Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 117 (2018) 50360-50369

ISSN: 2229-712X

Neutrosophic Soft Cubic Set on BCK/BCI Algebra

R. Anitha Cruz[1] and F. Nirmala Irudayam[2]

[1]R. Anitha Cruz, Departmentof Mathematics, Nirmala College for Women, Coimbatore, India.

[2] F. Nirmala Irudayam, Assistant Professor, Department of Mathematics, Nirmala College for Women,Coimbatore,India,

ARTICLE INFO

Article history:

Received: 23 February 2018;

Received in revised form: 25 March 2018;

Accepted: 5 April 2018;

Keywords

Neutrosophic soft cubic set (NSCS),

Neutrosophic soft cubic BCK/BCI (NSC BCK/BCI),

e-Neutrosophic soft cubic set (e-NSCS).

ABSTRACT

This paper aims of inducing the notion of neutrosophic soft cubic set to BCK/BCI-algebras. Here we focus on various characterizations of neutrosophic soft cubic BCK/BCI-algebras. We infer that the R-intersection of two neutrosophic soft cubic BCK/BCI-algebras is also a neutrosophic soft cubic BCK/BCI- algebras. We also investigate several properties of neutrosophic soft cubic subalgebras of BCK/BCI-algebras based on a given parameter.

© 2018 Elixir All rights reserved.

1. INTRODUCTION

Zadeh [12] introduced the concept of a fuzzy set theory in the year 1965.He also extended this concept of a fuzzy set by an interval-valued fuzzy set, i.e., a fuzzy set with an interval-valued membership function. Many real life situations depend on the existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets and theory of rough sets which deals with a lot of uncertainties. However, all of these theories have their own difficulties which are pointed out in [11]. Maji et al. [10] and Molodtsov [11] suggested that one reason for these difficulties may be due to the inadequacy of the parameterization tool. To overcome these difficulties, Molodtsov [10] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov [25] pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [9] described the application of soft set theory to a decision making problem. Maji et al. [10] also studied several operations on the theory of soft sets.

In 1966, Imai and Is´eki [20, 21] derived a new form of algebra called BCK-algebras and BCI-algebras. These concepts were elucidated in two different ways: One of them is based on set theory; another is from classical and non-classical propositional calculi. We also encounter the fact that there is a close relationship between the notions of the set difference in set theory and the implication functor in logical systems. Since then many researchers have worked in this area [19, 21, 22, 23, 24]. Jun et al. [18] applied the notion of soft sets to BCK/BCI-algebra and d-algebra.

Smarandache [14] coined neutrosophic logic and neutrosophic sets to deal with truth, indeterminate and falsehood. Jun et al. [5] introduced a new notion, called a (internal, external) cubic set, and investigated several properties. [5] dealt with P-union, P-intersection, R-union and R-intersection of cubic sets, and investigated several related properties and also expanded the concept of cubic sets to the neutrosophic cubic sets. Anitha & Nirmala [1] introduced neutrosophic soft cubic set (internal, external). [1] Dealt with P-union, P-intersection, R-union and R-intersection of neutrosophic soft cubic sets, and investigated several related properties.

This paper applies the notion of neutrosophic soft cubic set to BCK/BCI algebras, and introduces the idea of neutrosophic soft cubic BCK/BCI-algebras.

2. PRELIMINARIES

Definition 2.1[24]. In this section we include some of the elementary aspects that are necessary for this paper.

Algebra $(X; *, 0)$ of type (2, 0) is called a BCI-algebra if it satisfies the following axioms, for all $x, y, z \in X$

(i) $((x*y)*(x*z)*(z*y)=0$

(ii) $(x*(x*y)*y=0$

(iii) $x*x=0$

(iv) $x*y=0 \, and \, y*x=0 \Rightarrow x=y$

If a BCI-algebra X satisfies the following identity:

Tele: 9047714847

E-mail address: anithacruz@gmail.com

© 2018 Elixir All rights reserved

(v) $(\forall x \in X)(0 * x) = 0$ then X is called a BCK-algebra.

Any BCK/BCI-algebra X satisfies the following conditions, for all $x, y, z \in X$ λ

(a1) $(0 * x) = 0$

(a2) $((x * y) = 0 \Rightarrow (x * z) * (y * z) = 0, (z * y) * (z * x) = 0;$

(a3) $((x * y) * z = (x * z) * y),$

(a4) $((x * z) * (y * z) * (x * y) = 0.$

A fuzzy set in a set X is defined to be a function $\lambda : X \to I$, where I = [0, 1]. Denote by I^X the collection of all fuzzy sets in a set X. Define a relation $\leq$ on I^X as follows:

$(\forall \lambda, \mu \in I^X)(\lambda \leq \mu \Leftrightarrow (\forall x \in X)(\lambda(x) \leq \mu(x))).$

The join $(\vee)$ and meet $(\vee)$ of λ and μ are defined by

$\lambda \vee \mu(x) = \max\{\lambda(x), \mu(x)\},$

$\lambda \wedge \mu(x) = \min\{\lambda(x), \mu(x)\},$

respectively for all $x \in X$. The complement λ of denoted by λ^c , is defined by $(\forall x \in X)(\lambda^c(x) = 1 - \lambda(x))$.

For a family $\{\lambda_i \mid i \in \Lambda\}$ of fuzzy sets in X, we define the join $(\vee)$ and meet $(\wedge)$ operations as follows:

$\left(\underset{i \in \Lambda}{\vee} \lambda_i\right)(x) = \sup\{\lambda_i(x) \mid i \in \Lambda\},$

$\left(\underset{i \in \Lambda}{\wedge} \lambda_i\right)(x) = \inf\{\lambda_i(x) \mid i \in \Lambda\},$

respectively, for all $x \in X$.

Definition 2.2 ([6]).Let E be a universe. Then a fuzzy set μ over E is defined by X = { μ_x (x) / x: x ϵ E }where μ_x is called membership function of X and defined by $\mu_x : E \to [0,1]$. For each x ∈ E, the value $\mu_x(x)$ represents the degree of x belonging to the fuzzy set X.

Definition 2.3 ([3]). Let X be a non-empty set. By a cubic set, we mean a structure $\Xi = \{\langle x, A(x), \mu(x)\rangle \mid x \in X\}$ in which A is an interval valued fuzzy set (IVF) and μ is a fuzzy set. It is denoted by$\langle A, \mu\rangle$.

Definition 2.4 ([10]). Let U be an initial universe set and E be a set of parameters. Consider A ⊂ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F : A → P(U).

Definition 2.5 ([5]). Let X be an universe. Then a neutrosophic (NS) set λ is an object having the form

λ = {< x : T(x),I(x),F(x) >: x ∈ X}

where the functions T, I, F : X →]$^-$0, 1+[define respectively the degree of Truth, the degree of indeterminacy, and the degree of falsehood of the element x ∈ X to the set λ with the condition.

$^-0 \leq T(x) + I(x) + F(x) \leq 3^+$

Definition 2.6 ([8]). Let X be a non-empty set. An interval neutrosophic set (INS) A in X is characterized by the truth-membership function A_T, the indeterminacy-membership function A_I and the falsity-membership function A_F. For each point x ∈ X, $A_T(x), A_I(x), A_F(x) \subseteq [0,1]$.

Definition 2.7 [(1]).

Let X be an initial universe set. Let NC(X) denote the set of all neutrosophic cubic sets and E be the set of parameters. Let $A \subset E$ then $(P, A) = \{P(e_i) = \{<x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in X\} e_i \in A\}$, where $A_{e_i}(x) = \{<x, A^T_{e_i}(x), A^I_{e_i}(x), A^F_{e_i}(x) > / x \in X\}$, is an interval neutrosophic set , $\lambda_{e_i}(x) = \{<x, (\lambda^T_{e_i}(x), \lambda^I_{e_i}(x), \lambda^F_{e_i}(x) > / x \in X\}$ is a neutrosophic set. The pair (P, A) is termed to be the neutrosophic soft cubic set over X where P is a mapping given by $P: A \to NC(X)$.

