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Combined Effects of Soret-Dufour, Hall and Radiation on Unsteady MHD Flow of Dusty Fluid past Infinite Inclined Porous Plate

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ABSTRACT

The purpose of this paper is to present a numerical analysis of an unsteady three dimensional MHD flow of dusty fluid past an infinite inclined porous plate in presence of Soret effect, Dufour effect, radiation effect and Hall effect with variable temperature and concentration embedded in porous medium. At time $t' > 0$ the plate moves with constant velocity u_0 and at the same time, the plate temperature and concentration levels near the plate decreased exponentially with time t'. The governing boundary layer equations of flow problem are transformed into nonlinear partial differential equations using non-dimensional quantities and solved numerically by Crank-Nicolson finite difference method. The obtained results for velocity profiles along x' direction and z' direction, temperature profile and concentration profile are discussed through graphs and physical significance of quantities skin friction, Nusselt number, Sherwood number also discussed through tables. It is found that there are slightly change in velocity profile in x'-axis but measure change in z'-axis.

Introduction

Due to wide use in industry dusty fluids have gained considerable importance in last few years. Fluid flow with effect of magnetic field and heat transfer exists in magnetohydrodynamcis, nuclear accelerator, pumps and generators such type of fluids has applications in nuclear reactors, plasma studies etc. Recovery of crude oil from pores of reservoirs is an application of flow of fluids through porous medium. Saffman[9] analysed equations of motion for binary mixture of fluid and dust particles. Reptis et al[2] studied the effects of thermal radiation on MHD flow. Zueco and Ahmed[3] obtained perturbation solution of heat and mass transfer in ratating vertical channel with Hall current.

Kuamr et. al.[5] have studied on chemical reaction in steady mixed convection MHD viscous flow over shrinking sheet and Dubey et. al. [4] have studied on Effect of the dusty viscous fluid on unsteady free convective flow along a porous hot vertical plate with thermal diffusion and mass transfer solved by perturbation techniques. N. Pandya and A. K. Shukla [6] analysed effects of Soret, Dufour, Hall and radiation on an unsteady MHD flow past an inclined plate with viscous dissipation, chemical reaction and heat absorption & generation. N. Pandya and yadav [7] discussed Soret-Dufour effects on unsteady MHD flow of dusty fluid over inclined porous plate embedded in porous medium. Sharma et al. [9] discussed the effect of radiation on steady free convective flow along a uniform moving porous vertical plate in presence of heat source/sink and transverse magnetic field. Unsteady convective flow of a dusty fluid over rectangular channel was discussed by Dalal et al. [10]. Rana and Bhargava [11] used finite element and finite difference methods for nonlinear stretching sheet problem. Sparrow. And cess.[12]

Studied Effect of magnetic field on free convection heat transfer. Swati et al. [13] discussed the effects of thermal stratification on flow and heat transfer past a porous vertical stretching surface and exponentially stretching sheet embedded in a thermally stratified medium. Samuel and Zachary [14] studied the free convective flow along a heated vertical wall immersed in a thermally stratified environment. Gireesha et al. [15] discussed the MHD heat transfer effects on dusty fluid flow over a stretching sheet. Mohan Krishna et al. [16] analyzed the influence of radiation and chemical reaction on MHD convective flow with suction and heat generation effects. The flow of viscous Ag-water and Cu-water nanofluids past a stretching surface has been analyzed by Vajravelu et al. [17]. Muthucumarswamy [18] studied first order homogeneous chemical reaction on flow past infinite vertical plate. Kandasamy et al. [19] discussed heat and mass transfer effect along a wedge with heat source and concentration in the presence of suction/injection taking into account the chemical reaction of first order.

Gbadeyan et.al [20] Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco -elastic fluid in the presence of magnetic field, Alan and Rahman [21], examined Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction embedded in a porous medium for a hydrogen-air mixture as the nonchemical reacting fluid pair. Kim [22] presented the analysis for the free convection with mass transfer flow of a micropolar fluid through a porous medium bounded by a semi-infinite vertical permeable plate in the presence of a transverse magnetic field. Modather et al. [23] analyzed the convective flow of a micropolar fluid over an infinite moving porous plate in a saturated porous medium in the presence of a transverse magnetic field and obtained the analytical solutions.

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Numerical solution of MHD flow and heat transfer of dusty fluid over a linearly stretching sheet was given by Gireesha et al. [24]. Ramesh et al. [25] considered MHD boundary layer flow of dusty fluid over inclined stretching sheet. Later on, Gireesha et al. [26] examined thermal radiation effects of MHD flow of dusty fluid over exponentially stretching sheet. Rajesh et al.[27] have discussed Radiation and mass transfer effect on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Since the both the nano and dusty fluids are useful to enhance the thermal conductivity, Sandeep and Sulochana [28] studied the MHD flow of dusty nanofluids over a stretching surface. Heat and mass transfer of MHD unsteady Maxwell fluid flow through porous medium past a porous flat plate was analyzed by El-Dabe et. al. [29]. El-Dabe et. al. [30] discussed the dusty non-Newtonian flow between two coaxial circular cylinders. Alam and Rahman [31] studied the Dufour and Soret effects on steady MHD free convective heat and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium.

The objective of this work is to study combined effects of Soret-Dufour, radiation, Hall on unsteady MHD flow of dusty fluid past an inclined porous plate embedded in porous medium. Non-dimensional form of Partial differential equations have been solved by Crank-Nicolson implicit finite difference method. The obtained results for velocity, temperature and concentration are discussed through graphs.

Mathematical Analysis

An unsteady MHD flow of dusty fluid past an inclined porous plate with Soret-Dufour effect, Hall effect and radiation effect are taken. x'-axis is considered along plate, y'-axis is normal to it and z'-axis is perpendicular to $x'y'$ plane.. A uniform magnetic field B_0 is taken along y'-axis and plate is considered non-electric conducting. In beginning plate and fluid are at same temperature T'_∞ and concentration C'_∞. For $t' > 0$, the plate moves with velocity u_0 , its temperature and concentration increase exponentially with time. Magnetic Reynolds number is smaller than transversely applied magnetic field so induced magnetic field is negligible, Cowling [32].

$$J_x' = \frac{\sigma B_0}{1+m^2}(mu' - w') \qquad (1)$$

$$J_z' = \frac{\sigma B_0}{1+m^2}(u' + mw')$$

where J_x' and J_z' are electric current density, u' and w' are velocities along x'-axis and z'-axis respectively, m is Hall parameter.

