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Using Galerkin's Method of Finite Element for Predicting Oxygen Deficit in Rivers: A Case Study of Nworie River

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ABSTRACT

Using galerkin's method of finite element for predicting oxygen deficit in rivers is of great importance to water resources engineers. This is because of the trending discharge of untreated effluents in our water bodies which incidentally depreciates the dissolved oxygen needed by both man and aquatic organisms. As a numerical technique, it provides approximate solutions to physical problems with differential equations like the Streeter Phelps equation. In predicting the oxygen deficit along Nworie River in Imo State of Nigeria, a reconnaissance survey of the study area was conducted after which data were collected for physicochemical analysis of water samples; were determined using Galerkins weighted residual method. The study area was along Nworie river at three different locations namely: Amakohia head bridge; Assumpta-Holy Ghost college road; and Umezurike Hospital road. The result of the flow area after analysis gave 2.00, 5.28 and 5.32 m^2 respectively. The velocities of flow of the three locations as well gave 0.467, 0.417 and 0.411 m/s respectively. The average discharge of the river gave 1.362 cubic metre per second. Furthermore, the concentration of dissolved oxygen from the laboratory experimental results were used to compute the oxygen deficit at the discharge point using dissolved – oxygen model (Streeter Phelps equation) by applying Galerkin's weighted residual method approach. The oxygen deficit results at the five discretized nodes at location 1 (Amakohia head bridge) were: 5.728 mg/l at first node, 5.543 mg/l at the 2nd node, 5.356 mg/l at the 3rd node, 5.164 at the 4th node and 4.969 mg/l at the end node. Pearson's correlation coefficient and the measure of good fit using least square method denoted as 'r' value of 0.982 was computed during analysis. This showed that estimated dissolved oxygen using Galerkin's weighted residual (GWR) method result is strong, positively correlated to the measured values of dissolved oxygen. The results in this research are invaluable for water resources and irrigation Engineers in predicting the effect of water pollution on the downstream users.

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1.0 Introduction

Over the past decades, discharge of wastewater into the environment has brought about an important need to monitor the it. Also, environmental factors such as climate change, gradual decrease in water resources, and threatened habitats, both terrestrial and aquatic, are driving the need to monitor the environment and implement better policies to protect it. In Imo State, Owerri and its environs, water is an essential raw material for human life and a vital resource for industrial growth (Akagha, 2010; Umeham, 2014). Freshwater is beneficial to humans for drinking, irrigation, industrial uses, production of fish, and for such in-stream uses as recreation, transportation and waste disposal (Olatunji et al., 2012).

The area of study lies between latitudes 5˚ 25′–5˚ 30′N and longitudes 7˚ 00′–7˚ 05′E. It covers an area of 113.5 square kilometers (Nwagbara, et al., 2013) In its natural environment, it is characterized by impurities which reduce both oxygen assimilation by living organisms and the dissolution in the river. The quality of surface water at any point from the upstream to downstream is a primary consideration affecting its utilization. Studies have shown the need for effective water quality analysis to determine the dissolved oxygen value of rivers for users.

Oxygen deficit is a great problem to water users. This occurs in polluted water as a result of the activities of man in his everyday life. This indiscriminate use of water usually results in negative side effects such as eutrophication, increased incidence of water -borne diseases, organic and traces metal pollution, changes in species composition of aquatic biota and ultimately destruction of ecosystem integrity (Okoronkwo et al., 2013). Furthermore, industrial effluents like toxic chemicals and heavy metals pollute several surface waters. These metals also gain access into ecosystem through anthropogenic source and get distributed in the water body, suspended solids and sediments during the course of their mobility (Eneji et al., 2011). This pollution reduces the oxygen content of rivers. Oxygen is important to maintain an aerobic state of rivers as the end products of chemical and biochemical reactions in anaerobic systems produce aesthetically displeasing odours, colours and taste.

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When biodegradable organics are discharged into a stream, microorganisms convert the organics into new cells and oxidized waste products. During this process, dissolved oxygen (DO) is consumed.

According to Nwogazie (2008), the weighted residual methods (WRM), which include the orthogonal collocation, Bubnov – Galerkin, Subdomain, and Least-squares, are employed to deal directly with the governing equations of the physical problems. The weighted residuals in general utilize the concept of orthogonal projections of a residual of a differential equation onto a subspace spanned by certain weighing function. Stated differently, the unknown solution in all the WRM is approximated by a set of local basis functions containing adjustable constants or functions (Ames, 1977). These functions or constants are chosen by various criteria to give the "best" approximation for the selected family. For instance, the least – squares method requires higher order interpolation functions in general, even if the physical behaviour may be adequately described by lower order (linear) functions (Chung, 1978; Nwogazie, 2008). This restriction limits its use. The collocation is the simplest (WRM) to apply, but it has a drawback in terms of the number of nodes needed to achieve the same results as with the Galerkin method.

The most special interest in all the WRM is the Bubnov – Galerkin method (1915). The method (often referred to as Galerkin without Bubnov) is the most popular and widely used. Large numbers of non – linear fluid flow systems are easily transformed into "finite element equations" directly. The classical procedures of the Galerkin assume the weighing function and the trial function to be identical (Zienkiewicz, 1977). Like the variational principle, the Galerkin always yields a sysmmetric matrix equation for linear differential operators.

The Galerkin's weighted residual method is the basis of element derivation equations. The method requires that errors or residual between the approximate solution and the true solution be orthogonal to the functions used in the approximation. The principle is expressed mathematically as:

$$\int_{R} \mathbf{N}_{\boldsymbol{\beta}}\, \mathbf{L}\,(\emptyset)\mathbf{dR} = \mathbf{0} \qquad \boldsymbol{\beta} = \mathbf{i}, \mathbf{j}, \mathbf{k} \ldots\ldots \tag{1.0}$$

where,

$\mathbf{N}_{\boldsymbol{\beta}} =$ Shape function or approximation function;

$\emptyset = \textbf{unknown parameter and is approximated}$

$\emptyset = [\mathbf{N_i}, \mathbf{N_j}, \mathbf{N_k}, \ldots][\emptyset_{\mathbf{p}}]$;

L ($\emptyset$) = diffential equation governing $\emptyset$; **and**

R = region of interest

Equation (1.0) implies that the approximation function $\mathbf{N}_{\boldsymbol{\beta}}$ must be orthogonal to the residual between the approximate solution and the true solution over the region R. The residual in Equation (1.0) is taken as L ($\emptyset$).