Definition 2.8([1]).

Let X be a non-empty set. A neutrosophic soft cubic set (P, M) in X is said to be

- Truth-internal (briefly, T-internal) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (A^{-T}_{e_i}(x) \leq \lambda^T_{e_i}(x) \leq A^{+T}_{e_i}(x)), \quad (2.8.1)$$

- Indeterminacy-internal (briefly, I-internal) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (A^{-I}_{e_i}(x) \leq \lambda^I_{e_i}(x) \leq A^{+I}_{e_i}(x)), \quad (2.8.2)$$

- Falsity-internal (briefly, F-internal) if the following inequality is valid

$$(\forall x \in X, e_i \in E)(A^{-F}_{e_i}(x) \leq \lambda^F_{e_i}(x) \leq A^{+F}_{e_i}(x)). \quad (2.8.3)$$

If a neutrosophic soft cubic set in X satisfies (2.1), (2.2) and (2.3) we say that (P, M) is an internal neutrosophic soft cubic in X .

Definition 2.9([1]).

Let X be a non-empty set. A neutrosophic soft cubic set (P,M) in X is said to be

•Truth-external (briefly, T -external) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (\lambda^T_{e_i}(x) \notin (A^{-T}_{e_i}(x), A^{+T}_{e_i}(x))), \tag{2.4}$$

•indeterminacy-external (briefly, I -external) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (\lambda^I_{e_i}(x) \notin (A^{-I}_{e_i}(x), A^{+I}_{e_i}(x))), \tag{2.5}$$

•falsity-external (briefly, F -external) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (\lambda^F_{e_i}(x) \notin (A^{-F}_{e_i}(x), A^{+F}_{e_i}(x))). \tag{2.6}$$

If a neutrosophic soft cubic set (P,M) in *X* satisfies (2.4), (2.5) and (2.6), we say that (P,M) is an external neutrosophic soft cubic in *X*.

Definition 2.10 ([1]).

Let $(\mathrm{P},\mathrm{M}) = \{\mathrm{P}(\mathrm{e_i}) = \{<\mathrm{x}, \mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \lambda_{\mathrm{e_i}}(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{M}\}$ and

$(\mathrm{Q},\mathrm{N}) = \{\mathrm{Q}(\mathrm{e_i}) = \mathrm{B_i} = \{<\mathrm{x}, \mathrm{B}_{\mathrm{e_i}}(\mathrm{x}), \mu_{\mathrm{e_i}}(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{N}\}$ be two neutrosophic soft cubic sets in X. Let M and N be any two subsets of E (set of parameters), then we have the following

1. $(\mathrm{P},\mathrm{M}) = (\mathrm{Q},\mathrm{N})$ if and only if the following conditions are satisfied

a) M= N and

b) $\mathrm{P}(\mathrm{e_i}) = \mathrm{Q}(\mathrm{e_i})$ for all $e_i \in M$ if and only if $A_{e_i}(x) = B_{e_i}(x)$ and $\lambda_{e_i}(x) = \mu_{e_i}(x)$ for all $x \in X$ corresponding to each $e_i \in M$.

2. (P,M) and (Q,N) are two neutrosophic soft cubic set then we define and denote P- order as $(\mathrm{P},\mathrm{M}) \subseteq_{\mathrm{P}} (\mathrm{Q},\mathrm{N})$ if and only if the following conditions are satisfied

c) M $\subseteq$ N and

d) $\mathrm{P}(\mathrm{e_i}) \le_{\mathrm{P}} \mathrm{Q}(\mathrm{e_i})$ for all $e_i \in M$ if and only if $A_{e_i}(x) \subseteq B_{e_i}(x)$ and $\lambda_{e_i}(x) \le \mu_{e_i}(x)$ for all $x \in X$ corresponding to each $e_i \in M$.

3. (P,M) and (Q,N) are two neutrosophic soft cubic set then we define and denote P- order as $(\mathrm{P},\mathrm{M}) \subseteq_{\mathrm{R}} (\mathrm{Q},\mathrm{N})$ if and only if the following conditions are satisfied

e) M $\subseteq$ N and

f) $\mathrm{P}(\mathrm{e_i}) \le_{\mathrm{R}} \mathrm{Q}(\mathrm{e_i})$ for all $e_i \in M$ if and only if $A_{e_i}(x) \subseteq B_{e_i}(x)$ and $\lambda_{e_i}(x) \ge \mu_{e_i}(x)$ for all $x \in X$ corresponding to each $e_i \in M$.

Definition 2.11 ([1])

Let (P,M) and (Q,N) be two neutrosophic soft cubic sets (NSCS) in X where I and J are any two subsets of the parameteric set E. Then we define R-union of neutrosophic soft cubic set as $(P,M) \cup_R (Q,N) = (H,C)$ where $C = M \cup N$

$$H(e_i) = \begin{cases} P(e_i) & if\ e_i \in M-N \\ Q(e_i) & if\ e_i \in N-M \\ P(e_i) \vee_R Q(e_i) & if\ e_i \in M \cap N \end{cases}$$

where $P(e_i) \vee_R Q(e_i)$ is defined as

$$P(e_i) \vee_R Q(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda \wedge \mu_{\mathrm{e_i}})(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{M} \cap \mathrm{N}$$

where $\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})$ represent interval neutrosophic sets.

Hence

$$P^T(e_i) \vee_R Q^T(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda^{\mathrm{T}}_{\mathrm{e_i}} \wedge \mu^{\mathrm{T}}_{\mathrm{e_i}})(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{M} \cap \mathrm{N},$$

$$P^I(e_i) \vee_R Q^{\mathrm{I}}(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda^{I}_{\mathrm{e_i}} \wedge \mu^{I}_{\mathrm{e_i}})(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{M} \cap \mathrm{N},$$

$$P^F(e_i) \vee_R Q^F(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda^{\mathrm{F}}_{\mathrm{e_i}} \wedge \mu^{\mathrm{F}}_{\mathrm{e_i}})(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{M} \cap \mathrm{N}\ .$$

Definition 2.12 ([1])

Let (P,M) and (Q,N) be two neutrosophic soft cubic sets (NSCS) in X where M and N are any subsets of parameter's set E.

Then we define R-intersection of neutrosophic soft cubic set as $(P,M) \cap_R (Q,N) = (H,C)$ where $C = M \cap N$,

$H(e_i) = P(e_i) \wedge_R Q(e_i)$ and $e_i \in I \cap J$. Here $F(e_i) \wedge_R G(e_i)$ is defined as

$$P(e_i) \wedge_R Q(e_i) = H(e_i) = \{<\mathrm{x}, \min\{\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \vee \mu_{\mathrm{e_i}})(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{M} \cap \mathrm{N}\ .$$

where $\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})$ represent interval neutrosophic sets. Hence

$$P^T(e_i) \wedge_R Q^T(e_i) = \{<\mathrm{x}, \min\{\mathrm{A}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda^{T}_{\mathrm{e_i}} \vee \mu^{\mathrm{T}}_{\mathrm{e_i}})(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\}\ \mathrm{e_i} \in \mathrm{M} \cap \mathrm{N},$$

$P^I(e_i) \wedge_R Q^I(e_i) = \{<x, \min\{A^I_{e_i}(x), B^I_{e_i}(x)\}, (\lambda^I_{e_i} \vee \mu^I_{e_i})(x) > : x \in X\}\ e_i \in M \cap N$,

$P^F(e_i) \wedge_R Q^F(e_i) = \{<x, \min\{A^F_{e_i}(x), B^F_{e_i}(x)\}, (\lambda^F_{e_i} \vee \mu^F_{e_i})(x) > : x \in X\}\ e_i \in M \cap N$

Definition 2.12 ([2])

The complement of a neutrosophic soft cubic set

$(F, I) = \{F(e_i) = \{<x, A_{e_i}(x), \lambda_{e_i}(x) > : x \in X\}\ e_i \in I\}$ is denoted by $(F, I)^C$ and defined as

$(F, I)^C = \{(F, I)^c = (F^C, \neg I)\}$, where $F^C : \neg I \to NC(X)$ and

$(F, I)^C = \{(F(e_i))^C = \{<x, A^C_{e_i}(x), \lambda^C_{e_i}(x) > : x \in X\}\ e_i \in I\}$.

