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An Improve Poisson Approximation for the Generalized Binomial Distribution with Financial Application

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ABSTRACT

The aim of this study is to improve Poisson distribution for the approximation of a generalized binomial distribution and to combine the improved Poisson distribution with financial terms for the purpose of evaluating a European call and put option. It was found that the improved Poisson approximates generalized binomial when $\alpha \in [0,1]$, approximates generalized binomial sufficient enough when $n, \frac{B}{A+B}$ and $\alpha = 0$ and also that the problem of option for non-dividend paying stock can be approached using an improved Poisson distribution function equipped with some financial terms. The .numerical results obtained is the same as in [4], using the numerical data. Relevant connections of our work with the earlier work is pointed out.

1. INTRODUCTION

The improvement of distributions has become popular; this is because some of the existing distribution is not sufficient enough to approach real life problems or for modeling of real data set. Evaluating the value of an option has also become popular because the financial market have improved considerably, thus people can invest using various strategies or instrument to either reduce the risk of trading or investing and also to maximizes profit. The generalized binomial in this study was first presented by Dwass [6] in 1979.This distribution depends on four parameters A, B, n and α, where A and B are positive, n is a positive integer and α is an arbitrary real number. The details of the distribution can found in[6] .Dwass [6] gave its probability function of the form

$$\boldsymbol{P_x(x)} = \binom{n}{x} \frac{\boldsymbol{A(A-\alpha)\ldots\ldots A-(x-1)(B(B-\alpha)\ldots\ldots B-(n-x-1)\alpha)}}{\boldsymbol{(A+B)(A+B-\alpha)\ldots\ldots A+B-(n-1)\alpha)}} \tag{1}$$

$$= \binom{n}{x} \frac{A^{(x)}B^{(n-x)}}{(A+B)^{(n)}} \; x = 0,1 \ldots . n \tag{2}$$

And the mean and variance of are $\lambda = \frac{nA}{A+B}$ and $\sigma^2 = \frac{nAB(A+B-n\alpha)}{(A+B)^2(A+B-\alpha)}$ respectively.

The Poisson probability function with $\lambda = \frac{nA}{A+B}$ can be used to approximate the generalized binomial probability function, if $\boldsymbol{n}$ and A are small, approximate sufficiently enough if the bound obtained is small.

In view of that Wongkansam et al[15] gave a uniform bound on Poisson approximation to generalized binomial distribution as follows;

$$\left|\sum_{x\epsilon\Omega} P_X(x) - \sum_{x\in\Omega} \wp_\lambda\right| \le \left(1 - e^{-\lambda}\right) \frac{B(n-1)\alpha + A(A+B-\alpha)}{(A+B)(A+B-\alpha)} \tag{3}$$

Where $\subseteq \{0, \ldots n\}, \wp_\lambda(x) \frac{e^{-\lambda}\lambda^x}{x!}, \lambda = \frac{nA}{A+B}$ and $A \ge (n-1)(-\alpha)$, when $\alpha < 0$

Jaioun et al[8] improved Poisson for the approximation of binomial distribution as

$$b_{n,p}(x) = \frac{\lambda^x}{x!} e^{-n\left(\frac{\lambda}{n} - \frac{\lambda^2}{2n^2}\right)} \left\{ \frac{1}{1+\frac{x(x-1)}{2n}} + 0\left(\frac{1}{n^2}\right) \right\} \tag{4}$$

In this work we our interest is to improved Poisson probability function $\widetilde{\wp}_{\boldsymbol{\lambda}}(\boldsymbol{x})$ with mean $\tilde{\boldsymbol{\lambda}} = \frac{\boldsymbol{nA}}{\boldsymbol{B}}$ for approximating a generalized binomial probability function and criterion for the accuracy in of the form;

$$\left|\sum_{x\epsilon\Omega} P_X(x_0) - \sum_{x\in\Omega} \widetilde{\wp}_{\tilde{\lambda}}(x_0)\right|, \forall \; x_0 \in \mathbb{N} \cup \{0\} \tag{5}$$

If $\boldsymbol{\Omega} = \{\boldsymbol{x_0}\}$, where $x_0 \epsilon \{0, \ldots . n\}$, then we have $\left| P_X(x_0) - \widetilde{\wp}_{\tilde{\lambda}}(x_0)\right|$.