On account of infinite length in x' direction, flow variables are function of y' and t' only. Under usual Boussinesq approximation, governing equations are given by:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0 (const) \qquad (2)$$

$$\frac{\partial u'}{\partial y'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty)\cos(\alpha) + g\beta^*(C' - C'_\infty)\cos(\alpha) \qquad (3)$$

$$-\frac{\sigma B_0}{1+m^2}(u' + mw') - \frac{\nu u'}{K'} + \frac{KN_0}{\rho}(u'_d - u')$$

$$\frac{\partial w'}{\partial y'} + v'\frac{\partial w'}{\partial y'} = \nu\frac{\partial^2 w'}{\partial y'^2} + \frac{\sigma B_0}{1+m^2}(mu' - w')$$

$$-\frac{\nu w'}{K'} + \frac{KN_0}{\rho}(w'_d - w') \qquad (1)$$

$$m_1\frac{\partial u'_d}{\partial t'} = S_k(u' - u'_d) \qquad (2)$$

$$m_1\frac{\partial w'_d}{\partial t'} = S_k(w' - w'_d) \qquad (3)$$

$$\rho c_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \frac{\rho D_m K_T}{c_s}\frac{\partial^2 C'}{\partial y'^2} \qquad (4)$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D_m\frac{\partial^2 C'}{\partial y'^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \qquad (5)$$

where β is volumetric coefficient of thermal expansion, is β^* coefficient of volume expansion for mass transfer, u_d' and w_d' are the velocity of dust particles along x'-axis and y'-axis respectively, S_k is the stoke's resistance coefficient, v' is velocity along y'-axis, K' is permeability of porous medium, σ is electrical conductivity, D_m is molecular diffusivity, g is acceleration of gravity, K_T is thermal diffusion ratio, μ is viscosity, ρ is fluid density, k is thermal conductivity of fluid, C' and T' are dimensional concentration and temperature, C'_∞ and T'_∞ are concentration and temperature of free stream, N_0 is the number density of the dust particles which is constant, m_1 is the mass of dust particles, c_p is specific heat at constant pressure, q_r is radiative heat along y'-axis, ν is kinematic viscosity and T_m is mean fluid temperature.

Initial and Boundary condition for this problem are given as:

$$
\begin{aligned}
& t' \le 0 \; u' = 0 \quad w' = 0 \quad u'_d = 0 \quad w'_d = 0 \quad T' = T'_\infty \\
& C' = C'_\infty \quad \forall y', \\
& t' \ge 0 \quad u' = u_0 \quad w' = 0 \quad u'_d = u_0 \\
& w'_d = 0 \quad v' = -v_0 \quad T' = T'_\infty + (T'_w - T'_\infty)\mathrm{e}^{-At'} \\
& C' = C'_\infty + (C'_w - C'_\infty)\mathrm{e}^{-At'} \quad at \;\; y' = 0 \\
& u' = 0 \quad w' = 0 \quad u'_d = 0 \quad w'_d = 0 \\
& T' \to T'_\infty, \;\; C' \to C'_\infty, \quad y' \to \infty
\end{aligned} \tag{6}
$$

Where T'_w and C'_w are concentration and temperature respectively of plate and

$$A = \frac{v_0^2}{\nu}.$$

The radiative heat flux term by using the Roseland approximation is given by

$$q_r = -\frac{4\sigma}{3k_m}\frac{\partial T'^4}{\partial y'} \tag{7}$$

where σ and k_m are Stefan Boltzmann constant and mean absorption coefficient respectively. In this problem temperature difference within flow is very small, so that T'^4 may be expressed linearly with temperature. It is observed by expanding in a Taylor's series about T'_∞ and considering negligible higher order term, hence

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{8}$$

so, using equations 10 and 11, equation 7 is reduced

$$
\begin{aligned}
\rho c_p \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T'^3_\infty}{3k_m}\frac{\partial^2 T'}{\partial y'^2} \\
+ \frac{\rho D_m K_T}{c_s}\frac{\partial^2 C'}{\partial y'^2}
\end{aligned} \tag{9}
$$

In next step to obtain non-dimensional form of governing equations, we introduce following quantities:

$$
\begin{aligned}
& u = \frac{u'}{u_0}, \;\; t = \frac{t' v_0^2}{\nu}, \;\; \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \;\; C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\
& Gm = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0 v_0^2}, Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{u_0 v_0^2}, \\
& Du = \frac{D_m K_T (C'_w - C'_\infty)}{c_s c_p \nu (T'_w - T'_\infty)}, \;\; Sr = \frac{D_m K_T (T'_w - T'_\infty)}{T_m \nu (C'_w - C'_\infty)}, \\
& K = \frac{v_0^2 K'}{\nu^2}, \;\; \Pr = \frac{\mu c_p}{k}, \;\; M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \;\; R = \frac{4\sigma T'^3_\infty}{k_m k}, \\
& Sc = \frac{\nu}{D_m}, y = \frac{y' v_0}{\nu}, \;\; w = \frac{w'}{u_0}, \;\; u_d = \frac{u'_d}{u_0}, \\
& w_d = \frac{w'_d}{u_0}, \;\; B = \frac{\nu S_k N_0}{\rho u_0^2}, \;\; B = \frac{m_1 v_0^2}{\nu S_k}.
\end{aligned} \tag{10}
$$