The Equation (1.0) which is an expression of Galerkin's weighted residual method, can also be expressed as:

$$\int_{\mathbf{R}} (\textbf{Basic function})^{\mathbf{T}} (\textbf{Residual})\ \mathbf{dR} = \mathbf{0} \tag{2.0}$$

where,

R = region of interest or domain,

dR= derivative of R and

T = transpose (in matrix sense).

For one dimensional model, Equation ($\mathbf{2.0}$) becomes:

$$\int_{\mathbf{R}} \mathbf{N}_{\mathbf{k}}^{\mathbf{e}^{\mathbf{T}}} \left(\frac{\mathbf{d^2\phi}}{\mathbf{dx^2}} + \mathbf{F}\right) \mathbf{dx} = \mathbf{0} \tag{3.0}$$

where $\mathbf{N}_{\mathbf{k}}^{\mathbf{e}^{\mathbf{T}}} = \textbf{transpose of the weighing function (same as the transpose of the basis function for element e and node, k)}$

F = constant

ϕ = field variable

e = element number;

and $\mathbf{T} =$ transpose. (**Uzoigwe and Udeorji, 2016**)

To understand the effect of effluents discharged into Nworie River and make predictions about the extent to which this pollutant would travel along the stream, there is need to study the effluent discharge into the river. However, determination of physicochemical parameters (e.g. DO) of water sample at point of discharge had been a regular method of water analyses as the case with the method used byEneji et al. *(*2011) and Okoronkwo and Nwachukwu (2014). Although, this method gives the result of the parameter at that point but does not give a Hydrologist or water Engineer a better result of the effect of effluent discharge at some distance along the reach and downstream of the river except for regular collection of samples for analyses were performed. To determine or predict oxygen deficit along Nworie River would then require enormous time, energy and scarce resources getting samples at different points along the reach of the river as well as analysing the samples in the laboratory. At other times, it can be risky transporting on rivers with high currents or along their banks. These challenges demand for a better method of determining the oxygen deficit of water samples at discharge point, as well as predicting the quality along the reach of the river within some distance from the upstream to the downstream.

Another important challenge posed by previous studies on Nworie River is the fact that records or data on the flow rate (discharge) and velocity of flow of the river are not abundant. There is need to have abundant records to know the rate at which river flows. This is vital in determining or predicting the effect of effluents discharge downstream of a river.

Also, a popular method of determining the rate of transport of contaminant has long been applied in water recourses, which include the analytical method. However, this method is very rigorous and the solution is too hard to come by, an easier and precise method of solving problems relating to prediction of rate of transportation of contaminant has been presented, which is the Numerical method particularly the finite element method using Galerkins Weighted Residual Method. However, the main objective of this study is to predict oxygen deficit along Nworie River in Owerri, Imo State south eastern Nigeria, using Galerkins weighted residual method of finite element.

2.0 Materials and Methods

2.1 Materials

The materials used during the course of this study include collected water samples from Nworie River, a cooler filled with ice blocks for chilling of samples, and dye solution (sodium aluminum sulfosilicate). Several apparatuses were also involved in the sample collection and laboratory tests. These apparatuses include:

(i)Six (6) plastic containers with label,

(ii)Two pairs of staff, distance measuring tape and measuring rod,

(iii)Stop watch,

(iv)The laboratory Equipment: (pH meter, Suntex model TS-2, Multiparameter Bench Photometers HANNA Instruments (HI 83200), Comparator Instrument, Turbidity meter and handheld conductivity meter model HI98302 (HANNA), Beaker, Measuring cylinder, Round bottom flask, Hand glove, pipette, Wash bottle 0.45µm membrane and thermometer (v)Mobile Polaris Navigation

2.1.1 Collection of water Samples

The collected water samples from the three locations when observed appeared turbid or milky in coloration. Its temperature was between 29-30 ˚ C and a slightly irritating smell.

2.2 Methods

Three sample stations were established after a reconnaissance survey of the river as show in Plate 1

Plate 1. Locations of the three sites along Nworie Stream where samples were collected.

(i)Source: Mobile Polaris Navigation

The stations selected are:

(ii)Station1 was Egbeada Federal Housing New Road Bridge Head.

(iii)Station 2 was Ware House/Asumpta Avenue Road Head Bridge Owerri.

(iv)Station 3 was Umezurike Hospital Road Head Bridge Owerri.

2.2.1 Discharge of Dye solution (sodium aluminum sulfosilicate).

Dye solution of sodium aluminium sulfosilicate was discharged at a point from the upstream along the bridge head at Egbeada sample location. Then at 2.4m interval, five samples were collected for laboratory analysis. This was to demonstrate the transport of effluent and determine the oxygen deficit along the river.

2.2.2 Water collected after discharge of Dye solution (Sodium Aluminum Sulfosilicate) at Egbeada Federal Housing New Road Bridge Head.

The collected water sample as dye solution which was discharged into the river appeared to be bluish. As the effluent flows and mixes with the river, the bluish coloration diminishes until a point is reached where it becomes unnoticed.

Samples of water and sediments were collected in the early hours of the day into five 10 litre containers from these stations during the months of May to June 2016. These samples were labelled, packed inside an ice cooled container and transported to Federal University of Technology Owerri (FUTO) laboratory for analysis. The standard methods for the examination of water and wastewater in accordance with the American Public Health Association (APHA, 1992), were used for all the physicochemical analysis.

The methods used in achieving the specific objectives of this study are as follow:

(i)Water quality analysis of samples from Nworie River.

