no repeated blocks and have three intersection numbers x = 0 , y = 2 and z = 3 and α_1 = 15 , α_2 = 5 and b* -1 – α_1 – α_2 = 1 holds the all relations of theorem 3.5.

Corollary 3.7 D_* is a quasi-symmetric design with parameter set (v, b_* , r_* , k , λ_* ; χ , y ,z) have three intersection numbers χ = 0 , y = t - 1 and z = t . Then D_* holds the following relations :

1. $v r_* = b_* k$,
2. $\lambda_* (v-1) = r_* (k-1)$,
3. $r_*^2 (k-1)+ r_* \lambda_* = b_* k \lambda_*$,
4. $\alpha_1 y + \alpha_2 z + (b_* -1- \alpha_1 – \alpha_2) \chi = k (r_*- 1)$,
5. $\alpha_1 y(y-1) + \alpha_2 z(z-1) + (b_* -1- \alpha_1 – \alpha_2) \chi (\chi-1) = k (k- 1)(\lambda_*-1)$,
6. $k(r_*-1) (y + z -1) – yz (b_*- 2) – \chi (y + z – \chi) = k(k-1)(\lambda_*-1)$,
7. z – y divides k – y and $r_* - \lambda_*$,

Example 3.8 D_* is a quasi-symmetric design with parameters v = 13 , b_* = 26 , r_* =12 , k = 6 , λ_* = 5 have three intersection numbers χ= 0, y = 2 and z = 3 and α_1 = 6 , α_2 = 18 and (b_* -1 – α_1 – α_2) = 1 holds the all relations of corollary 3.7.

Corollary 3.9 D* is a quasi – symmetric design with parameters (v, b*,r*,k, λ*; χ , y ,z).

Then, $\alpha_1 y(y-\chi) = \alpha_2 z(\chi-z) + k^2 (\lambda^*-1) + k \{ \chi (1-r^*) - \lambda^*+ r^*\}$.

Proof : By theorem 3.5 relation (4) and (5)

$\alpha_1 y + \alpha_2 z + (b^* -1- \alpha_1 – \alpha_2) \chi = k (r^*- 1)$,

$\alpha_1 y(y-1) + \alpha_2 z(z-1) + (b^* -1- \alpha_1 – \alpha_2) \chi (\chi-1) = k (k- 1) (\lambda^*-1)$,

On solving above two relations of theorem 3.5 , we get

$\alpha_1 y(y-\chi) = \alpha_2 z(\chi-z) + k^2 (\lambda^*-1) + k \{\chi (1- r^*) - \lambda^*+r^*\}$.

Corollary 3.10 D_* with parameters (v, b_* , r_* , k , λ_* ; χ , y ,z) is a quasi – symmetric design.

Then, $\alpha_1 y (y- \chi) = \alpha_2 z (\chi-z) + k^2 (\lambda_* - 1) + k \{ \chi (1- r_*) - \lambda_*+ r_*\}$.

Proof :- Solve this corollary with the help of relations (4) and (5) of corollary 3.7.

Corollary 3.11 Let D* be a quasi – symmetric design with parameter set (v, b*, r*, k, λ*; χ, y, z).Then $\alpha_1 r^{*2} + b_1 r^* + c_1 = 0$, where

$\alpha_1 = (k-1) yz$

$b_1 = \{yz -k^2(y + z-1)\} \lambda^*$

$c_1 = \{k^2(\lambda^*-1) + k (y+z–\lambda^*) -2yz + \chi (y + z -\chi)\} \lambda^* k$

Proof :-- From above theorem 3.5-(3), $(k-1)r^{*2}+ \lambda^* r^*- b^* k \lambda^*=0$.

Hence, $(k-1) yz r^{*2} + \lambda^* r^* yz – b^* yz k \lambda^*= 0$.

From theorem 3.5-(6) $b^* yz= 2yz +k(r^*-1)(y+z-1) – k(k-1)(\lambda^*-1) – \chi (y+z-\chi)$.

Substituting the value of b*yz in above equation we get

$(k-1) y z r^{*2}+ \lambda^* r^* yz- \{ k (r^*-1)(y+z-1) + 2yz -k(k-1)(\lambda^*-1)- \chi(y+z-\chi)\} = 0$

Hence, $(k-1) yz r^{*2}+\{yz-k^2(y+z-1)\} \lambda^* r^*+\{k^2(\lambda^*-1)+ k (y+z-\lambda^*) - 2yz + \chi (y+z-\chi)\} \lambda^* k=0$.