$(F, I)^c =$

$\{<x, ([1-A^{+T}_{e_i}, 1-A^{-T}_{e_i}], [1-A^{+I}_{e_i}, 1-A^{-I}_{e_i}], [1-A^{+F}_{e_i}, 1-A^{-F}_{e_i}]), (1-\lambda^T_{e_i}, 1-\lambda^I_{e_i}, 1-\lambda^F_{e_i}) > x \in X\}\ e_i \in I$.

3. NUETROSOPHIC SOFT CUBIC SET ON BCK/BCI ALGEBRA

Definition 3.1. A Neutrosophic soft cubic set (P, M) over U is said to be Neutrosophic soft cubic BCK/BCI algebra over U based on a parameter e_i if there exists $e_i \in M$ such that

$$A^{T,I,F}_{e_i}(x*y) \geq \min\{A^{T,I,F}_{e_i}(x), A^{T,I,F}_{e_i}(y)\}, \quad (3.1.1)$$

$$\lambda^{T,I,F}_{e_i}(x*y) \leq \max\{\lambda^{T,I,F}_{e_i}(x), \lambda^{T,I,F}_{e_i}(y)\} \quad (3.1.2)$$

where

$A^{T,I,F}_{e_i}(x) = A_{e_i}(x) = \{<x, A^T_{e_i}(x), A^I_{e_i}(x), A^F_{e_i}(x) > / x \in U\}$

$= \{<x, [A^{-T}_{e_i}(x), A^{+T}_{e_i}(x)], [A^{-I}_{e_i}(x), A^{+I}_{e_i}(x)], [A^{-F}_{e_i}(x), A^{+F}_{e_i}(x)] > / x \in U\}$

and $\lambda^{T,I,F}_{e_i}(x) = \lambda_{e_i}(x) = \{<x, (\lambda^T_{e_i}(x), \lambda^I_{e_i}(x), \lambda^F_{e_i}(x) > / x \in U\}$

$$A^{T,I,F}_{e_i}(x*y) \geq \min\{A^{T,I,F}_{e_i}(x), A^{T,I,F}_{e_i}(y)\} = \begin{Bmatrix} A^T_{e_i}(x*y) \geq \min\{A^T_{e_i}(x), A^T_{e_i}(y)\}, \\ A^I_{e_i}(x*y) \geq \min\{A^I_{e_i}(x), A^I_{e_i}(y)\}, \\ A^F_{e_i}(x*y) \geq \min\{A^F_{e_i}(x), A^F_{e_i}(y)\} \end{Bmatrix}$$

$$\lambda^{T,I,F}_{e_i}(x*y) \leq \max\{\lambda_{e_i}(x), \lambda_{e_i}(y)\} = \begin{Bmatrix} \lambda^T_{e_i}(x*y) \leq \max\{\lambda^T_{e_i}(x), \lambda^T_{e_i}(y)\}, \\ \lambda^I_{e_i}(x*y) \leq \max\{\lambda^I_{e_i}(x), \lambda^I_{e_i}(y)\}, \\ \lambda^F_{e_i}(x*y) \leq \max\{\lambda^F_{e_i}(x), \lambda^F_{e_i}(y)\} \end{Bmatrix}$$

If (P, M) is a NSCS BCK/BCI algebra over U based on all parameter, we say that (P, M) is a NSCS BCK/BCI algebra over U.

Example3.2. Consider a BCK-algebra $U = \{0, a, b, c\}$ with the following Cayley table.

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Consider a set of parameters A={e_1,e_2,e_3}. Let (P,M) be a Neutrosophic soft cubic set over U which is represented as the following tabular form.

*	P(e_1)		P(e_2)		P(e_3)	
	<Ae_1(x*y),	λ e_1(x*y) >	<Ae_2(x*y),	λ e_2(x*y) >	<Ae_3(x*y),	λ e_3(x*y) >
0	[.7,.9][.3,.5][.4,.6]	[.1,.3,.5]	[.6,.8][.4,.6][.2,.4]	[.1,.3,.4]	[.2,.5][.1,.2][.2,.4]	[.4,.8,.7]
a	[.6,.8][.1,.2][.1,.4]	[.3,.5,.6]	[.5,.8][.3,.4][.2,.4]	[.1,.3,.5]	[.6,.8][.2,.3][.1,.4]	[.3,.5,.6]
b	[.3,.8][.2,.5][.1,.3]	[.2,.4,.7]	[.1,.5][.3,.6][.1,.2]	[.5,.4,.6]	[.3,8][.2,.5][.1,.3]	[.3,.7,.6]
c	[.1,.5][.3,.5][.2,.4]	[.1,.3,.5]	[.2,.3][.1,.2][.1,.3]	[.2,.3,.4]	[.5,.6][.3,.4][.1,.3]	[.2,.4,.5]

Then (P,M) is a NSCS BCK-algebra over U based on parameters $e_1$and e_2 but it is not a NSCS BCK-algebra over U based on the parameters e_3 since

$\lambda^{T,I,F}_{e_3}(a*b)$ = [0.4,0.8,0.7] $\nleq$ [0.3,0.7,0.6]= $\max\{\lambda_{e_i}(a), \lambda_{e_i}(b)\}$ and/or

$A^{T,I,F}_{e_3}(a*b)$ =[0.2,0.5][0.1,0.2][0.2,0.4] $\ngeq$ [0.3,0.8][0.2,0.3][0.1,0.3]= $\min\{A^{T,I,F}_{e_i}(a), A^{T,I,F}_{e_i}(b)\}$,

Proposition 3.3.

If (P, M) is a NSCS BCK/BCI algebra over U based on a parameter e_i in M then

$A^{T,I,F}_{e_i}(0) \geq A^{T,I,F}_{e_i}(x)$ and $\lambda^{T,I,F}_{e_i}(0) \leq \lambda^{T,I,F}_{e_i}(x)$ for all $x \in U$ ·

Proof.

For any $x \in U$ we have

$$A_{e_i}^{T,I,F}(0) = A_{e_i}^{T,I,F}(x * x) \geq \min\{A_{e_i}^{T,I,F}(x), A_{e_i}^{T,I,F}(x)\}$$
$$= \min\left\{[A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)], [A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)], [A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)]\right\},$$
$$\min\left\{[A_{e_i}^{-T}(y), A_{e_i}^{+T}(y)], [A_{e_i}^{-I}(y), A_{e_i}^{+I}(y)], [A_{e_i}^{-F}(y), A_{e_i}^{+F}(y)]\right\}$$
$$= [A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)], [A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)], [A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)]$$
$$= A_{e_i}^{T,I,F}(x)$$

and

$$\lambda_{e_i}^{T,I,F}(0) = \lambda_{e_i}^{T,I,F}(x * x) \leq \max\{\lambda_{e_i}^{T,I,F}(x), \lambda_{e_i}^{T,I,F}(x)\}$$
$$= \max\{(\lambda_{e_i}^{T}(x), \lambda_{e_i}^{I}(x), \lambda_{e_i}^{F}(x)), (\lambda_{e_i}^{T}(x), \lambda_{e_i}^{I}(x), \lambda_{e_i}^{F}(x))\}$$
$$= (\lambda_{e_i}^{T}(x), \lambda_{e_i}^{I}(x), \lambda_{e_i}^{F}(x)) = \lambda_{e_i}^{T,I,F}(x) \text{ for all } x \in U.$$

Corollary 3.4.