In terms of the above non-dimensional quantities the governing equations read

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+GrCos(\alpha)\theta+GmCos(\alpha)C \tag{11}$$

$$+B_1(u_d-u)-\left(\frac{M}{1+m^2}+\frac{1}{K}\right)u-\left(\frac{mM}{1+m^2}\right)w$$

$$\frac{\partial w}{\partial t}-\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}+B_1(w_d-w) \tag{12}$$

$$-\left(\frac{M}{1+m^2}+\frac{1}{K}\right)w+\left(\frac{mM}{1+m^2}\right)u$$

$$B\frac{\partial u_d}{\partial t}=u-u_d \tag{13}$$

$$B\frac{\partial w_d}{\partial t}=w-w_d \tag{14}$$

$$\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left(1+\frac{4R}{3}\right)\frac{\partial^2\theta}{\partial y^2}+Du\frac{\partial^2 C}{\partial y^2} \tag{15}$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+Sr\frac{\partial^2\theta}{\partial y^2} \tag{16}$$

With the initial and boundary conditions,

$$t\le 0 \quad u=0 \quad w=0 \quad u_d=0 \quad w_d=0 \quad \theta=0 \quad C=0 \quad \forall y \tag{17}$$

$$t\ge 0 \quad u=1 \quad w=0 \quad u_d=1 \quad w_d=0 \quad \theta=e^{-t} \quad C=e^{-t} \quad at\ y=0$$

$$u=0 \quad w=0 \quad u_d=0 \quad w_d=0 \quad \theta\to 0 \quad C\to 0 \quad y\to\infty$$

Now it is important to calculate the physical quantities of primary interest, which are skin-friction coefficients τ_1 and τ_2 along wall $\boldsymbol{x}$-axis and $\boldsymbol{z}$-axis respectively, Nusselt number Nu and Sherwood number Sh. Non-dimensional form of these physical quantities are:

$$\tau_1=\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{18}$$

$$\tau_2=\left(\frac{\partial w}{\partial y}\right)_{y=0}$$

$$Nu=-\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$Sh=-\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Method of solution

The unsteady, nonlinear pair of equations 14-19 with conditions 20 are solved by using Crank-Nicolson implicit finite-difference scheme. Consider a rectangular region with y varying from $\mathbf{0}$ to $\boldsymbol{y_{max}}(=\mathbf{4})$, where $\boldsymbol{y}$ max corresponds to $\boldsymbol{y}=\mathbf{1}$ at which lies well outside the momentum, energy and concentration boundary layers. The region to be examined in $(\boldsymbol{y},\boldsymbol{t})$ space is covered by a rectilinear grid with sides parallel to axes with $\boldsymbol{y}$ and $\boldsymbol{\Delta t}$, the grid spacing in $\boldsymbol{y}$, and $\boldsymbol{t}$ directions, respectively. The grid points $(\boldsymbol{y},\boldsymbol{t})$ are given by $(\boldsymbol{i\Delta y},\boldsymbol{j\Delta t})$. The finite-difference equations corresponding to 14-19 are given by

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}-\frac{u_{i+1,j}-u_{i,j}}{\Delta y}=\left(\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}+u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}}{2(\Delta y)^2}\right)+B_1\left(\left(\frac{(u_d)_{i,j+1}+(u_d)_{i,j}}{2}\right)-\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)\right)$$

$$+Gr\cos(\alpha)\left(\frac{\theta_{i,j+1}+\theta_{i,j}}{2}\right)+Gr\cos(\alpha)\left(\frac{C_{i,j+1}+C_{i,j}}{2}\right)-\left(\frac{M}{1+m^2}+\frac{1}{K}\right)\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)-\left(\frac{mM}{1+m^2}\right)\left(\frac{w_{i,j+1}+w_{i,j}}{2}\right) \tag{19}$$

$$\frac{w_{i,j+1}-w_{i,j}}{\Delta t}-\frac{w_{i+1,j}-w_{i,j}}{\Delta y}= \tag{20}$$
$$\left(\frac{w_{i-1,j}-2w_{i,j}+w_{i+1,j}+w_{i-1,j+1}-2w_{i,j+1}+w_{i+1,j+1}}{2\left(\Delta y\right)^2}\right)$$
$$+B_1\left(\left(\frac{\left(w_d\right)_{i,j+1}+\left(w_d\right)_{i,j}}{2}\right)-\left(\frac{w_{i,j+1}+w_{i,j}}{2}\right)\right)$$
$$-\left(\frac{M}{1+m^2}+\frac{1}{K}\right)\left(\frac{w_{i,j+1}+w_{i,j}}{2}\right)+\left(\frac{mM}{1+m^2}\right)\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)$$

$$B\left(\frac{\left(u_d\right)_{i,j+1}-\left(u_d\right)_{i,j}}{\Delta t}\right)=\left(\left(\frac{u_{i,j+1}+u_{i,j}}{2}\right)-\left(\frac{\left(u_d\right)_{i,j+1}+\left(u_d\right)_{i,j}}{2}\right)\right) \tag{21}$$

$$B\left(\frac{\left(w_d\right)_{i,j+1}-\left(w_d\right)_{i,j}}{\Delta t}\right)=\left(\left(\frac{w_{i,j+1}+w_{i,j}}{2}\right)-\left(\frac{\left(w_d\right)_{i,j+1}+\left(w_d\right)_{i,j}}{2}\right)\right) \tag{22}$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}-\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y}=\frac{1}{\Pr}\left(1+\frac{4R}{3}\right) \tag{23}$$
$$\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}+\theta_{i-1,j+1}-2\theta_{i,j+1}+\theta_{i+1,j+1}}{2\left(\Delta y\right)^2}\right)$$
$$+Du\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}+C_{i-1,j+1}-2C_{i,j+1}+C_{i+1,j+1}}{2\left(\Delta y\right)^2}\right)$$

$$\frac{C_{i,j+1}-C_{i,j}}{\Delta t}-\frac{C_{i+1,j}-C_{i,j}}{\Delta y}= \tag{24}$$
$$\frac{1}{Sc}\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}+C_{i-1,j+1}-2C_{i,j+1}+C_{i+1,j+1}}{2\left(\Delta y\right)^2}\right)$$
$$+Sr\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}+\theta_{i-1,j+1}-2\theta_{i,j+1}+\theta_{i+1,j+1}}{2\left(\Delta y\right)^2}\right)$$