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And we are also interested in a particular type of derivative of security considered in Osu et al [2] and Samson Egege at el [11] , and a particular CRR binomial model proposed by Chandra et al [4] Most recently models for evaluating option price are

$$C_{(0)} = \frac{1}{R^n}\sum_{x=0}^{n}\binom{n}{x}\frac{\hat{A}^x\hat{B}^{n-x}}{(\hat{A}+\hat{B})^{(n)}}\ Max[u^x d^{n-x}S_{(0)} - K, 0] \quad (6)$$

Where (1.4) can be also express as of the form

$$C_{(0)} = \frac{1}{R^n}\sum_{x=0}^{n}\binom{n}{x}\frac{\hat{A}^x\hat{B}^{n-x}}{(\hat{A}+\hat{B})^{(x)}(\hat{A}+\hat{B})^{(n-x)}}C_t(x). \quad (7)$$

Where $C_t(x) = Max[u^x d^{n-x}S_{(0)} - K, 0]$,$R$ is the interest rate ,$\frac{\hat{A}}{\hat{A}+\hat{B}}$ and $\frac{\hat{B}}{\hat{A}+\hat{B}}$ are the neutral probability ,u and d is the rate at which the price move up and down respectively and k is the strike price.

$$C_{(0)} = \frac{1}{(1+r)^n}\sum_{x=0}^{n}\binom{n}{x}\frac{\left(\check{r}/_{\check{r}+\check{b}}\right)^x\left(\check{b}/_{\check{r}+\check{b}}\right)^{n-x}}{\left(\check{r}/_{\check{r}+\check{b}}+\frac{\check{b}}{\check{r}+\check{b}}\right)^n}C_T(x) \quad (8)$$

Where$C_T(x) = \max[u^x d^{n-x}S_{(0)} - K]$,$(1+r)$is the Oduro et al [8] gave binomial model for a two-step binomial as

$$f = e^{-2r\Delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}]. \quad (9)$$

With payoff $= [0, S_T - K]$ and neutral probability $p = \frac{e^{rt}-d}{u-d}$

Nyustern[1] gave a Block –Sholes model written as a function of five variables S, K, T, r and σ^2 as

$$C = SN(d_1) - ke^{-rt}N(d_2), \quad (10)$$

Where $d_1 = \frac{\text{In}\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$, where *S* is current value of the underlying asset ,K is the strike price of the option, t is the life to expiration of the option , r is the riskless interest rate corresponding to the life of the option and σ^2 is the variance in the In($value$)of the underlying asset

Chandra et al [4] developed CRR binomial model for the case of two period of the form

$$C_{(0)} = e^{-r\Delta t}\left[\check{p}^2 C_{uu} + 2\check{p}(1-\check{p})C_{ud} + (1-\check{p})^2 C_{dd}\right] \quad (11)$$

$$P_{(0)} = e^{-r\Delta t}\left[\check{p}^2 P_{uu} + 2\check{p}(1-\check{p})P_{ud} + (1-\check{p})^2 P_{dd}\right] \quad (12)$$

With neutral probability $\check{p} = \frac{e^{-r\Delta t}-d}{u-d}$.

Instead in our case, we generated a model by equipping an improved Poisson with financial terms of the form

$$C = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\binom{N}{x_0}e^{-\lambda}\frac{\lambda^{x_0}}{N^{x_0}}e^{\lambda}\left(\frac{B}{A+B}\right)^N fS(N) \quad (13)$$

and

$$P = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\binom{N}{x}e^{-\lambda}\frac{\lambda^{x_0}}{N^x}e^{\lambda}\left(\frac{B}{A+B}\right)^N fS(N) \quad (14)$$

For the call and put options with fS(N), the payoff and $\lambda = \frac{nA}{B}$

The proposed model and the improved distribution applied in finance is of the form

$$C = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\binom{N}{x_0}e^{-\lambda}\frac{\tilde{\lambda}^{x_0}}{n^x}e^{\lambda}\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N \max\left[u^{x_0}d^{N-x_0}S_{(0)}, -k, 0\right] \quad (15)$$

This work seek to approach the problem of option pricing contained in Chandra et al [4]

2.METHOD

The Generalized Binomial distribution in this study was first presented by Dwass [6] (1979). It is a discrete distribution that depends on four parameters *A*, *B*, *n* and α , where A and B are positive , *n* is a positive integer and α is an arbitrary real number ,satisfying $(n-1) \leq A + B$. And Tereapabolarn [13] gave Dwass identity of the form $x^{(i)} = x(x-\alpha)\ldots\ldots(x-(i-1)\alpha)$.