(ii)flow area, velocity of flow and discharge of the river

(iii) Determining the transport of contaminant (oxygen deficit along Nworie River) using Galerkins weighted residual method.

(iv)Determine the Streeter Phelp's constants for Nworie River.

The Dissolved-Oxygen Model of the Nworie Stream is given as proposed by Streeter and Phelps in 1925. The model predicts changes in the deficit, D as a function of Biological Oxygen Demand (BOD) exertion and stream reparation. The rate of change in the deficit is given as

$$\frac{dD}{dt} = K_1 L_t - K_2 D \qquad (4.0)$$

Or

$$\frac{dD}{dt} + K_2 D - K_1 L_t = 0, \; D(0) = D_0 \qquad (5.0)$$

The application of the Galerkin's weighted residual finite element method to Equation (2.2) yields:

$$\int_0^L N^T \left(\frac{dD}{dt} + K_2 D - K_1 L_t\right) dt = 0 \qquad (6.0)$$

Or

$$\int_0^L N^T \left(\frac{dD}{dt}\right) dt + \int_0^L N^T (K_2 D)\, dt - \int_0^L N^T (K_1 L_t)\, dt = 0 \qquad (7.0)$$

Where the upper limit of the integration, L stands for length of an element and $N^T$is the transpose of the basis function.

3.0 Result Presentation

The results of the tests conducted and the numerical studies were presented in Figures 1 to 9.

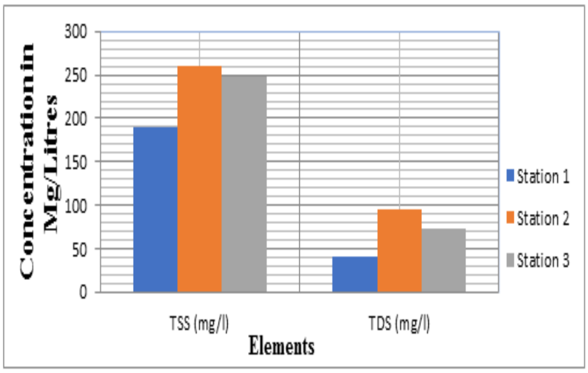

Figure 1. Total suspended solid (TSS) and total dissolved solid (TDS) in Nworie River.

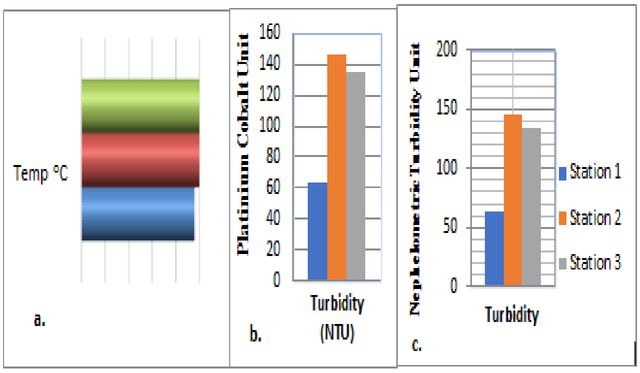

Figure 2a, b and c. Temperature, Colour of Water and Turbidity of Nworie River.

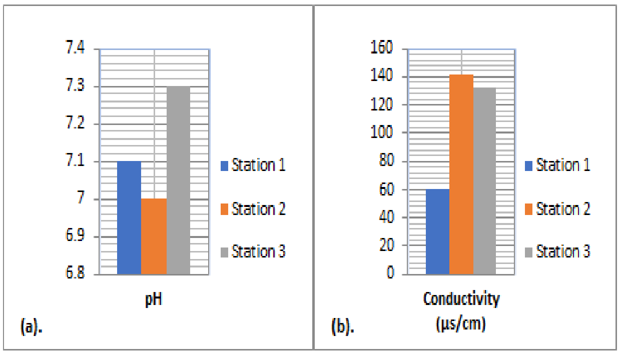

Figure 3a & b. Chemical Characteristics of Nworie River pH and Conductivity.

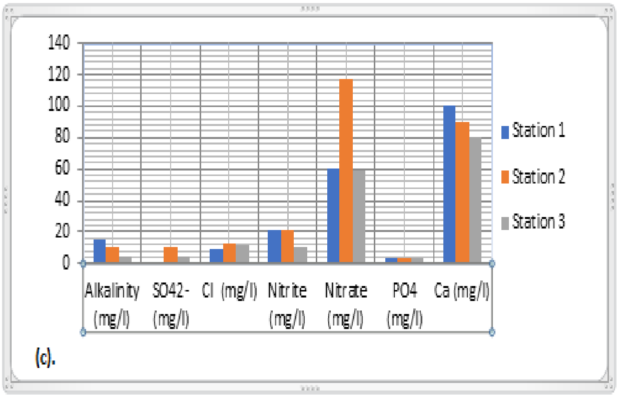

Figure 3c. Chemical Characteristics of Nworie River.

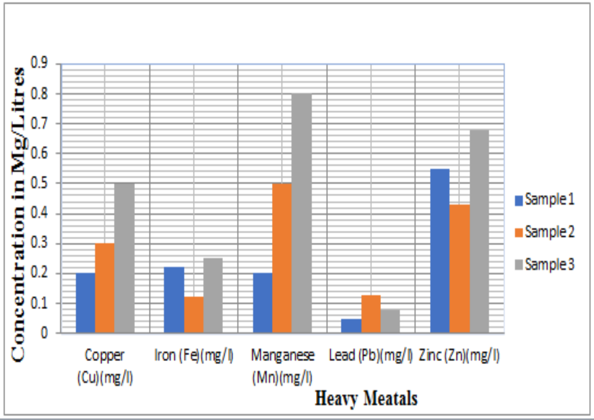

Figure 4. Heavy metals characteristics of the three sampling points along Nworie River.