with boundary and initial conditions

$$u_{i,0}=0\quad w_{i,0}=0\quad \left(u_d\right)_{i,0}=0\quad \left(w_d\right)_{i,0}=0\quad \theta_{i,0}=0\quad C_{i,0}=0\quad \forall i \tag{25}$$
$$u_{0,j}=1\quad w_{0,j}=0\quad \left(u_d\right)_{0,j}=1\quad \left(w_d\right)_{0,j}=0\quad \theta_{0,j}=\mathrm{e}^{-j\Delta t}\quad C_{0,j}=\mathrm{e}^{-j\Delta t}$$
$$u_{n,j}=0\quad w_{n,j}=0\quad \left(u_d\right)_{n,j}=0\quad \left(w_d\right)_{n,j}=0\quad \theta_{n,j}\to 0\quad C_{n,j}\to 0$$

where $\boldsymbol{\Delta t = t_{j+1} - t_j}$ and $\boldsymbol{\Delta y = y_{j+1} - y_j}$, index $\boldsymbol{i}$ and $\boldsymbol{j}$ refer to $\boldsymbol{y}$ and time $\boldsymbol{t}$ respectively. After getting values of $\boldsymbol{u}$, $\boldsymbol{u_d}$, $\boldsymbol{w}$, $\boldsymbol{w_d}$, θ and $\boldsymbol{C}$ at $\boldsymbol{t}$, we have solved above equations for values $\boldsymbol{t + \Delta t}$ and process as follows: We obtain these values to substitute $\boldsymbol{i = 1, 2, 3, \ldots\ldots, n-1}$ where *n* refers to ∞ then equations 26 and 27 give tridiagonal system of equations with initial and boundary conditions in equation 28 are solved using Thomos algorithm as discussed in Carnahan et al.[8], we have been found values of θ and $\boldsymbol{C}$ for all values of $\boldsymbol{y}$ at $\boldsymbol{t + \Delta t}$. Equation 21 to 24 are solved by same to substitute these values of θ and $\boldsymbol{C}$, we get solution for $\boldsymbol{u}$, $\boldsymbol{u_d}$, $\boldsymbol{w}$ and $\boldsymbol{w_d}$ till desired time $\boldsymbol{t}$. calculation were execute for $\boldsymbol{\Delta y = 0.1}$, $\boldsymbol{\Delta t = 0.001}$ and repeated till $\boldsymbol{y = 4}$.

Result and Discussion

In order of analysis, we see numerical results for velocity profiles $\boldsymbol{u}$, $\boldsymbol{w}$ along $\boldsymbol{x}$-axis and $\boldsymbol{z}$-axis, temperature profile θ and concentration profile $\boldsymbol{C}$ with help of graphs by assigning numerical values of thermal Grash of number $\boldsymbol{Gr}$, solutal Grashof number $\boldsymbol{Gm}$, Schmidt number $\boldsymbol{Sc}$, Prandtl number $\boldsymbol{Pr}$, Soret number $\boldsymbol{Sr}$, Dufour number $\boldsymbol{Du}$, magnetic parameter $\boldsymbol{M}$, dusty fluid parameter $\boldsymbol{B_1}$, dusty particle parameter $\boldsymbol{B}$, Hall parameter $\boldsymbol{m}$, radiation parameter $\boldsymbol{R}$, inclination angle α .

Velocity profiles $\boldsymbol{u}$ and $\boldsymbol{w}$ in figure 1 and figure 2 decrease as dust fluid parameter increases, in figures 3 and 4 it is seen that velocity profiles *u* and *w* decrease when inclination angle α increases. On increasing thermal Grash of number and solutan Grash of number, velocity profiles $\boldsymbol{u}$ and $\boldsymbol{w}$ increase in figures 7, 8, 5 and 6. It is seen that as an increase of magnetic parameter leads to decrease of both the velocity profile $\boldsymbol{u}$ decreases in figure 9 while velocity profile *w* increases in figure 10.

Figures 11 and 12 represent the effects of Hall parameter on velocity profiles $\boldsymbol{u}$ and $\boldsymbol{w}$. With an increase of Hall parameter both velocities increases. Also as an increase of radiation parameter $\boldsymbol{R}$ leads to an increase on velocity $\boldsymbol{u}$ and temperature profile in figures 17 and 18. Figure 13 and 14 depict that velocity profiles u and w decrease when Schmidt number Sc increases. Velocity profiles u, w and Concentration profile C increase as Soret number increases in figures 15, 16 and 19 respectively while temperature profile θ decreases in figure 26. Temperature profile θ increase slightly in figure 20 as Dufour number Du increases.

From figures 21 and 22 we find that velocity profiles u and w increase as time t increases. Concentration profile C and Temperature profile θ first decrease after some distance from plate increase when time t increases in figures 23 and 24 respectively. On increasing Schmidt number Sc Concentration profile C decreases. It is evident from figure 27 that on increasing radiation parameter R increase concentration profile C first decreases after some distance it increases. Velocity profile u decreases when dust particle parameter B increases.

It is seen from table(1) that B, B_1 and Sc increase then skin friction coefficients τ_1 and τ_2 decrease while Gm, Gr, m, Sr and t increase then skin friction coefficients τ_1 and τ_2 increase and on increasing M and R, τ_1 decreases while τ_2 increases.

On the other hand it is seen from table(2) when Du, Sc and R increase then Nusselt number Nu decreases and Sherwood number Sh increases and on increasing Sr, Nusselt number Nu increases and Sherwood number Sh decreases while on increasing t Nusselt number Nu and Sherwood number Sh decreases.

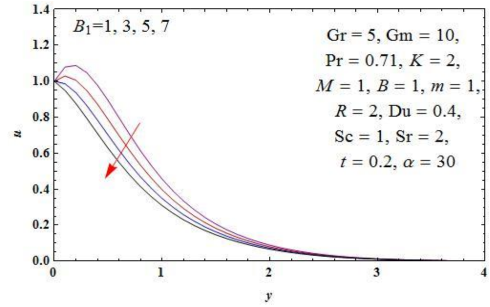

Fig 1. Velocity profile u for different value of B_1.