Corollary 2.1

Let X be the generalized Binomial random variable. Then following Terepabolan [13] ,its probability function is of the form

$$P_X(x) = \binom{n}{x}\frac{[A(A-\alpha)\ldots\ldots(A)-(x-1)\alpha][B(B-\alpha)\ldots\ldots(B-(n-x-1)\alpha)]}{(A+B)(A+B-\alpha)\ldots\ldots\ldots(A+B-(n-1)\alpha)} = \binom{n}{x}\frac{A^{(x)}B^{(n-x)}}{(A+B)^{(n)}}, x = 0, \ldots\ldots n \quad (16)$$

Special Cases

i. If $\alpha = 0$, it reduces to Binomial distribution with parameters n and $\frac{A}{A+B}$

ii. If $\alpha < 0$ the result of (1.7) is pòlya distribution with parameters A, B, nand α.

iii.If $\alpha > 0$ it reduces to hypergeometric distribution with parameters A, B, n and α and , some integers $\frac{A}{\alpha}$ and $\frac{B}{\alpha}$

Where if $\alpha = 0$, the generalized binomial reduces binomial distribution with parameters,*A*, *B*and . And its probability function is given as

$$P_x(x_0) = \binom{n}{x_0}\left(\frac{A}{A+B}\right)^{x_0}\left(\frac{B}{A+B}\right)^{n-x_0}. \quad (17)$$

Where if $\alpha < 0$, the generalized binomial reduces to Pólya distribution with probability function of the form.

$$P_{\mathbb{Y}}(x_0) = \frac{\binom{\frac{A}{-\alpha}+x_0-1}{x_0}\binom{\frac{B}{-\alpha}+n-x_0-1}{n-x_0}}{\binom{\frac{A}{-\alpha}+\frac{B}{-\alpha}+n-1}{n}}.$$

Where if $\alpha > 0$, it reduces to hyper geometric distribution with probability function

$$P_H(x_0) = \frac{\binom{\frac{A}{\alpha}}{x_0}\binom{\frac{B}{\alpha}}{n-x_0}}{\binom{\frac{A}{\alpha}+\frac{B}{\alpha}}{n}} \tag{18}$$

This can be easily proved.

2.1 ASSUMPTIONS FOR THE PROPOSED MODEL

In what follows, we assume the following:

1. The initial values of the stock is S_0
2. At the end of the period , the price is either going up or down by a fixed factor $u = e^{\sigma\sqrt{\Delta t}}$ or go down by a factor $d = e^{-\sigma\sqrt{\Delta t}}$
3. The price of an option is dependent on the following

a) The strike price *K*

b) The expire time *T*

c) The risk free rate *r*

d) The underlying price S_0

e) Volatility σ

4. $e^{\sigma\sqrt{\Delta t}} > e^{r\Delta t} > e^{-\sigma\sqrt{\Delta t}} > 0$
5. The stock pays no dividends
6. The continuous compounded interest rate $\boldsymbol{r}$ such that $B(0,T) = e^{r\Delta t}$
7. The length of each period Δt can be positive number
8. From market data for stock price one can estimate the stock price volatility σ per one time unit.(typically one year)
9. Set $N = \frac{T}{\Delta t}$.

2.2 OPTION PRICING PARAMETERS

1. The current stock price S_0 *: which is the prevailing market price of the stock at expiration*

2. The strike priceK: which is the predetermined price at which the holder will exercised right.

3. The time to expirationT : which is the time duration the holder has to exercise right.

*4. The risk-free interest rate **r**:which is the rate of investment on the stock*

5. The volatility of the stock price σ*: which measures the uncertainty of movement in the market*

Change in the above parameters affects the price of the option discussed in Osu et at [2] *and oduro* [10]

Lemma 2.1*:* Let $x \in N \cup \{0\}$ for $n > 0$ (Dongping Hu et at {5]);

$$\frac{1}{\prod_{i=0}^{x-1}\left(1-\frac{1}{n}\right)} = \prod_{i=0}^{x-1}\left(1+\frac{i}{n}+\left(\frac{i}{n}\right)^2+\cdots\right) = 1+\frac{x(x-1)+0\left(\frac{1}{n^2}\right)}{2n} \tag{19}$$