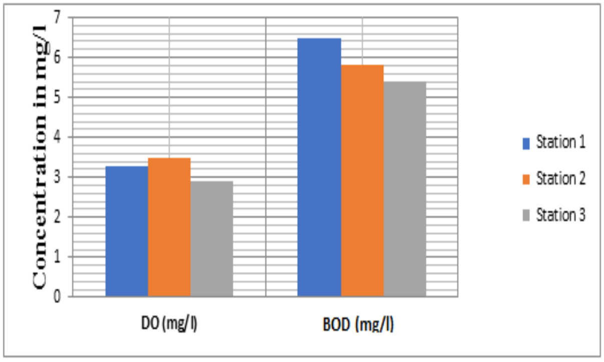

Figure 5. Dissolved Oxygen and Biochemical Oxygen Demand of Nworie River.

3.1 Water contaminants in Nworie River

Figures 1 to 5 depicted the values of the physicochemical parameters in Nworie River. At the end of the laboratory analysis the physicochemical characteristics of the river as presented in Figure 1 and 2 indicate the follow

a) The Temperature in degree Celsius from the three sample stations of Nworie River.

b) The Colour of water (Platinum Cobalt Unit) from the three sample stations of Nworie River.

c) The Turbidity (Nephelometric Turbidity Unit) from the three sample stations of Nworie River.

Also, the chemical characteristics of the river were plotted on bar chart as shown in Figures 3 a, b, and c.

a) The pH of water from the three sample stations of Nworie River.

b) The Conductivity (µs/cm) of water from the three sample stations of Nworie River.

c) The bar chart showing alkalinity, sulphate, nitrite, nitrate, phosphate and calcium (all in mg/l) respectively.

The heavy metals characteristics of the river were also analyzed and presented on a bar chart shown in Figure 4.

The other characteristics of the river that were also analyzed include the dissolved oxygen (DO) and biochemical oxygen demand of the river. The values were also plotted and shown on the bar chart in Figure 5.

3.1.2 Dissolved Oxygen of sample collected after discharge of dye solution (Sodium Aluminum Sulfosilicate) at Egbeada Federal Housing New Road Bridge Head (Station one).

The dissolved oxygen (DO) results from point 1 to point 5 along the river, after application of dye solution as pollutant were 3.30 mg/l, 3.50 mg/l, 3.55 mg/l, 3.65 mg/l and 3.80 mg/l respectively.

3.1.3 Presentation of results of flow area, velocity and discharge of the river

The flow area, velocity and discharge of the river were calculated and the results in section 3.1.3.1 and 3.1.3.2

3.1.3.1 Results of flow area

The flow area of the river at location 1, 2 and 3 are all shown in Figures 7, 8 and 9 respectively in Appendix C. Their values were 2.0, 5.28 and 5.32 square meters respectively.

3.1.3.2 Velocity and flow rate results of Nworie River

The velocity of flow of the river at the three sample locations are shown in Table 3.1

Average flow rate $= \frac{(Q_1 + Q_2 + Q_3)}{3} = \frac{0.794 + 1.652 + 1.640)}{3}$

$$= \frac{4.086}{3} = 1.362 \text{ m}^3/\text{s}$$

3.1.4 Presentation of results of the model from Galerkins weighted residual method

Evaluating the individual terms of Equation 7, using linear interpolation (basis) function gave the result of the oxygen deficit (D), Dissolved oxygen and BOD of the model using Galerkins weighted residual method as presented in Table 2

3.1.5 Validation of the Galerkin's Weighted Residual (GWR) Model

The model was validated using pearson's correlation coefficient and by finding the measure of the goodness of fit using the "method of least square correlation respectively.

3.1.5.1 Result of Pearson's Correlation Coefficient of the Relationship between Laboratory Dissolved Oxygen and the Estimated Dissolved Oxygen using GWR method.

The result of Pearson's correlation coefficient of the relationship between laboratory dissolved oxygen and the estimated dissolved oxygen using GWR method was presented in Table 3

$$\mathbf{r} = \frac{5\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[N\Sigma x^2 - (\Sigma x)^2][N\Sigma y^2 - (\Sigma y)^2]}} \qquad (8.0)$$

$$r = \frac{5*65.651-(18.38)(17.800)}{\sqrt{[5*63.505-(17.800)^2][5*67.925-(18.380)^2]}}$$

$$\mathbf{r = 0.9822}$$

3.1.5.2 Measure of the Goodness of Fit of the Estimated Do and Observed Do using least square method

The relationship between the estimated DO and observed DO was plotted on a line graph as shown in Figure 6.

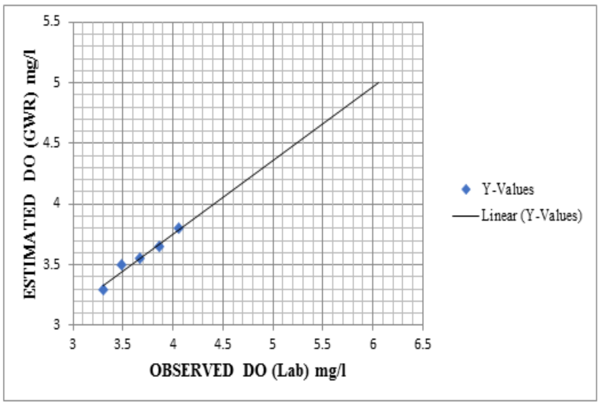

Figure 6. Graph of Estimated DO against Observed DO.