If (P,M) is a NSCS BCK/BCI algebra over U then $A_{e_i}^{T,I,F}(0) \geq A_{e_i}^{T,I,F}(x)$ and $\lambda_{e_i}^{T,I,F}(0) \leq \lambda_{e_i}^{T,I,F}(x)$ for all $e_i \in M$ and $x \in U$.

Proof.

Let (P,M) be NSCS BCK/BCI algebra over U. For a parameter $e_i \in M$, $r \in [0,1]$ and $[s,t]^{T,I,F} \in [I]^{T,I,F}$ we define a set

$$\cup_{e_i}(P,M)_{[s,t]}^{r} := \{x \in U \,/\, A_{e_i}^{T,I,F}(x) \geq [s,t]^{T,I,F}, \lambda_{e_i}^{T,I,F}(x) \leq r^{T,I,F}\}$$

If we put

$$A_{e_i}(P,M)_{[s,t]} := \{x \in U \,/\, A_{e_i}^{T,I,F}(x) \geq [s,t]^{T,I,F}\} \text{ and}$$
$$\lambda_{e_i}(P,M)^{r} := \{x \in U \,/\, \lambda_{e_i}^{T,I,F}(x) \leq r^{T,I,F}\}$$

i.e

$$A_{e_i}(P,M)_{[s,t]} := \{x \in U \,/\, A_{e_i}^{T}(x) \geq [s,t]^{T}, A_{e_i}^{I}(x) \geq [s,t]^{I}, A_{e_i}^{F}(x) \geq [s,t]^{F}\} \text{ and}$$
$$\lambda_{e_i}(P,M)^{r} := \{x \in U \,/\, \lambda_{e_i}^{T}(x) \leq r^{T}, \lambda_{e_i}^{I}(x) \leq r^{I}, \lambda_{e_i}^{F}(x) \leq r^{F}\}$$

then

$$\cup_{e_i}(P,M)_{[s,t]}^{r} := A_{e_i}^{T,I,F}(P,M)_{[s,t]} \cap \lambda_{e_i}^{T,I,F}(P,M)^{r}\}$$

Theorem 3.5.

For a NSCS (P,M) over U, the following are equivalent

1. (P,M) is a NSCS BCK/BCIalgebra over U based on the parameter $e_i \in M$.
2. The sets $A_{e_i}(P,M)_{[s,t]} := \{x \in U \,/\, A_{e_i}^{T,I,F}(x) \geq [s,t]^{T,I,F}\}$ and $\lambda_{e_i}(P,M)^{r} := \{x \in U \,/\, \lambda_{e_i}^{T,I,F}(x) \leq r^{T,I,F}\}$ are sub algebra of U for $r \in [0,1]$ and $[s,t]^{T,I,F} \in [I]^{T,I,F}$ whenever they are nonempty.

Proof.

Assume that (P,M) is a NSCS BCK/BCIalgebra over U based on the parameter $e_i \in M$.

Let $x, y \in U$ and $[s,t]^{T,I,F} \in [I]^{T,I,F}$ be such that $x, y \in A_{e_i}(P,M)_{[s,t]}$. Then $A_{e_i}^{T,I,F}(x) \geq [s,t]^{T,I,F}$ and $A_{e_i}^{T,I,F}(y) \geq [s,t]^{T,I,F}$.

It follows from (3.1) that

$$A_{e_i}^{T,I,F}(x * y) \geq \min\{A_{e_i}^{T,I,F}(x), A_{e_i}^{T,I,F}(y)\} \geq \min\{[s,t]^{T,I,F}, [s,t]^{T,I,F}\} = [s,t]^{T,I,F}$$

Hence $x * y \in A_{e_i}(P,M)_{[s,t]}$, and therefore $A_{e_i}(P,M)_{[s,t]}$ is subalgebra of U.

Now let $x, y \in U$ and $[r]^{T,I,F} \in [0,1]$ be such that $x, y \in \lambda_{e_i}(P,M)^{r}$, then $\lambda_{e_i}^{T,I,F}(x) \leq r^{T,I,F}$ *and* $\lambda_{e_i}^{T,I,F}(y) \leq r^{T,I,F}$.

Using (3.2) we have $\lambda_{e_i}^{T,I,F}(x * y) \leq \max\{\lambda_{e_i}^{T,I,F}(x), \lambda_{e_i}^{T,I,F}(y)\} \leq r$ and so $x, y \in \lambda_{e_i}(P,M)^{r}$.

Therefore $\lambda_{e_i}(P,M)^{r}$ is a sub algebra of U.

Conversely. Suppose that the sets $A_{e_i}(P,M)_{[s,t]} := \{x \in U \,/\, A_{e_i}^{T,I,F}(x) \geq [s,t]^{T,I,F}\}$ and $\lambda_{e_i}(P,M)^{r} := \{x \in U \,/\, \lambda_{e_i}^{T,I,F}(x) \leq r^{T,I,F}\}$ are sub algebras of U for all $r \in [0,1]$ and $[s,t]^{T,I,F} \in [I]^{T,I,F}$ whenever they are nonempty.

Assume that there exists $a, b \in U$ such that $A_{e_i}^{T,I,F}(a * b) \ngeq \min\{A_{e_i}^{T,I,F}(a), \lambda_{e_i}^{T,I,F}(b)\}$. If we take

$A^{T}_{e_i}(a)=[s^{T}_{a},t^{T}_{a}]$, $A^{I}_{e_i}(a)=[s^{I}_{a},t^{I}_{a}]$, $A^{F}_{e_i}(a)=[s^{F}_{a},t^{F}_{a}]$ and $A^{T}_{e_i}(b)=[s^{T}_{b},t^{T}_{b}]$, $A^{I}_{e_i}(b)=[s^{I}_{b},t^{I}_{b}]$, $A^{F}_{e_i}(b)=[s^{F}_{b},t^{F}_{b}]$

and $A^{T}_{e_i}(a*b)=[s^{T}_{0},t^{T}_{0}]$, $A^{I}_{e_i}(a*b)=[s^{I}_{0},t^{I}_{0}]$, $A^{F}_{e_i}(a*b)=[s^{F}_{0},t^{F}_{0}]$

then, $[s^{T}_{0},t^{T}_{0}]\geq \min\{[s^{T}_{a},t^{T}_{a}],[s^{T}_{b},t^{T}_{b}]\} = [\min\{s^{T}_{a},s^{T}_{b}\},\min\{t^{T}_{a},t^{T}_{b}\}]$,

$[s^{I}_{0},t^{I}_{0}]\geq \min\{[s^{I}_{a},t^{I}_{a}],[s^{I}_{b},t^{I}_{b}]\} = [\min\{s^{I}_{a},s^{I}_{b}\},\min\{t^{I}_{a},t^{I}_{b}\}]$,

$[s^{F}_{0},t^{F}_{0}]\geq \min\{[s^{F}_{a},t^{F}_{a}],[s^{F}_{b},t^{F}_{b}]\} = [\min\{s^{F}_{a},s^{F}_{b}\},\min\{t^{F}_{a},t^{F}_{b}\}]$.