Corollary 3.12 Let D_* with parameter set (v, b_* , r_* , k , λ_* ; χ , y ,z) is a quasi – symmetric design . Then $\alpha_1 r_*^2 + b_1 r_* + c_1 = 0$, where $\alpha_1 = (k-1) yz$, $b_1 = \{ yz -k^2(y + z-1)\} \lambda_*$ and $c_1 = \{ k^2 (\lambda_*-1) + k (y + z– \lambda_*) -2yz + \chi (y + z –\chi)\} \lambda_* k$.

Corollary 3.13 If D* with parameter set (v, b*, r*, k, λ*; χ, y, z) is a quasi – symmetric design , then **(i)** r* = v – 1, **(ii)** λ* = k-1, **(iii)** b* = 2(r *+ 1) **(iv)** if k ≥ 3 then λ* ≥ 2 .

Proof . (i) From equation (8) r* = 4t – 2 = v – 1.

(ii) Since λ* (v-1) = r*(k-1) and from above result (i) r*= v-1.Hence, λ* = k-1.

(iii) Since b*= 2v and from (ii) v = r* + 1. Hence , b* = 2(r* + 1).

(iv) From (iii) k = λ* +1 and if k ≥ 3 then λ*+1 ≥ 3. Hence, λ* ≥ 2.

Corollary 3.14 If D_* is a quasi – symmetric design with parameter set (v, b_* , r_* , k , λ_* ; χ , y ,z) , then **(i)** $r_* = v – 1$, **(ii)** $\lambda_* = k-1$, **(iii)** $b_*= 2(r_*+ 1)$ **(iv)** if k ≥ 3 then $\lambda_* \geq 2$.

4. RESULTS

The following table-1 provide a list of parameters which can be obtained by using theorem 2.1 and theorem 3.1. and table-2 provide a list of parameters which can be obtained by using theorem 3.3.

Table 1.

S.No.	t	v = b = 4t-1	r = k = 2t-1	$\lambda = t-1$	$b^* = 2b$	$r^* = 2r$	$\lambda^* = 2\lambda$	χ	$y = \lambda$	$z = \lambda + 1$	$b^* - 1 - \alpha_1 - \alpha_2$	α_1	α_2
1	2	7	3	1	14	6	2	0	1	2	1	9	3
2	3	11	5	2	22	10	4	0	2	3	1	15	5
3	5	19	9	4	38	18	8	0	4	5	1	27	9
4	6	23	11	5	46	22	10	0	5	6	1	33	11
5	7	27	13	6	54	26	12	0	6	7	1	39	13
6	8	31	15	7	62	30	14	0	7	8	1	45	15
7	11	43	21	10	86	42	20	0	10	11	1	63	21
8	12	47	23	11	94	46	22	0	11	12	1	69	23
9	15	59	29	14	118	58	28	0	14	15	1	87	29
10	17	67	33	16	134	66	32	0	16	17	1	99	33
11	18	71	35	17	142	70	34	0	17	18	1	105	35

Table 2.

S.No.	t	v = 4t+1	$b_* = 8t+2$	$r_* = 4t$	k = 2t	$\lambda_* = 2t-1$	χ	y = t–1	z = t	$b_* - 1 - \alpha_1 - \alpha_2$	α_1	α_2
1	**3**	**13**	**26**	**12**	**6**	**5**	**0**	**2**	**3**	**1**	**6**	**18**
2	**4**	**17**	**34**	**16**	**8**	**7**	**0**	**3**	**4**	**1**	**8**	**24**
3	**7**	**29**	**58**	**28**	**14**	**13**	**0**	**6**	**7**	**1**	**14**	**42**
4	**9**	**37**	**74**	**36**	**18**	**17**	**0**	**8**	**9**	**1**	**18**	**54**
5	**10**	**41**	**82**	**40**	**20**	**19**	**0**	**9**	**10**	**1**	**20**	**60**
6	**13**	**53**	**106**	**52**	**26**	**25**	**0**	**12**	**13**	**1**	**26**	**78**

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