From Table 3, the relationship between the two variables was determined using the "method of least square".

$$\boldsymbol{y = a_0 + a_1 x} \quad (9.0)$$

$$\mathbf{DO(GWR) = a_0 + a_1 DO(Lab)}$$
(10.0)

$$\boldsymbol{a_1 = \frac{n\sum xy - \sum x \sum y}{\sum x^2 - (\sum x)^2}} = -0.0043$$

$$\boldsymbol{a_0 = \frac{\sum y - a_1 \sum x}{n}} = 3.691308$$

$$\boldsymbol{DO(GWR) = 3.691308 - 0.0043 DO(Lab)}$$
(11.0)

$$\boldsymbol{r^2 = \frac{a_0\sum y + a_1 \sum xy - (\sum y)^2/n}{\sum y^2 - (\sum y)^2/n}} = 0.9647$$

∴ r = 0.982

3.2 Discussion

3.2.1. Physicochemical characteristic of Nworie River

From Figure 2a, the in-situ average temperature of the three locations gave 24.67^{o} C, with the individual temperature as 24.0^{0}C, 25.0^{0}C and 25.0^{0}C respectively. Thus, the temperature of the river was acceptable for temperature of freshwater habitat, because it was within the ambient temperature. Also, from the physical analysis of the samples, the mean color of water gave 568.67 PCU or HU (platinum cobalt units or Hazen Units- Hazen units are defined in terms of a platinum cobalt standard APHA Method 2120B (1992)). However, from Figure 2b, the individual sample points gave 431, 640, and 635 PCU/HU respectively. Comparing the sample results with the WHO set standard of 15 PCU/HU, an apparent color was observed resulting from the combined effect of true color and particulate matter or organic matters in the river.

Turbidity of the river gave 63.5, 146.30 and 134.70 all in NTU respectively. This result was in consonant with similar work of Umunnakwe et al. (2015), in which their average Turbidity gave 112.2 NTU. It was also observed that the three sample results exceeded the WHO set standard of 50 NTU. These observations could be as a result of the dredging and sand mining activities taking place within the river and possibly from surface run-offs from communities, thus, showing that the river is filled with particles or suspended solids which may include organic materials.

Total suspended solids (TSS) are the main causes of turbidity. Therefore, from the result presented in Figure 1, the samples from the three locations gave 190.5 mg/l, 260.0 mg/l and 248 mg/l respectively. Comparing mean result (232.83 mg/l) with the recorded value of 263.33 mg/l from the research of Umunnakwe et al., (2015) and the WHO set standard, showed a similar phenomenon in the river. They both exceeded the WHO margin and hence contain particles in water larger than 2 microns. This is an indication of increased sediment load which reduces the depth of the river thus endangering those aquatic species requiring specific depth for survival. Besides, the loaded rivers are murky or clouded and thus reduces light penetration. This phenomenon is attributed to inefficient waste management in Owerri (Acholonu, 2008)

The chemical characteristic of the river was also presented and plotted on a bar chart in Figure 3a, b and c respectively. The result showed the pH, conductivity, alkalinity, sulphate chlorine nitrate, nitrite and phosphate ions present in the river. The measure of hydrogen ion concentration (pH) gave an average of 7.1, while the three respective samples stations gave 7.1, 7.0 and 7.3. The conductivity of the river which measures the water's ability to conduct electricity gave a mean value of 111.1µs/cm (micro-siemen per cubic meter). It was observed that sample location 2 (Asumpta/Control head bridge) had the highest conductivity of 141.0 $\boldsymbol{\mu s/cm}$, followed by location 3 with 132.50 $\boldsymbol{\mu s/cm}$. Egbeada sample 60. $\boldsymbol{\mu s/cm}$, appeared to be below the WHO set standard of 100 $\boldsymbol{\mu s/cm}$. Nwagbara et al., (2013) had a slight lower mean result of 103.5 $\boldsymbol{\mu s/cm}$in their research on hydro-geochemical analysis of water samples from Nworie River, Owerri South-eastern Nigeria. However, Umunnakwe et al., (2011) also recorded a mean value of 113. $\boldsymbol{\mu s/cm}$ in their work on "preliminary assessment of some physicochemical parameters during dredging of Nworie River, Owerri". These results, similarly were observed to be relatively slightly higher than the acceptable limits. Furthermore, the significant high conductivity value observed at sample location 2 and 3 was an indicator that a discharge or some other sources of pollution had entered the water at these locations. Also, the higher values of conductivity may be due to increased ionic compounds. (e.g. Ca, Mg, Na, and Cl) (Waziri, et al., *(*2012)).

The alkalinity mean value was recorded to be 10.0 mg/l, the sample at Egbeada was 15.0 mg/l, decreasing downstream to 10.0 mg/l at Assumpta control head bridge (sample location 2) and finally 5.0mg/l at Umezurike Hospital head bridge (sample location 3). However, comparing these results with the WHO set standard of 200 mg/l, showed that the values were within safe limits for aquatic biodiversity survival. A slight difference was however reported by Umunnakwe et al., (2011 and 2015) in their works, in which their mean values were 24.933 mg/l and 12.57 mg/l respectively.

The sulphate result from Figure 3c showed low concentrations at the three sample locations when compared with the WHO set standard of 250 mg/l. However, it was observed that sample location 2 had the highest concentration (10 mg/l) in relation to the three locations. This could be as a result of certain municipal effluent discharge within the location; although it does not pose a threat to the aquatic organisms. Increased effluent discharge has resulted to a high concentration of nitrate compounds in the river. This was notable in the result of the three sample locations that is, 265.8 mg/l, 520.30 mg/l and 259.16 mg/l respectively. The WHO set standard of 40 mg/l was exceeded at all the point of collection. The chlorine ion in the river from the result showed location 2 with highest concentration of 13.2 mg/l, followed by location 3 with 11.5 mg/l and location 1 being the least of 9.0 mg/l. The WHO set standard of 200 was not exceeded at all the point of collection. The discharge of untreated inorganic waste into the river at location 2 brought about the increase in the level of chlorine concentration. Thus, a state-wide chlorine policy is needed to promote consistency and improve clarity for dischargers and Water board permit writers (TRC and TCO 2005).

In the aquatic environment, the most common ionic (reactive) forms of inorganic nitrogen are ammonium, (nitrite (NO_2) and nitrate (NO_3) (Julio, et al; 2005). This statement was justified from the result of nitrate (NO_3^-) concentration in the three sample locations, which gave 60 mg/l, 117.5 mg/l, and 58.5 mg/l respectively. However, the mean concentration gave 78.67mg/l which exceeded the WHO set standard of 40mg/l. Although, the three locations indicated pollution of nitrate, however, the effect of high concentration of nitrate was felt in sample location 2, where euthrophication was felt as a result of nitrogen from nitrate ion. Thus, as plants take up these nutrients this brought about a decrease at the downstream at location 3 (58.5 mg/l). This was as a result of agricultural runoff, refuse dump runoff or contamination with human or animal wastes (WHO 2011). Nitrite concentration was also evident from the result in Figure 3c where the three samples gave 21mg/l, 21mg/l and 10mg/l respectively. However, there was no pollution of nitrite owing to the fact that the three sample concentrations were below the WHO set standard for nitrites in fresh water.