Hence we have the following three cases:

(i) (a) $s^{T}_{0}\geq \min\{s^{T}_{a},s^{T}_{b}\}\ and\ t^{T}_{0}\leq \min\{t^{T}_{a},t^{T}_{b}\}$

(b) $s^{I}_{0}\geq \min\{s^{I}_{a},s^{I}_{b}\}\ and\ t^{I}_{0}\leq \min\{t^{I}_{a},t^{I}_{b}\}$

(c) $s^{F}_{0}\geq \min\{s^{F}_{a},s^{F}_{b}\}\ and\ t^{T}_{0}\leq \min\{t^{F}_{a},t^{F}_{b}\}$

(ii) (a) $s^{T}_{0}< \min\{s^{T}_{a},s^{T}_{b}\}\ and\ t^{T}_{0}\geq \min\{t^{T}_{a},t^{T}_{b}\}$

(b) $s^{I}_{0}< \min\{s^{I}_{a},s^{I}_{b}\}\ and\ t^{I}_{0}\geq \min\{t^{I}_{a},t^{I}_{b}\}$

(c) $s^{F}_{0}< \min\{s^{F}_{a},s^{F}_{b}\}\ and\ t^{T}_{0}\geq \min\{t^{F}_{a},t^{F}_{b}\}$

(iii) (a) $s^{T}_{0}< \min\{s^{T}_{a},s^{T}_{b}\}\ and\ t^{T}_{0}< \min\{t^{T}_{a},t^{T}_{b}\}$

(b) $s^{I}_{0}< \min\{s^{I}_{a},s^{I}_{b}\}\ and\ t^{I}_{0}< \min\{t^{I}_{a},t^{I}_{b}\}$

(c) $s^{F}_{0}< \min\{s^{F}_{a},s^{F}_{b}\}\ and\ t^{T}_{0}< \min\{t^{F}_{a},t^{F}_{b}\}$

For the First case we have

$$A^{T}_{e_i}(a)=[s^{T}_{a},t^{T}_{a}] \geq [\min\{s^{T}_{a},s^{T}_{b}\},\min\{t^{T}_{a},t^{T}_{b}\}] \geq [\min\{s^{T}_{a},s^{T}_{b}\},t^{T}_{0}],$$

$$A^{I}_{e_i}(a)=[s^{I}_{a},t^{I}_{a}] \geq [\min\{s^{I}_{a},s^{I}_{b}\},\min\{t^{I}_{a},t^{I}_{b}\}] \geq [\min\{s^{I}_{a},s^{I}_{b}\},t^{I}_{0}],$$

$$A^{F}_{e_i}(a)=[s^{F}_{a},t^{F}_{a}] \geq [\min\{s^{F}_{a},s^{F}_{b}\},\min\{t^{F}_{a},t^{F}_{b}\}] \geq [\min\{s^{F}_{a},s^{F}_{b}\},t^{F}_{0}],$$

$$A^{T}_{e_i}(b)=[s^{T}_{b},t^{T}_{b}] \geq [\min\{s^{T}_{a},s^{T}_{b}\},\min\{t^{T}_{a},t^{T}_{b}\}] \geq [\min\{s^{T}_{a},s^{T}_{b}\},t^{T}_{0}],$$

$$A^{I}_{e_i}(b)=[s^{I}_{b},t^{I}_{b}] \geq [\min\{s^{I}_{a},s^{I}_{b}\},\min\{t^{I}_{a},t^{I}_{b}\}] \geq [\min\{s^{I}_{a},s^{I}_{b}\},t^{I}_{0}],$$

$$A^{F}_{e_i}(b)=[s^{F}_{b},t^{F}_{b}] \geq [\min\{s^{F}_{a},s^{F}_{b}\},\min\{t^{F}_{a},t^{F}_{b}\}] \geq [\min\{s^{F}_{a},s^{F}_{b}\},t^{F}_{0}],$$

And so $a*b\in A_{e_i}(P,M)_{[\min\{s_a^{T,I,F},s_b^{T,I,F}\},t_0^{T,I,F}]}$ but $a*b\notin A_{e_i}(P,M)_{[\min\{s_a^{T,I,F},s_b^{T,I,F}\},t_0^{T,I,F}]}$, a contradiction. By the similar way (ii) and (iii) induce a contradiction. Thus $A^{T,I,F}_{e_i}(x*y)\geq \min\{A^{T,I,F}_{e_i}(x),A^{T,I,F}_{e_i}(y)\}$ and for all $x,y\in U$.

Now Suppose (3.2) is false. Then $\lambda^{T,I,F}_{e_i}(a*b)>r\geq \max\{\lambda^{T,I,F}_{e_i}(a),\lambda^{T,I,F}_{e_i}(b)\}$ for some $x,y\in U$ and $r\in[0,1]$ which implies $a,b\in \lambda_{e_i}(P,M)^{r_0}$ but $a*b\notin A_{e_i}(P,M)^{r_0}$, this is a contradiction and hence (3.2) is valid. Therefore (P,M) is a NSCS BCK/BCI algebra over U based on the parameter $e_i\in M$.

Corollary 3.6.

If a NSCS (P,M) over U is a BCK/BCIalgebra over U based on parameter $e_i\in M$ then $\bigcup_{e_i}(P,M)=\{x\in U / A^{T,I,F}_{e_i}(x)\geq[s,t],\lambda^{T,I,F}_{e_i}(x)\leq r\}$ is a sub algebra of U for all $r\in[0,1]$ and $[s,t]^{T,I,F}\in[I]^{T,I,F}$ whenever it is nonempty.

Theorem 3.7.

The R-intersection of two NSCS BCK/BCI-algebra over U is also a NSCS BCK/BCIalgebra over U.

Proof.

Let (P,M) and (Q,N) be NSCS BCK/BCI-algebra over U and let $(H,C)=(P,M)\cap_R(Q,N)$ be the R-intersection of (P,M) and (Q,N). Then $C=M\cup N$ .For any $e_i\in C$, if $e_i\in M\setminus N$ then

$H(e_i) = P(e_i)\wedge_R Q(e_i)$ and $e_i\in I\cap J$.

$H(e_i) = \{<x,C_{e_i}(x),\gamma_{e_i}(x)>: x\in X\}$ $e_i\in M\cap N$

$$\begin{aligned} C_{e_i}(x*y) &= P_{e_i}(x*y) \\ &= P_{e_i}^{T,I,F}(x*y) \\ &\geq \min\{A_{e_i}^{T,I,F}(x), A_{e_i}^{T,I,F}(y)\} \\ &= \min\{C_{e_i}^{T,I,F}(x), C_{e_i}^{T,I,F}(y)\} \end{aligned}$$

and

$$\begin{aligned} \gamma_{e_i}(x*y) &= P_{e_i}(x*y) \\ &= P_{e_i}^{T,I,F}(x*y) \\ &\leq \max\{\lambda_{e_i}^{T,I,F}(x), \lambda_{e_i}^{T,I,F}(y)\} \\ &== \max\{\gamma_{e_i}^{T,I,F}(x), \gamma_{e_i}^{T,I,F}(y)\} \end{aligned}$$

$\text{for all } x, y \in U$ .Similarly, if $e_i \in N \setminus M$ then

$C_{e_i}(x*y) \geq \min\{C_{e_i}^{T,I,F}(x), C_{e_i}^{T,I,F}(y)\} \text{and} \gamma_{e_i}(x*y) \leq \max\{\gamma_{e_i}^{T,I,F}(x), \gamma_{e_i}^{T,I,F}(y)\}$ $\text{for all } x, y \in U$.