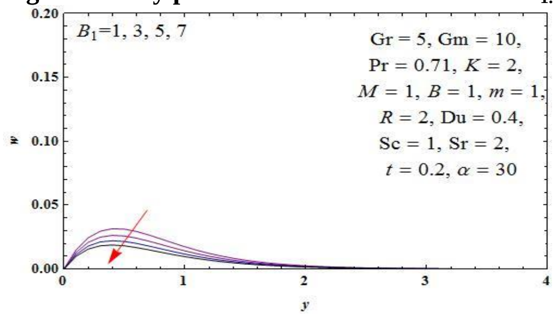

Fig 2. Velocity profile w for different value of B_1.

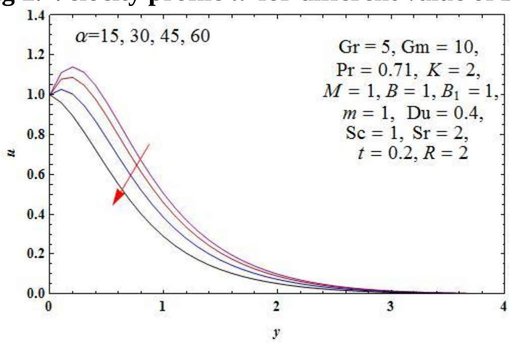

Fig 3. Velocity profile u for different value of α.

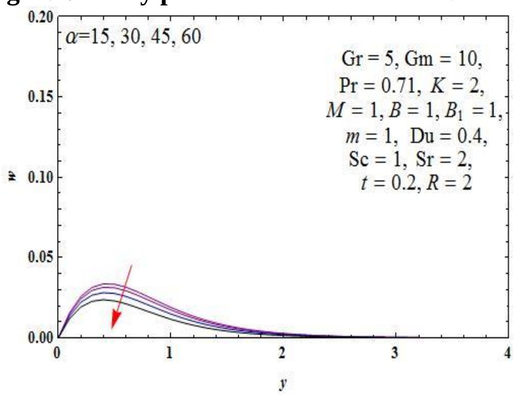

Fig 4. Velocity profile w for different value of α.

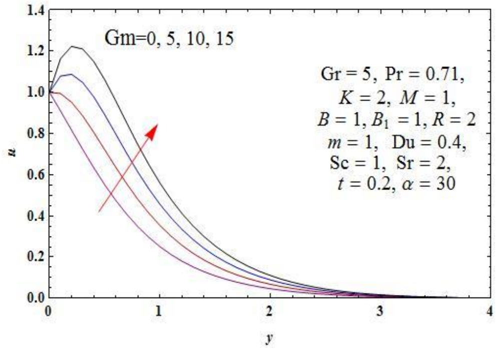

Fig 5. Velocity profile u for different value of Gm.

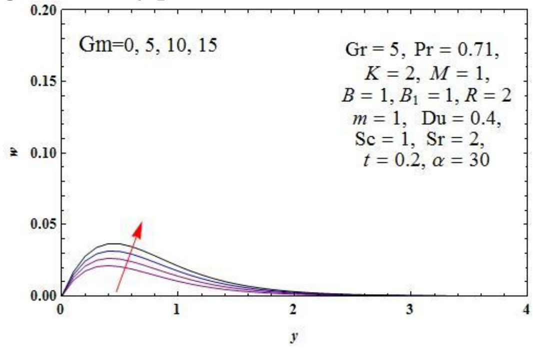

Fig 6. Velocity profile w for different value of Gm.

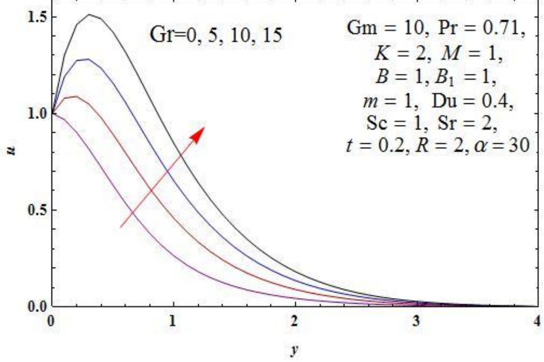

Fig 7. Velocity profile u for different value of Gr.

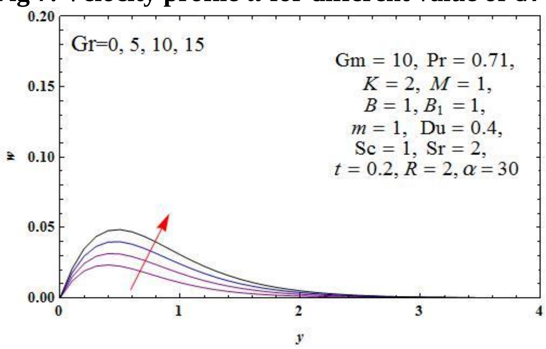

Fig 8. Velocity profile w for different value of Gr.

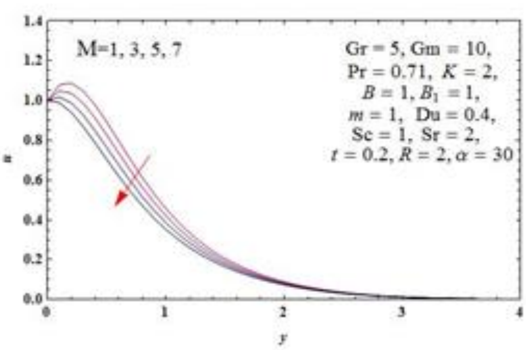

Fig 9. Velocity profile u for different value of M

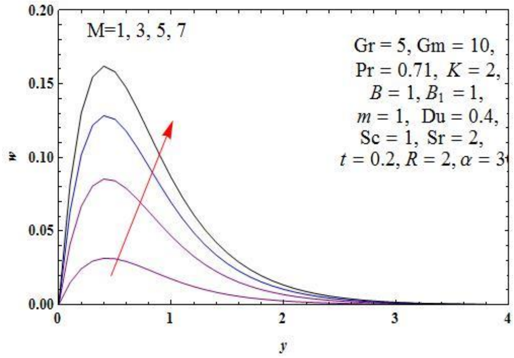

Fig 10. Velocity profile w for different value of M.