The result in Figure 3c also showed the concentration of phosphate ion of the three sample locations in Nworie river having 3.30 mg/l, 3.34mg/l and 3.27 mg/l respectively. Relating the result with other research by Umunnakwe et al. (2015), showed it was significantly high when compared with the mean value of 0.163 mg/l. Furthermore, it was also observed to be slightly higher when compared with their previous work (Umunnakwe et al., (2011)) which gave a mean value of 2.427 mg/l. This indicates that the river was prone to soil erosion, as that is a major contributor of phosphorus to rivers. Phosphorus tends to attach to soil particles and, thus, moves into surface water bodies from runoff (USGS, (2016)). The effect of this was the eutrophication that was observed at some part on the surface of the river.

Calcium concentration was observed from the result presented in Figure 3c, with its concentration decreasing along the reach of the river from 100mg/l in station 1, 90mg/l and 80 mg/l respectively. The three samples were observed to be above the safe set limit standard by WHO of 70 mg/l. This phenomenon could be as a result of runoffs, washing calcium from areas where roads and bridges are under construction into the river (Lehigh, (2011))

3.2.2 Heavy Metal Concentration in Nworie River

The third sub-group or subdivision of the physicochemical analysis result is the heavy metal concentration. These were further presented on a bar chart in Figure 4

From the experimental result, it was observed that Zn^{2+} ion had the highest mean concentration of 0.553 mg/l of heavy metals. Considering the concentrations from the upstream flow at location 1 to location 3, it gave 0.55mg/l, 0.48 mg/l and 0.68 respectively. However, none of these three points exceeded the WHO set standard of 5 mg/l.

The next heavy metal with a higher mean concentration after that of zinc was manganese ion with 0.500 mg/l, having their individual sample concentration at the three locations as 0.2 mg/l, 0.5mg/l and 0.8mg/l respectively. It was observed that the third sample at Umezurike Hospital Head Bridge had the highest concentration of Manganese ion. This was due to the anthropogenic activities occurring around the environs of the sample location, where municipal wastes as well as hospital waste are discharged as effluent into the river. However, comparing the WHO set standard of 0.5mg/l with the three sample point results, it was observed that sample at location 3 exceeded with an increase of 0.3mg/l concentration. Although sample at location 2 did not exceed the WHO set standard, being at the threshold point shows that necessary action must be taken in order for it not to exceed the limit that would hamper the lives of species of organisms in the river. An overall guidance value for protection of 95% of species with 50% confidence was derived at 0.2 mg/l for soft waters for the freshwater environment (IPC 2004) The third most abundant heavy metal concentration resulted from the water analysis was copper ion (Cu^{2+}) with a mean concentration of 0.333 mg/l, while having 0.333 mg/l, while having its upstream concentration of 0.2mg/l at sample point 1, mid-stream concentration of 0.3 mg/l, and a downstream of 0.5 mg/l at point 3. The WHO set standard for (Cu^{2+}) was 1.0mg/l, which was not exceeded by any of the samples from the river. Thus, the observed result is acceptable as copper is an essential nutrient at low concentration, but toxic to aquatic organisms at higher concentrations.

Iron, usually (Fe^{3+}) is a common constituent of river water. From the result, it was observed to be the fourth abundant heavy metal, with mean concentration resulting to 0.197 mg/l. taking a look at the three sample points, their individual concentrations were 0.22, 0.12 and 0.25 mg/l respectively. The three points were observed to be lower than the WHO set standard of 0.3 mg/l which indicates that Iron readings were safely and non- deleterious to most organisms and would not cause any damage in the river system within margin.

The least heavy metal abundant in Nworie river was Lead (Pb). The chart showed the mean concentration to be 0.083 mg/l and the sample points 1, 2 and 3 having 0.05, 0.12, and 0.08 mg/l respectively. Also, by comparing the various points with the WHO set standard, it was discovered that at point 1, the concentration of Pb^{2+} met the WHO set standard, point 2 was recorded to be highly concentrated with lead, while point 3 was slightly above the set standard and thus was fairly concentrated. This indicates that the Pb^{2+} readings at point 2 (Assumpta/ Warehouse head bridge) and 3 (Umezurike Hospital Head Bridge) are highly toxic and harmful within margin to organisms and to humans if the water is used for domestic purpose. This could be as a result of wastes from illegal dumping and sediment run-off in the river. Heavy metals gained access into the aquatic environment from natural and anthropogenic sources and distributed in the water bodies, suspended solids and sediments from the course of their transportation (Olajire and Imeokparia, 2000). Reports have shown that heavy metal pollution of ecosystems occurs more in sediments and aquatic animals than in water samples (Luinnik and Zubenko, 2000). Alinnor and Obiji (2010) working on Nworie River identified Fe, Cd and Mn from fresh fish species Tilapia guineens is with mean values 3.275, 0.048 and 0.103 ppm whereas Pb was below detection limit.

3.2.3 Biochemical Oxygen Demand (BOD) and Dissolved oxygen (DO) of Nworie River

The fourth group of subdivision of the physicochemical analysis result is the two parameters, i.e. Biochemical Oxygen Demand (BOD) and Dissolved oxygen (DO). These were further presented on a bar chart in Figure 5.

A mean value of DO of 3.233 mg/l was recorded and the three samples 1, 2 and 3 were: 3.30, 3.50 and 2.90 mg/l. Nwagbara et al., (2013) reported a higher mean value of 4.48 mg/l. Comparing these results with the WHO set standard of 4.0 mg/l, showed that the river falls below the range of WHO standard. This may be due to the process of eutrophication in those areas, which enhanced the growth of aquatic vegetation or phytoplankton. Algal blooms disrupt normal functioning of ecosystem causing variety of problems which include; lack of oxygen needed for fish and shell fish to survive. Also, the FEPA standard for DO is 6-8 mg/l to sustain aquatic life including fish (FEPA, 1991).