Suppose that $e_i \in M \cap N$ .Then

$$\begin{aligned} C_{e_i}(x*y) &= P(e_i) \wedge_R Q(e_i)\,(x*y) \\ &\geq \min\{A_{e_i}^{T,I,F}(x), B_{e_i}^{T,I,F}(y)\} \\ &= \min\{C_{e_i}^{T,I,F}(x), C_{e_i}^{T,I,F}(y)\} \\ &= \min\{C_{e_i}(x), C_{e_i}(y)\} \end{aligned}$$

and

$$\begin{aligned} \gamma_{e_i}(x*y) &= P(e_i) \wedge_R Q(e_i)\,(x*y) \\ &\leq \max\{\lambda_{e_i}^{T,I,F}(x), \mu_{e_i}^{T,I,F}(y)\} \\ &= \max\{\gamma_{e_i}^{T,I,F}(x), \gamma_{e_i}^{T,I,F}(y)\} \\ &= \max\{\gamma_{e_i}(x), \gamma_{e_i}(y)\} \end{aligned}$$

$\text{for all } x, y \in U$ .Therefore $(H,C) = (P,M) \cap_R (Q,N)$ is a NSCS BCK/BCI algebra over U.

4. SUBALGEBRAS OF BCK/BCI ALGEBRAS BASED ON NEUTROSOPHIC SOFT CUBIC SET

Definition 4.1 .

A Neutrosophic soft cubic set (P,M) over U is said to be Neutrosophic soft cubic BCK/BCIalgebra over U based on a parameter e_i (briefly, e-Neutrosophic soft cubic subalgebra over U) if there exists $e_i \in M$ such that

$$A_{e_i}(x*y) \geq \min\{A_{e_i}(x), A_{e_i}(y)\}, \tag{4.1}$$

$$\lambda_{e_i}(x*y) \leq \max\{\lambda_{e_i}(x), \lambda_{e_i}(y)\} \tag{4.2}$$

Where

$$\begin{aligned} A e_i(x) = A_{e_i}^{T,I,F}(x) &= \{< x, A_{e_i}^T(x), A_{e_i}^I(x), A_{e_i}^F(x) > / x \in U\} \\ &= \{< x, [A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)], [A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)], [A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)] > / x \in U\} \end{aligned}$$

And

$$\lambda e_i(x) = \lambda_{e_i}^{T,I,F}(x) = \{< x, (\lambda_{e_i}^T(x), \lambda_{e_i}^I(x), \lambda_{e_i}^F(x) > / x \in U\}$$

$$A_{e_i}^{T,I,F}(x*y) \geq \min\{A_{e_i}^{T,I,F}(x), A_{e_i}^{T,I,F}(y)\} = \left\{ \begin{array}{l} A_{e_i}^{T}(x*y) \geq \min\{A_{e_i}^{T}(x), A_{e_i}^{T}(y)\}, \\ A_{e_i}^{I}(x*y) \geq \min\{A_{e_i}^{I}(x), A_{e_i}^{I}(y)\}, \\ A_{e_i}^{F}(x*y) \geq \min\{A_{e_i}^{F}(x), A_{e_i}^{F}(y)\} \end{array} \right\}$$

$$\lambda_{e_i}^{T,I,F}(x*y) \leq \max\{\lambda_{e_i}(x), \lambda_{e_i}(y)\} = \left\{ \begin{array}{l} \lambda_{e_i}^{T}(x*y) \leq \max\{\lambda_{e_i}^{T}(x), \lambda_{e_i}^{T}(y)\}, \\ \lambda_{e_i}^{I}(x*y) \leq \max\{\lambda_{e_i}^{I}(x), \lambda_{e_i}^{I}(y)\}, \\ \lambda_{e_i}^{F}(x*y) \leq \max\{\lambda_{e_i}^{F}(x), \lambda_{e_i}^{F}(y)\} \end{array} \right\}$$

If (P,M) is a NSCS BCK/BCI algebra over U based on all parameter , we say that (P,M) is a NSCS BCK/BCI algebra over U.

Theorem 4.2.

Let (P,M) & (Q,N) be NSCS subalgebras over U. If M&N are disjoint, then the R-union of (P,M) & (Q,N) is a NSCS over U.

Proof.

By definition 2.11 we can write as $(P,M)\cup_R (Q,N)=(H,C)$ where $C=M\cup N$

$$H(e_i)=\begin{cases} P(e_i) & if\ e_i\in M-N \\ Q(e_i) & if\ e_i\in N-M \\ P(e_i)\vee_R Q(e_i) & if\ e_i\in M\cap N \end{cases}$$

Since $M\cap N=\varphi$ either $e_i\in M-N$ or $e_i\in N-M$ for all $e_i\in C$. If $e_i\in M-N$ then $H(e_i)=P(e_i)$ is NSCS sub algebra over U. If $e_i\in N-M$ then $H(e_i)=Q(e_i)$ is a NSCS sub algebra over U. Hence $(P,M)\cup_R (Q,N)=(H,C)$ is a NSCS sub algebra over U.

Theorem 4.3.

Given a parameter $e_i\in M$ a NSCS (P,M) over U is an e-NSCS subalgebra over U if and only if the sets

$$A_{e_i}[\delta_1,\delta_2]:=\{x\in U \mid A_{e_i}^{T,I,F}(x)\geq[\delta_1^{T,I,F},\delta_2^{T,I,F}]\}, \tag{5}$$

$$\lambda_{e_i}(t)=\{x\in U \mid \lambda_{e_i}^{T,I,F}(x)\leq t^{T,I,F}\} \tag{6}$$

are subalgebras of U for all $[\delta_1^{T,I,F},\delta_2^{T,I,F}]\in I^{T,I,F}$ and $t^{T,I,F}\in[0,1]$

Proof .

Assume that a NSCS (P,M) over U is an e-NSCS subalgebra over U and let $x,y\in U$.

If $x,y\in U$ $A_{e_i}[\delta_1,\delta_2]$ for all $[\delta_1^{T,I,F},\delta_2^{T,I,F}]\in I^{T,I,F}$ then

$A_{e_i}^{T,I,F}(x)\geq[\delta_1^{T,I,F},\delta_2^{T,I,F}]$ and $A_{e_i}^{T,I,F}(y)\geq[\delta_1^{T,I,F},\delta_2^{T,I,F}]$

From (4.1) and (4.2)

$$\begin{aligned}
&A_{e_i}^{T,I,F}(x*y)\geq\min\{A_{e_i}^{T,I,F}(x),A_{e_i}^{T,I,F}(y)\}\\
&=\min\Big\{[A_{e_i}^{-T}(x),A_{e_i}^{+T}(x)],[A_{e_i}^{-I}(x),A_{e_i}^{+I}(x)],[A_{e_i}^{-F}(x),A_{e_i}^{+F}(x)]\Big\},\\
&\quad\min\Big\{[A_{e_i}^{-T}(y),A_{e_i}^{+T}(y)],[A_{e_i}^{-I}(y),A_{e_i}^{+I}(y)],[A_{e_i}^{-F}(y),A_{e_i}^{+F}(y)]\Big\}\\
&=\min\{\delta_1^{T}(x),\delta_1^{I}(x),\delta_1^{F}(x)\},\min\{\delta_2^{T}(y),\delta_2^{I}(y),\delta_2^{F}(y)\}\\
&=\min\Big\{[\delta_1^{T}(x),\delta_2^{T}(y)],[\delta_1^{I}(x),\delta_2^{I}(y)],[\delta_1^{F}(x),\delta_2^{F}(y)]\Big\}\\
&=[\delta_1^{T,I,F}(x),\delta_2^{T,I,F}(y)]
\end{aligned}$$

Hence $x,y\in A_{e_i}[\delta_1,\delta_2]$. Now if $x,y\in\lambda_{e_i}(t)$ for all $t=t^{T},t^{I},t^{F}\in[0,1]$ then

$\lambda_{e_i}(x)\leq t,\ and\ \lambda_{e_i}(y)\leq t$.