Proof

In this work we show that lemma2.1 hold by mathematical induction

For $x = 1$

$$\frac{1}{\prod_{i=0}^{1-1}\left(1-\frac{i}{n}\right)} = \prod_{i=0}^{1-1}\left(1+\frac{i}{n}+\left(\frac{i}{n}\right)^2+\cdots\ldots\right) = 1+\frac{1(1-1)}{2n}+0\left(\frac{1}{n^2}\right) = 1$$

Let $x = k \in N$ such that $\frac{1}{\prod_{i=0}^{k-1}\left(1-\frac{1}{n}\right)} = 1+\frac{k(k-1)}{2n}+0\left(\frac{1}{n^2}\right)$.

Thus for $x = k+1$

$$\frac{1}{\prod_{i=0}^{k}\left(1-\frac{1}{n}\right)} = \left\{1+\frac{k(k-1)}{2n}+0\left(\frac{1}{n^2}\right)\right\}\left(1+\frac{k}{n}\right)$$
$$= 1+\frac{k}{n}+\frac{k(k-1)}{2n}-\frac{k(k-1)}{2n}\frac{k}{n}+O\left(\frac{1}{n^2}\right)-O\left(\frac{1}{n^2}\right)\cdot\frac{k}{n}$$
$$= 1+\frac{k}{n}+\frac{k(k-1)}{2n}-\frac{k^2(k-1)}{2n^2}+O\left(\frac{1}{n^2}\right)-O\frac{k}{n^2}$$
$$= 1+\frac{k}{n}+\frac{k(k-1)+}{2n}O\left(\frac{1}{n^2}\right) = 1+\frac{2k+k^2-k}{2n}+O\left(\frac{1}{n^2}\right)$$
$$= 1+\frac{k(k+1)}{2n}+O\left(\frac{1}{n^2}\right) \tag{20}$$

For $x = k+1$ we obtain

$$= 1+\frac{x(x-1)}{2n}+O\left(\frac{1}{n^2}\right) \tag{21}$$

Lemma 2.2 For $u = \frac{1}{d} = e^{\sigma\sqrt{\Delta t}}$, a risk free interest rate $e^{2r} = e^{r\Delta t}$ for two period and $\frac{\breve{A}}{\breve{A}+\breve{B}} = \frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}$ holds

If $\sum_{j=1}^{3}\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}} = 1$

Lemma 2.3: If $e^{\sigma\sqrt{\Delta t}} > e^{r\Delta t} > e^{-\sigma\sqrt{\Delta t}} > 0$ and no arbitrary principle exist thus the following holds

1. $E_{\frac{\breve{A}}{\breve{A}+\breve{B}}}(S_2) = e^{2r\Delta t}S_0$
2. $\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)_j > 0$ where $j = 1,2,\ldots n$

3. MAIN RESULTS

In what follows we state;

Theorem 3.1: For $x_0 \in \mathbb{N} \cup \{0\}$ and $\lambda = \frac{NA}{B}$ then we have

$$\mathrm{Gbd}(A, B, N, \alpha) \cong \widetilde{\wp_\lambda}(x) + O\left(\frac{1}{N^2}\right) \tag{22}$$

and for $\alpha = 0$ $N, \frac{B}{A+B}$ large $Gbd(A, B, n) \cong \widetilde{\wp_\lambda}(x_0)$ Where $\widetilde{\wp_\lambda}(x_0) = \wp_\lambda e^{\lambda}\left(\frac{B}{A+B}\right)^N \frac{1}{1+\frac{x_0(x_0-1)}{2N}} + O\left(\frac{1}{N^2}\right)$

Proof

For $x_0 = 0$, $Gbd(A, B, N, \alpha) = \left(\frac{B}{A+B}\right)^N = \widetilde{\wp_\lambda}(0) + O\left(\frac{1}{N^2}\right)$