The BOD mean value was recorded as 5.9 mg/l, being high at station 1 and decreased along the reach i.e. 6.5, 5.8 and 5.4 mg/l respectively. However, a lower mean value of 3.0 and 3.23 mg/l was reported by Umunnakwe et al., (2015); **Uzoigwe and Udeorji**, (**2016**) in their research. Thus, comparing both mean relatively, it is clear that the recent research recorded a high BOD, indicating the presence of organic pollutant in the river.

3.2.4 Dissolved Oxygen of Water collected after discharged of Dye solution (Sodium Aluminum Sulfosilicate) at Egbeada Federal Housing New Road Bridge Head (Location one).

The result of dissolved oxygen of the river at Egbeada Federal Housing Head bridge (sample location 1) was presented in Section 3.1.2. This was necessary for comparison with the result generated using Galerkin's weighted residual method. A selected length of 12m was discretized into 5 points and samples were collected and analysed in the laboratory. The values of the five points were: 3.30, 3.50, 3.55, 3.65, 3.80 mg/l respectively. They were observed to increase downstream as the effluent diffused, bringing about reoxygenation of the river while deoxygenation was seen at the upstream.

3.2.5 Cross-sectional Area

The result of flow area for the three sample locations along the river is shown in Figures 7, 8 and 9 respectively. In Figure 7 the graph shows the relationship between the widths against its depth of the river in location 1. It was observed from the graph that the depth was uniform considering the point where the water sample was collected and the velocity flow rate measured (lined channel). The average area of location 1 was calculated to be 2.0 square metres.

In Figure 8, the cross-sectional area of flow of the river at location 2 (Assumpta-Control Road Head bridge) was plotted with the values of the depth against width. The resulting graph plotted was used to calculate the flow area. The average area was calculated to be 5.28 square metres.

Also, in Figure 9, the flow of the river at location 3 was plotted with the values of the depth against width. The average area of location 3 was calculated to be 5.32 square meters.

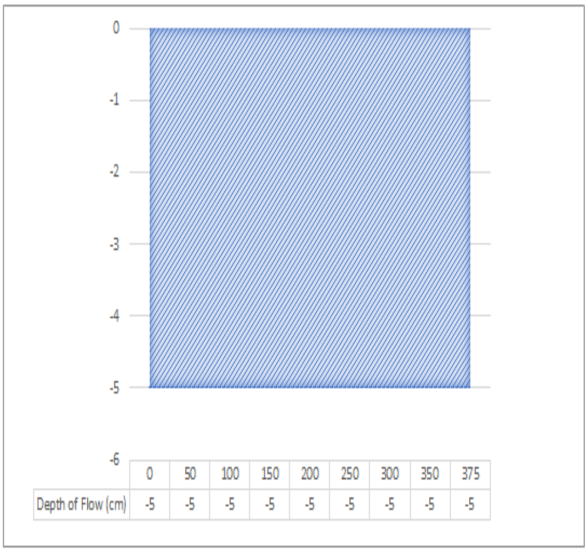

Figure 7. Flow Area of Location 1.

3.2.6 Velocity and flow rate of Nworie River

From Table 1, the velocity of flow of water in column 5 was calculated by dividing the flow length (column 2) with the average time of flow of floater (column 4). This resulted to 0.467, 0.417 and 0.411 m/s respectively. Furthermore, the flow rate of the three sample locations along Nworie River were calculated using the flow area (column 3) multiplied by the velocity of flow (column 5) and the velocity correction

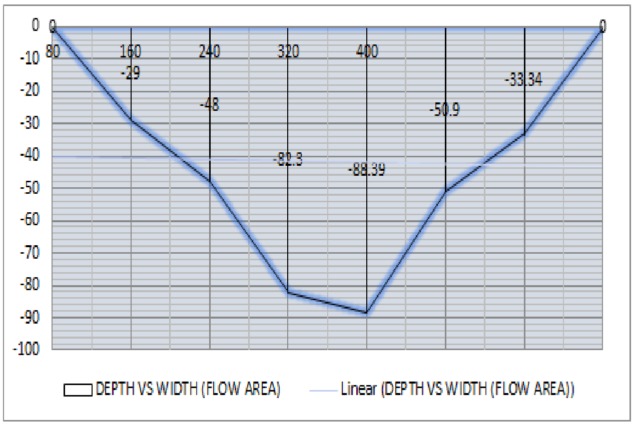

Figure 8. Flow Area of Location 2.

$$\textbf{Flow area} = \textbf{number of squares x the area which one square represents}$$

$$\mathbf{A = 66\ squares\ x\ 0.08\ m^2}$$

$$\mathbf{= 5.28m^2}$$

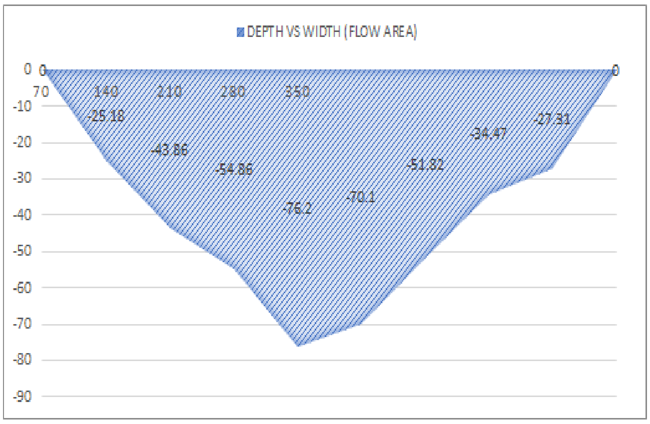

Figure 9. Flow Area of Location 3.