From (4.2) we have

$$\begin{aligned}
&\lambda_{e_i}(x*y)\leq\max\{\lambda_{e_i}(x),\lambda_{e_i}(y)\}\\
&\leq\max\{\lambda_{e_i}^{T,I,F}(x),\lambda_{e_i}^{T,I,F}(y)\}\\
&=\max\{\lambda_{e_i}^{T}(x),\lambda_{e_i}^{I}(x),\lambda_{e_i}^{F}(x)\},\max\{\lambda_{e_i}^{T}(y),\lambda_{e_i}^{I}(y),\lambda_{e_i}^{F}(y)\}\\
&=\max\{(\lambda_{e_i}^{T}(x),\lambda_{e_i}^{T}(y)),(\lambda_{e_i}^{I}(x),\lambda_{e_i}^{I}(y)),(\lambda_{e_i}^{F}(x)\lambda_{e_i}^{F}(y))\}\\
&=t^{T},t^{I},t^{F}\\
&=t^{T,I,F}=t
\end{aligned}$$

And so $x,y\in\lambda_{e_i}(x*y)$. Therefore $A_{e_i}^{T,I,F}[\delta_1,\delta_2]$ and $\lambda_{e_i}^{T,I,F}(t)$ sub algebra over U.

Conversely suppose that $A_{e_i}^{T,I,F}[\delta_1,\delta_2]$ and $\lambda_{e_i}^{T,I,F}(t)$ are subalgebra over U for all $[\delta_1^{T,I,F},\delta_2^{T,I,F}]\in I^{T,I,F}$ and $t^{T,I,F}\in[0,1]$.

Assume that there exist $a,b\in U$ such that $A_{e_i}^{T,I,F}(a*b)\geq\min\{A_{e_i}^{T,I,F}(a),A_{e_i}^{T,I,F}(b)\}$.

Let

$A_{e_i}^{T,I,F}(a)=[\gamma_1^{T,I,F},\gamma_2^{T,I,F}]$, $A_{e_i}^{T,I,F}(b)=[\gamma_3^{T,I,F},\gamma_4^{T,I,F}]$, $A_{e_i}^{T,I,F}(a*b)=[\delta_1^{T,I,F},\delta_2^{T,I,F}]$ then

$$\begin{aligned}
&[\delta_1^{T,I,F},\delta_2^{T,I,F}]\leq\min\Big\{[\gamma_1^{T,I,F},\gamma_2^{T,I,F}],[\gamma_3^{T,I,F},\gamma_4^{T,I,F}]\Big\}\\
&=\min[\gamma_1^{T,I,F},\gamma_3^{T,I,F}],\min[\gamma_2^{T,I,F},\gamma_4^{T,I,F}]
\end{aligned}$$

Hence $\delta_1^{T,I,F} \le \min[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}]$ and $\delta_2^{T,I,F} \le \min[\gamma_2^{T,I,F}, \gamma_4^{T,I,F}]$

Taking

$$[\tau_1, \tau_2] = \frac{1}{2}\left(A_{e_i}^{T,I,F}(a*b) + \min\{A_{e_i}^{T,I,F}(a) + A_{e_i}^{T,I,F}(b)\}\right)$$

$$= \frac{1}{2}\left([\delta_1^{T,I,F}, \delta_2^{T,I,F}] + \min\{[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}], [\gamma_2^{T,I,F}, \gamma_4^{T,I,F}]\}\right)$$

$$= \frac{1}{2}\left(\delta_1^{T,I,F} + \min[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}]\right) + \frac{1}{2}\left(\delta_2^{T,I,F} + \min[\gamma_2^{T,I,F}, \gamma_4^{T,I,F}]\right)$$

It follows that

$$\min[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}] > \tau_1 = \frac{1}{2}\left(\delta_1^{T,I,F} + \min[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}]\right) > \delta_1^{T,I,F}$$

$$\min[\gamma_2^{T,I,F}, \gamma_4^{T,I,F}] > \tau_2 = \frac{1}{2}\left(\delta_2^{T,I,F} + \min[\gamma_2^{T,I,F}, \gamma_4^{T,I,F}]\right) > \delta_2^{T,I,F}$$

And so that $\min[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}], \min[\gamma_2^{T,I,F}, \gamma_4^{T,I,F}] > [\tau_1, \tau_2] > [\delta_1^{T,I,F}, \delta_2^{T,I,F}] = A_{e_i}^{T,I,F}(a*b) \cdot\ a*b \notin A_{e_i}^{T,I,F}[\tau_1, \tau_2]$

On the other hand we know that

$$A_{e_i}^{T,I,F}(a) = [\gamma_1^{T,I,F}, \gamma_2^{T,I,F}] \ge \min\{[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}], [\gamma_2^{T,I,F}, \gamma_4^{T,I,F}]\} \ge [\tau_1, \tau_2]$$

$$A_{e_i}^{T,I,F}(b) = [\gamma_3^{T,I,F}, \gamma_4^{T,I,F}] \ge \min\{[\gamma_1^{T,I,F}, \gamma_3^{T,I,F}], [\gamma_2^{T,I,F}, \gamma_4^{T,I,F}]\} \ge [\tau_3, \tau_4]$$

which implies that $a, b \in A_{e_i}^{T,I,F}[\tau_1, \tau_2]$.This is a contradiction and so $A_{e_i}^{T,I,F}(x*y) \ge \min\{A_{e_i}^{T,I,F}(x), A_{e_i}^{T,I,F}(y)\}$ for all $x, y \in U$.

Now assume that $\lambda_{e_i}^{T,I,F}(a*b) > \max\{\lambda_{e_i}^{T,I,F}(a), \lambda_{e_i}^{T,I,F}(b)\}$ for some $a, b \in U$ then there exist $t_0^{T,I,F} \in [0,1]$ such that $\lambda_{e_i}^{T,I,F}(a*b) \ge t_0^{T,I,F} > \max\{\lambda_{e_i}^{T,I,F}(a), \lambda_{e_i}^{T,I,F}(b)\}$ .Hence $a, b \in t_0^{T,I,F}$ but $a, b \notin t_0^{T,I,F}$ This is a contradiction and therefore $\lambda_{e_i}(x*y) \le \max\{\lambda_{e_i}(x), \lambda_{e_i}(y)\}$ for all $x, y \in U$. Consequently, (P, M) is an e-NSCS subalgebra over U.

Proposition 4.4.

Given a parameter $e_i \in M$ if a NSCS (P, M) over U is an e-NSCS subalgebra over U then $A_{e_i}^{T,I,F}(0) \ge A_{e_i}^{T,I,F}(x)$ and $\lambda_{e_i}^{T,I,F}(0) \le \lambda_{e_i}^{T,I,F}(x)$ for all $x \in U$ ·

Proof.

For any $x \in U$ we have

$$A_{e_i}^{T,I,F}(0) = A_{e_i}^{T,I,F}(x*x) \ge \min\{A_{e_i}^{T,I,F}(x), A_{e_i}^{T,I,F}(x)\}$$

$$= \min\left\{\begin{matrix}[[A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)], [A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)], [A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)]], \\ [[A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)], [A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)], [A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)]]\end{matrix}\right\}$$

$$= [A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)], [A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)], [A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)]$$

$$= A_{e_i}^{T,I,F}(x)$$

$$\lambda_{e_i}^{T,I,F}(0) = \lambda_{e_i}^{T,I,F}(x*x) \le \max\{\lambda_{e_i}^{T,I,F}(x), \lambda_{e_i}^{T,I,F}(x)\}$$

$$= \max\{(\lambda_{e_i}^{T}(x), \lambda_{e_i}^{I}(x), \lambda_{e_i}^{F}(x)), (\lambda_{e_i}^{T}(x), \lambda_{e_i}^{I}(x), \lambda_{e_i}^{F}(x))\}$$

$$= (\lambda_{e_i}^{T}(x), \lambda_{e_i}^{I}(x), \lambda_{e_i}^{F}(x) = \lambda_{e_i}^{T,I,F}(x)$$

for all $x \in U$.