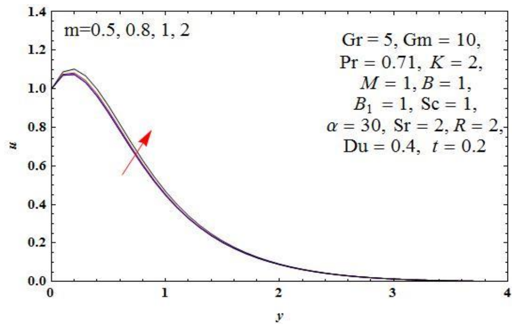

Fig 11. Velocity profile u for different value of m.

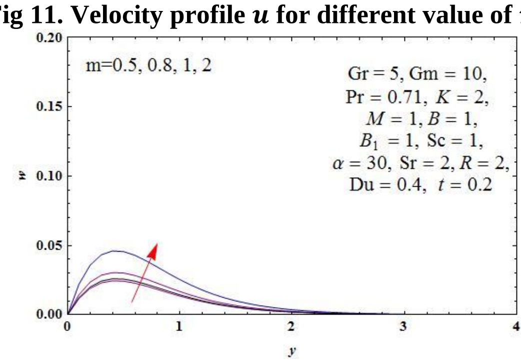

Fig 12. Velocity profile w for different value of m.

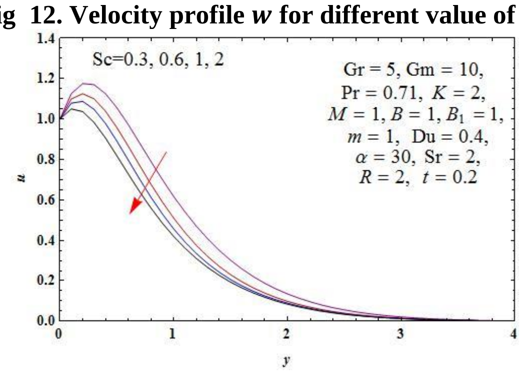

Fig 13. Velocity profile u for different value of Sc.

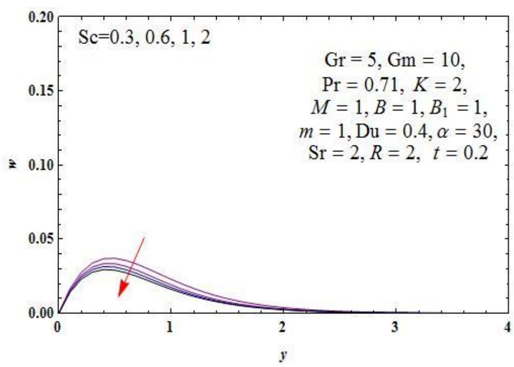

Fig 14. Velocity profile w for different value of Sc.

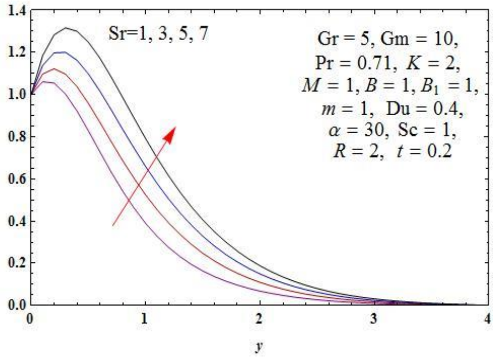

Fig 15. Velocity profile u for different value of Sr.

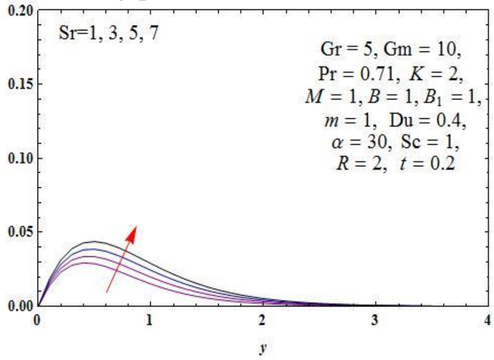

Fig 16. Velocity profile w for different value of Sr.

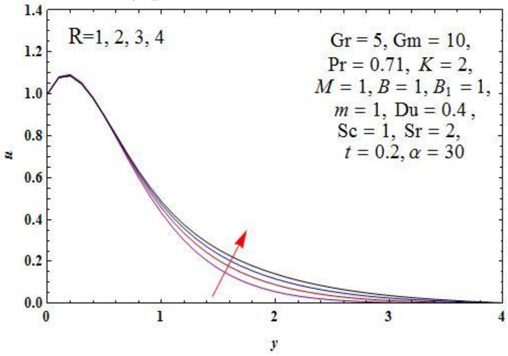

Fig 17. Velocity profile u for different value of R.

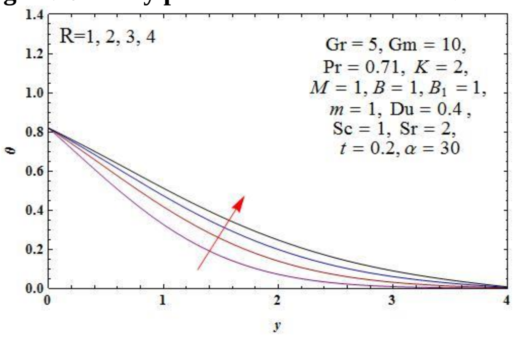

Fig 18. Temperature profile θ for different value of R.

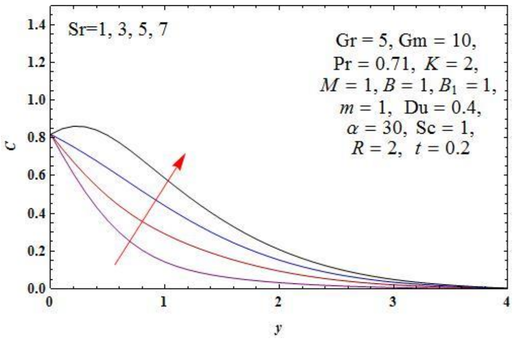

Fig 19. Concentration profile C for different value of Sr.