Thus

$$Gbd(A, B, n, \alpha) = \binom{N}{x_0} \frac{A(A-\alpha).....A-(x_0-1)(B(B-\alpha).....B-(N-x_0-1)\alpha)}{(A+B)(A+B-\alpha)......A+B-(N-1)\alpha)} \tag{23}$$

$$= \binom{N}{x_0} \frac{A^{(x_0)} B^{(N-x_0)}}{(A+B)^{(N)}}$$

$$= \binom{N}{x_0} \left(\frac{N^{A}/_{A+B}}{N}\right)^{x_0} \left(\frac{B}{A+B}\right)^N \left(\frac{B}{A+B}\right)^{-x_0}$$

$$= \binom{N}{x_0} \left(\frac{N^{A}/_{A+B}}{N}\right)^{x_0} \frac{\left(\frac{B}{A+B}\right)^N}{\left(\frac{B}{A+B}\right)^{x_0}}$$

$$= \binom{N}{x_0} \left(\frac{N^{A}/_{A+B}}{N}\right)^{x_0} \frac{\left(\frac{B}{A+B}\right)^N}{\left(\frac{N^{A}/_{A+B}}{\lambda}\right)^{x_0}}$$

$$= \binom{N}{x_0} \left(\frac{N^{x_0}\left(\frac{A}{A+B}\right)^{x_0}}{N^{x_0}}\right) \left(\frac{B}{A+B}\right)^N \times \frac{\lambda^{x_0}}{N^{x_0}\left(\frac{A}{A+B}\right)^{x_0}}$$

$$= \binom{N}{x_0} \lambda^{x_0} \frac{N^x}{N^x} \left(\frac{\frac{A}{A+B}}{N\frac{A}{A+B}}\right)^{x_0} \left(\frac{B}{A+B}\right)^N$$

$$= N_{C_{x_0}} \frac{e^{-\lambda}\lambda^{x_0} e^{\lambda}}{N^{x_0}} \left(\frac{B}{A+B}\right)^N \tag{24}$$

$$= \frac{N(N-1)(N-2).....(N-x+1)(N-x)..(2)(1)}{x_0(x_0-1)(x_0-2).....(2)(1).(n-x_0)..(2)(1)} \frac{e^{-\lambda}\lambda^{x_0}e^{\lambda}}{N^{x_0}} \left(\frac{B}{A+B}\right)^N$$

$$\frac{N(N-1)(N-2).......(N-x+1)e^{-\lambda}\lambda^{x_0}e^{\lambda}}{N^{x_0}x_0!} \left(\frac{B}{A+B}\right)^N = \frac{\left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right)........\left(1-\frac{x-1}{N}\right)e^{-\lambda}\lambda^{x_0}e^{\lambda}}{x!} \left(\frac{B}{A+B}\right)^N$$

$$\frac{\lambda^{x_0}}{x_0!} \prod_{i=0}^{x_0-1}\left(1-\frac{i}{N}\right)\left(\frac{B}{A+B}\right)^N = \frac{e^{-\lambda}\lambda^{x_0}e^{\lambda}}{x!}\left(\frac{B}{A+B}\right)^N \frac{1}{\prod_{i=0}^{x_0-1}\left(1-\frac{i}{N}\right)} \text{ by lemma 2.1}$$

We obtain

$$\cong \frac{\frac{e^{-\lambda}\lambda^{x}e^{\lambda}}{x!}\left(\frac{B}{A+B}\right)^N}{1+\frac{x(x-1)}{2N}} + O\left(\frac{1}{N^2}\right). \tag{25}$$

For $\alpha = 0$ and $N, \frac{B}{A+B}$ large $O\left(\frac{1}{N^2}\right) \to 0$

By corollary 1.1 we obtained Implies

$$Gbd(A, B, N) \cong \frac{\frac{e^{-\lambda}\lambda^{x_0}e^{\lambda}}{x_0!}\left(\frac{B}{A+B}\right)^N.}{1+\frac{x_0(x_0-1)}{2N}} \tag{26}$$

With

$$E(X) = \frac{\lambda e^{\lambda}\left(\frac{B}{A+B}\right)^N}{a} \text{ and } Var(X) = ae^{\lambda}\left(\frac{B}{A+B}\right)^N \frac{\left[(\lambda^2+\lambda)\text{-}\lambda^2 e^{2\lambda}\left(\frac{B}{A+B}\right)^{2N}\right]}{a^2} \text{ where } a = 1 + \frac{x_0(x_0-1)}{2N} \tag{27}$$