Theorem 4.5.

Let (P, M) be a e-NSCS sub algebra over U for a parameter $e_i \in M$. If there is a sequence in U such that and $\lim_{n\to\infty} A_{e_i}^{T,I,F}(x_n) = [1,1]$ and $\lim_{n\to\infty} \lambda_{e_i}^{T,I,F}(x_n) = 0$ then $A_{e_i}^{T,I,F}(0) = [1,1]$ and $\lambda_{e_i}^{T,I,F}(0) = 0$.

Proof.

Since $A_{e_i}^{T,I,F}(0) \ge A_{e_i}^{T,I,F}(x)$ and $\lambda_{e_i}^{T,I,F}(0) \le \lambda_{e_i}^{T,I,F}(x)$ for all $x \in U$,

We have

$A_{e_i}^{T,I,F}(0) \ge A_{e_i}^{T,I,F}(x)$,

$\lambda_{e_i}^{T,I,F}(0) \le \lambda_{e_i}^{T,I,F}(x)$,

for every positive integer n. Note that

$[1,1] \geq A_{e_i}^{T,I,F}(0) \geq \lim_{n\to\infty} A_{e_i}^{T,I,F}(x_n) = [1,1]$ and $0 \leq \lambda_{e_i}^{T,I,F}(0) \leq \lambda_{e_i}^{T,I,F}(x_n) = 0$.

Hence $A_{e_i}^{T,I,F}(0) = [1,1]$ and $\lambda_{e_i}^{T,I,F}(0) = 0$.

Theorem 4.6.

Given a parameter $e_i \in M$ if a NSCS (P,M) over U is an e-NSCS subalgebra over U then the sets $U_{A_{e_i}^{T,I,F}} := \{x \in U \mid A_{e_i}^{T,I,F}(x) = A_{e_i}^{T,I,F}(0)\}$ and $U_{\lambda_{e_i}^{T,I,F}} := \{x \in U \mid \lambda_{e_i}^{T,I,F}(x) = \lambda_{e_i}^{T,I,F}(0)\}$ are subalgebra of U.

Proof.

Let $x, y \in U$ .If $x, y \in U_{A_{e_i}^{T,I,F}(0)}$ then $A_{e_i}^{T,I,F}(x) = A_{e_i}^{T,I,F}(0) = A_{e_i}^{T,I,F}(y)$. Hence

$A_{e_i}^{T,I,F}(x*y) \geq \min\{A_{e_i}^{T,I,F}(x), A_{e_i}^{T,I,F}(y)\} = \min\{A_{e_i}^{T,I,F}(0), A_{e_i}^{T,I,F}(0)\} = A_{e_i}^{T,I,F}(0)$ and

$\lambda_{e_i}^{T,I,F}(x*y) \leq \max\{\lambda_{e_i}^{T,I,F}(x), \lambda_{e_i}^{T,I,F}(y)\} = \max\{\lambda_{e_i}^{T,I,F}(0), \lambda_{e_i}^{T,I,F}(0)\} = \lambda_{e_i}^{T,I,F}(0)$

Combining this and Proposition 4.5, we have $A_{e_i}^{T,I,F}(x*y) = A_{e_i}^{T,I,F}(0)$ and $\lambda_{e_i}^{T,I,F}(x*y) = \lambda_{e_i}^{T,I,F}(0)$. This shows that $x*y \in U_{A_{e_i}^{T,I,F}}$ and $x*y \in U_{\lambda_{e_i}^{T,I,F}}$. Therefore $U_{A_{e_i}^{T,I,F}}$ *and* $U_{\lambda_{e_i}^{T,I,F}}$ neutrosophic soft cubic subalgebras of U.

Corollary 4.8.

Given a parameter $e_i \in M$ if a NSCS (P,M) over U is an e-NSCS subalgebra over U then the set $U_{A_{e_i}^{T,I,F}} \cap U_{\lambda_{e_i}^{T,I,F}}$ is subalgebra of U.

Proof.

The proof is straight forward.

References

[1] R. Anitha Cruz and F. Nirmala Irudayam, Neutrosophic Soft Cubic Set, International Journal of Mathematics Trends andTechnology,46(2)(2017) 88-94.
[2] R. Anitha Cruz and F. Nirmala Irudayam, More on P-union and P- intersection of Neutrosophic Soft Cubic Set, Neutrosophic Sets and Systems (17)74-87.
[3] R. Anitha Cruz and F. Nirmala Irudayam, More on R-union and R- intersection of Neutrosophic Soft Cubic Set, Mathematical Journal of Interdisciplinary Sciences 6(1)2017.
[4] Y. B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008) 1408–1413.
[5] Y. B. Jun, C. S. Kim and K. O. Yang, Cubic sets, Ann. Fuzzy Math. Inform. 4(1) (2012) 83–98.
[6] Y. B. Jun, K. J. Lee, and M. S. Kang, "Cubic structures applied to ideals of BCI-algebras," Computers and Mathematics with Applications,.62(9)(2011)3334–3342,.
[7] Y. B. Jun, K. J. Lee, C. H. Park, Soft set theory applied to ideals in d-algebras, Comput. Math. Appl. 57 (2009) 367–378.
[8] P. K. Maji, A. R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002) 1077–1083.
[9] P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[10] D. Molodtsov, Soft set theory - First results, Comput. Math. Appl. 37 (1999) 19–31.
[11] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Inform. Sci. 8 (1975) 199–249
[12] L. A. Zadeh, "Fuzzy sets," Information and Control, 8(1965) 378–352,
[13] F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophy Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA (2002).
[14] M. B. Gorzałczany, "A method of inference in approximate reasoning based on interval-valued fuzzy sets," Fuzzy Sets and Systems21(1),(1987)1–17.
[15] P.K.Maji, A.R.Roy, and R.Biswas,"Anapplicationofsoftsetsinadecisionmakingproblem,"ComputersandMathematicswith Applications,44,(8-9)(2002)1077–1083.
[16] D. Chen, E. C. C. Tsang, D. S. Yeung, and X. Wang, "The parameterization reduction of soft sets and its applications," Computers and Mathematics with Applications, 49,(5-6)(2005)757–763,.
[17] K.Gong, Z.Xiao, and X.Zhang, "The objective soft set with its operations,"ComputersandMathematicswithApplications,60(8) (2010)2270–2278.
[18] Y. B. Jun, K. J. Lee, and C. H. Park, "Fuzzy soft set theory applied to BCK/BCI-algebras," Computers and Mathematics withApplications,59(2010).3180–3192,.
[19] Y.S. Huang, BCI-algebra, Science Press, Beijing, China, 2006.
[20] Y. Imai and K. Is´eki, On axiom systems of propositional calculi, XIV, Proc. Japan Academy, 42 (1966), 1922.
[21] K. Is´eki, An algebra related with a propositional calculus, Japan Acad., 42 (1966) 26-29.
[22] K. Is´eki, On BCI-algebras, Math. Sem. Notes, 8 (1980), 125-130.
[23] K. Is´eki and S. Tanaka, An introduction to the theory of BCK-algebras, Math. Japonica, 23(1) (1978), 1-26.
[24] J. Meng, Jie and Y. B. Jun, BCK-algebras, Kyung Moon Sa Co., Seoul, 1994.
[25] D. A. Molodtsov, The Theory of Soft Sets, URSS Publishers, Moscow, Russia, 2004(Russian).