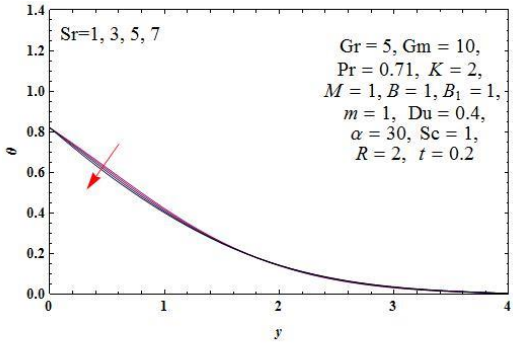

Fig 20. Temperature profile θ for different value of Sr.

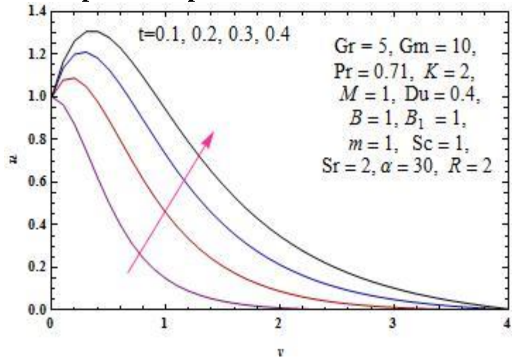

Fig 21. Velocity profile u for different value of t.

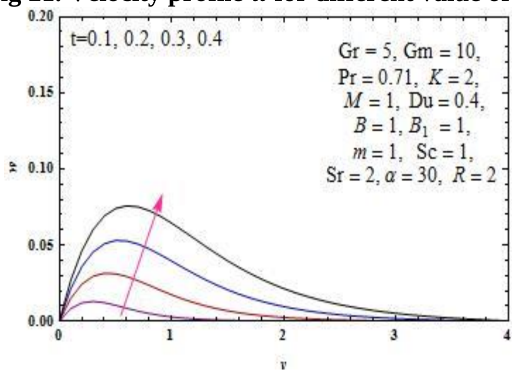

Fig 22. Velocity profile w for different value of t.

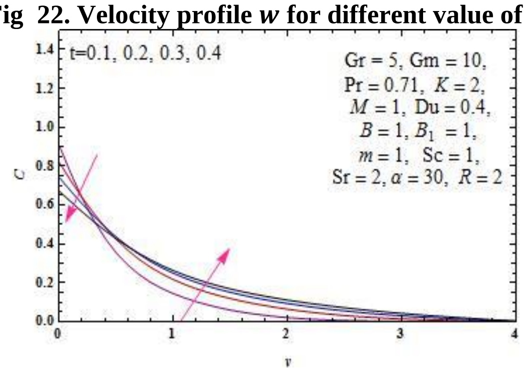

Fig 23. Concentration profile C for different value of t.

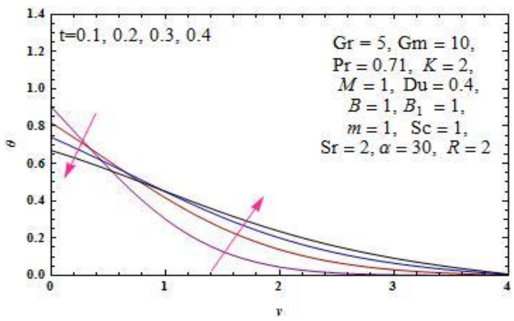

Fig 24. Temperature profile θ for different value of t.

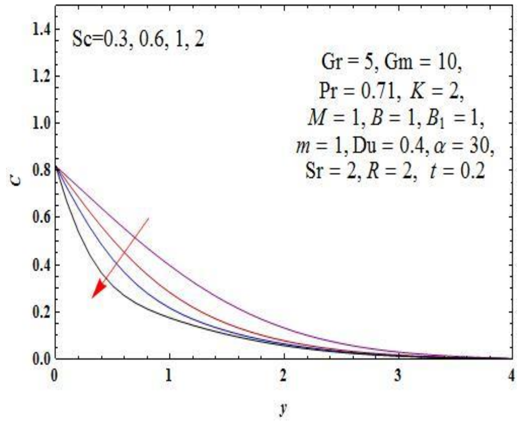

Fig 25. Concentration profile C for different value of Sc.

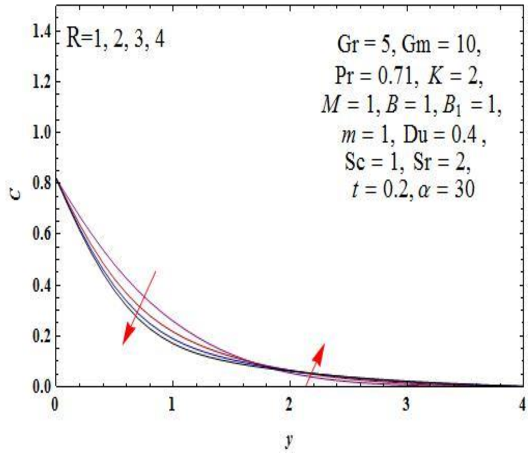

Fig 27. Concentration profile C for different value of R.

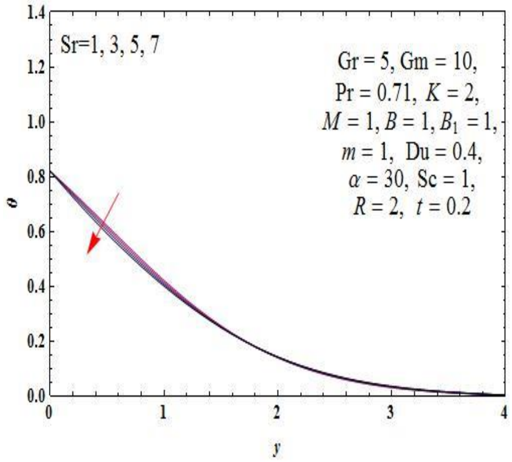

Fig 26. Concentration profile θ for different value of t.