Flow area of location 3 is given as:

$$\textbf{Area} = \textbf{number of squares x the area which one square represents}$$

$$\mathbf{A = 76\ squares\ x\ 0.07\ m^2 = 5.32m^2}$$

factor (column 6) as shown in Table 1. The average flow rate resulted to 1.362 cubic meter per second

3.2.7 Discussion of result from Galerkin Weighted Residual (GWR) Analysis

All the estimated oxygen deficits from GWR finite elements were presented in Table 2. The results of the oxygen deficit at the first node which is the point of effluent discharge gave 5.728 mg/l. As pollutants moves a distance of one fourth of the distance under consideration as our linear global domain element, reoxygenation occurs. This brought about a reduction in the oxygen deficit along the downstream of the river with the following results at the respective nodes: 5.543 mg/l at the 2nd node, 5.356 mg/l at the 3rd node, 5.164mg/l at the 4th node and 4.968 mg/l at the end node. The dissolved oxygen along the river was calculated by subtracting the value of each oxygen deficit from the saturation dissolved oxygen (S.DO) of 9.028 mg/l. The results were observed to be increasing as the oxygen deficit reduces. This phenomenon is similar to the result recorded by Uzoigwe and Udeorji, (2016).

3.2.8 Verification of Galerkin's Weighted Residual (GWR) Model

To validate the determined/estimated dissolved oxygen from GWR finite element, pearson's correlation coefficient, and measure of the goodness of fit of the estimated DO and observed DO using least square method were computed and discussed in proceeding sections.

3.2.8.1 Pearson's Correlation Coefficient of the Relationship between Laboratory Dissolved Oxygen and the Estimated Dissolved Oxygen using GWR method.

Table 3 relates the statistical measures of the laboratory dissolved oxygen and estimated dissolved oxygen.

The computed correlation coefficient 'r' value gave 0.982 which showed that estimated dissolved oxygen using Galerkin's weighted residual method result is a high positive linearly correlated to the observed value or the laboratory dissolved oxygen. The implication of this high computed value is that the estimated dissolved oxygen variable being finite element method provides a good approximate solution to differential equations of initial and/or boundary values problems in engineering and mathematical physics that is close to laboratory result.

3.2.8.2 Goodness of Fit of the Estimated Do and Observed Do using least square method

The measure of the degree of correlation between the two variables denoted as r was computed and gave 0.982 (98.2%) as the case with Pearson method. This also verified the Galerkins Weighted Residual equation and gives a strong proof that the method can be effective in determining pollutant transport of contaminant.

4.0 Conclusion

The study demonstrated the variation of oxygen deficits along Nworie River. Water samples from the river were analyzed and were shown to be physically, chemically polluted and contain organic pollutant as indicated by the BOD increase when compared with previous research on the river. It was also filled with suspended solid which was highly noticeable from the color of the river. The study also had shown the average flow rate or discharge of the river to be 1.362 cubic meter per second. Since DO is one of the most important constituents of natural water systems; as fish and other aquatic animal species require oxygen, it is important to know the value of river life sustainability. Therefore, the study had shown the average DO to be 3.23 mg/L, however, research from NSIDC, (2013) showed that rivers must have a minimum of 2 mg/L DO to maintain higher life forms, although most game fish require 4 mg/L. Also, from the Galerkins Weighted Residual model, dissolved oxygen was predicted and the results were observed to be increasing as the oxygen deficit reduces. Furthermore, there is also similar increase from the DO of the five samples collected at location one after the dye solution was discharged and analyzed at the laboratory.

This observation may be due to dilution of flowing river, low temperatures mitigating the occurrence of reduce solubility of oxygen and aeration of the river by atmospheric wind.

In conclusion, the study further validated the predicted oxygen deficit using pearson's correlation coefficient and least square method of correlation coefficient. There is a strong relationship between the model result and the laboratory result of DO. Therefore, Galerkin's weighted residual finite element method should be used in determining or predicting the effect of effluent discharge downstream of a river.

Table 1. Velocity and flow rate of Nworie River.

S/N	Flow Length (m)	Area (m^2)	Average Time of flow of floater	Velocity of flow (m/s)	Velocity Correction Factor	Flow rate Q (cm^3/s)
Location 1	12.30	**2.00**	26.33	0.467	0.85	0.784
Location 2	10.00	**5.28**	24.00	0.417	0.75	1.652
Location 3	10.00	**5.32**	24.33	0.411	0.75	1.640

Table 2. Determined Oxygen deficit, DO and BOD along Nworie River using GWR finite element method.

Node No.	Oxygen Deficit (D) mg/l	Dissolved Oxygen (DO) mg/l	BOD mg/l
1	**5.728**	3.300	6.500
2	**5.543**	3.485	6.067
3	**5.356**	3.672	5.662
4	**5.164**	3.864	5.280
5	**4.969**	4.059	4.930

Legend DO=S.Do – D = 9.028 – D.

Table 3. Relationship between Estimated DO and Observed DO.

DO (GWR)(Y)	3.300	3.485	3.672	3.864	4.059	$\boldsymbol{\Sigma Y = 18.38}$
DO (Lab) (X)	3.30	3.50	3.55	3.65	3.80	$\boldsymbol{\Sigma X}$ =17.800
(xy)	10.89	12.1975	13.0356	14.1036	15.4242	$\boldsymbol{\Sigma xy}$ = 65.651
y^2	10.89	12.14523	13.48358	14.9305	16.47548	$\boldsymbol{\Sigma Y^2}$ = 67.925
x^2	10.89	12.25	12.6025	13.3225	14.44	$\boldsymbol{\Sigma X^2}$ = 63.505

4.1 Recommendation

Having achieved the objectives of this study, there is great need to make a formal recommendation regarding the alternative(s) or better ways in attaining more accurate results as well as bringing about development in water resources engineering. The following recommendations needed are:

i.Further study on wider reach or stretch of the river should be covered.

ii.Further research and on the effect of river area, speed and discharge on dissolved oxygen content of the river. Should be carried out to bridge the gap of inadequate information contained in this study.

iii.Comparative research on other methods of modeling.

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