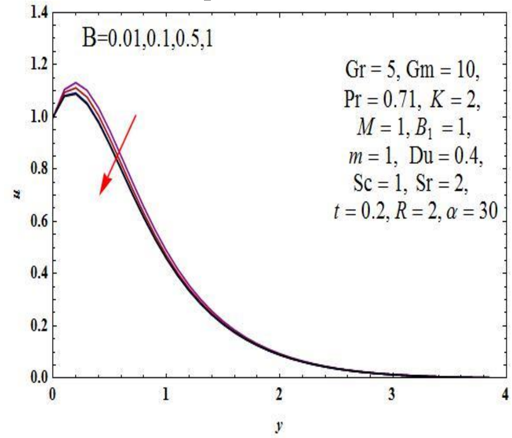

Fig 28. Velocity profile u for different value of B.

Table 1. Skin friction coefficients τ_1 and τ_2 for different values of parameters.

Gr	*Gm*	*B*	*B₁*	*K*	*M*	*m*	*Pr*	*Du*	*R*	*Sc*	*Sr*	*t*	τ_1	τ_2
0	10	1	1	2	1	1	0.71	0.4	2	1	2	0.2	-0.326438	0.11982
10	10	1	1	2	1	1	0.71	0.4	2	1	2	0.2	1.8998	0.178009
15	10	1	1	2	1	1	0.71	0.4	2	1	2	0.2	3.01292	0.207104
5	0	1	1	2	1	1	0.71	0.4	2	1	2	0.2	-0.901021	0.110183
5	5	1	1	2	1	1	0.71	0.4	2	1	2	0.2	-0.0571695	0.129549
5	15	1	1	2	1	1	0.71	0.4	2	1	2	0.2	1.63053	0.168281
5	10	0.01	1	2	1	1	0.71	0.4	2	1	2	0.2	1.05054	0.159558
5	10	0.1	1	2	1	1	0.71	0.4	2	1	2	0.2	0.935203	0.15361
5	10	0.5	1	2	1	1	0.71	0.4	2	1	2	0.2	0.812645	0.149651
5	10	1	3	2	1	1	0.71	0.4	2	1	2	0.2	0.289627	0.128803
5	10	1	5	2	1	1	0.71	0.4	2	1	2	0.2	-0.140398	0.112559
5	10	1	7	2	1	1	0.71	0.4	2	1	2	0.2	-0.516262	0.0993158
5	10	1	1	2	3	1	0.71	0.4	2	1	2	0.2	0.479253	0.411476
5	10	1	1	2	5	1	0.71	0.4	2	1	2	0.2	0.17894	0.631834
5	10	1	1	2	7	1	0.71	0.4	2	1	2	0.2	-0.111044	0.815521
5	10	1	1	2	1	0.5	0.71	0.4	2	1	2	0.2	0.699515	0.116313
5	10	1	1	2	1	0.8	0.71	0.4	2	1	2	0.2	0.754306	0.144007
5	10	1	1	2	1	2	0.71	0.4	2	1	2	0.2	0.878174	0.122083
5	10	1	1	2	1	1	0.71	0.4	1	1	2	0.2	0.816819	0.148499
5	10	1	1	2	1	1	0.71	0.4	3	1	2	0.2	0.779255	0.149537
5	10	1	1	2	1	1	0.71	0.4	4	1	2	0.2	0.779132	0.15017
5	10	1	1	2	1	1	0.71	0.4	2	0.3	2	0.2	1.25908	0.166051
5	10	1	1	2	1	1	0.71	0.4	2	0.6	2	0.2	0.99196	0.155653
5	10	1	1	2	1	1	0.71	0.4	2	2	2	0.2	0.503307	0.141435
5	10	1	1	2	1	1	0.71	0.4	2	1	1	0.2	0.610809	0.14221
5	10	1	1	2	1	1	0.71	0.4	2	1	3	0.2	0.971943	0.155829
5	10	1	1	2	1	1	0.71	0.4	2	1	5	0.2	1.37635	0.170361
5	10	1	1	2	1	1	0.71	0.4	2	1	7	0.2	1.83979	0.186002
5	10	1	1	2	1	1	0.71	0.4	2	1	2	0.1	-0.367509	0.0819344
5	10	1	1	2	1	1	0.71	0.4	2	1	2	0.3	1.35091	0.210005
5	10	1	1	2	1	1	0.71	0.4	2	1	2	0.4	1.6555	0.265282

Table 2. Nusselt number and Sherwood number Nu and Sh respectively for different values of parameters.

Gr	*Gm*	*B*	B_1	*K*	*M*	*m*	*Pr*	*Du*	*R*	*Sc*	*Sr*	*t*	*Nu*	*Sh*
5	10	1	1	2	1	1	0.71	0.05	2	1	2	0.2	0.440289	0.856995
5	10	1	1	2	1	1	0.71	0.2	2	1	2	0.2	0.420855	0.88549
5	10	1	1	2	1	1	0.71	0.6	2	1	2	0.2	0.359525	0.97767
5	10	1	1	2	1	1	0.71	0.4	1	1	2	0.2	0.522893	0.754969
5	10	1	1	2	1	1	0.71	0.4	3	1	2	0.2	0.327144	1.00505
5	10	1	1	2	1	1	0.71	0.4	4	1	2	0.2	0.286627	1.04896
5	10	1	1	2	1	1	0.71	0.4	2	0.3	2	0.2	0.426168	0.443568
5	10	1	1	2	1	1	0.71	0.4	2	0.6	2	0.2	0.41089	0.666976
5	10	1	1	2	1	1	0.71	0.4	2	2	2	0.2	0.346615	1.54392
5	10	1	1	2	1	1	0.71	0.4	2	1	1	0.2	0.381551	1.08201
5	10	1	1	2	1	1	0.71	0.4	2	1	3	0.2	0.404248	0.754054
5	10	1	1	2	1	1	0.71	0.4	2	1	5	0.2	0.434776	0.320997
5	10	1	1	2	1	1	0.71	0.4	2	1	7	0.2	0.478928	-0.293975
5	10	1	1	2	1	1	0.71	0.4	2	1	2	0.1	0.651856	1.41094
5	10	1	1	2	1	1	0.71	0.4	2	1	2	0.3	0.267512	0.688322
5	10	1	1	2	1	1	0.71	0.4	2	1	2	0.4	0.19039	0.53439

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