Equation (27) can be combine with financial terms to determine the call and put price of an option, provided the following is satisfy

I. $\sum_{x=0}^{n} N_{C_{x_0}} \frac{e^{-\lambda}\lambda^{x_0}e^{\lambda}}{N^{x_0}}\left(\frac{B}{A+B}\right)^N = 1$

II. $\left(\frac{B}{A+B}\right)^N > 0$

III. $1 - \frac{B}{A+B} = \frac{A}{A+B}$

IV. $N > 0$

V. $\frac{A}{A+B} = \frac{\breve{A}}{\breve{A}+\breve{B}}$ and $1 - \frac{A}{A+B} = 1 - \frac{\breve{A}}{\breve{A}+\breve{B}}$

Theorem 3.2: let C and P be the value of a European derivation security whose payoff is $f(S(N))$ then if $C = e^{-r\Delta t}\left[\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 C_{uu} + 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)C_{ud} + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2\right]$ exist implies $C = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{n}\frac{e^{-\lambda}\lambda^{x}e^{\lambda}}{n^x}\left(\frac{B}{A+B}\right)^n f(S(N))$ and $P = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{n}\frac{e^{-\lambda}\lambda^{x}e^{\lambda}}{n^x}\left(\frac{B}{A+B}\right)^n f(S(N))$.

Where C *and* P the cost of call and put is is option respectively and $f(S(N))$ is the payoff. $\frac{\breve{A}}{\breve{A}+\breve{B}}$ and $\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)$ is the neutral probabilities.

Proof

We show that $C = e^{-r\Delta t}\left[\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 C_{uu} + 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)C_{ud} + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2\right]$

For N= $\mathbf{2}$ we define $\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_1 = \left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2, \left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_2 = 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)$ and $\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_3 = \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2$

Then

$$E_{\frac{\breve{A}}{\breve{A}+\breve{B}}} = \left[\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 f_{uu}S(2) + 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)f_{ud}S(2) + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)f_{dd}S(2)\right]$$

$$= \left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 C_{uu} + 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)C_{ud} + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 C_{dd}$$

$$= C\left[\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 u^2 + 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)du + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 d^2\right]$$

$$= C\left[\frac{\breve{A}}{\breve{A}+\breve{B}}u + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)d\right]^2$$

$$E_{\frac{\breve{A}}{\breve{A}+\breve{B}}} = Ce^{r\Delta t}(27)$$

Making C the subject we obtained

$$C = e^{-r\Delta t}\left[\left[\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 f_{uu}S(2) + 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)f_{ud}S(2) + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)f_{dd}S(2)\right]\right] = \left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 C_{uu} + 2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)C_{ud} + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2 C_{dd}$$

$C = e^{-r\Delta t}\left[\frac{\breve{A}}{\breve{A}+\breve{B}}u + \left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)d\right]^2 f(S(2))$ by binomial expansion

$$C = e^{-r\Delta t}\sum_{x=0}^{N}\binom{N}{x}\frac{\breve{A}^{x_0}B^{N-x_0}}{(\breve{A}+\breve{B})^N}f(S(2))$$

$$C = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\binom{N}{x}\frac{\widehat{A}^{x_0}\widehat{B}^{N-x_0}}{(\widehat{A}+\widehat{B})^{(x_0)}(\widehat{A}+\widehat{B})^{(n-x_0)}}Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]$$

Where $N \in \mathbb{N}$ and $x_0 \in \mathbb{N}$

$$= \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\left[\binom{N}{x_0}\left(\frac{N^{\breve{A}}/_{\breve{A}+\breve{B}}}{n}\right)^{x_0}\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^N\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^{-x_0}\right]Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]$$

$$= \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\left[\binom{N}{x}\left(\frac{N^{\breve{A}}/_{\breve{A}+\breve{B}}}{N}\right)^{x_0}\frac{\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^N}{\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^{x_0}}\right]Max\left[u^{x_0}d^{N-x_{0s}}S_{(0)}-K,0\right]$$

$$= \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\left[\binom{N}{x}\left(\frac{N^{\breve{A}}/_{\breve{A}+\breve{B}}}{N}\right)^{x_0}\frac{\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N}{\left(\frac{N^{\breve{A}}/_{\breve{A}+\breve{B}}}{\lambda}\right)^{x_0}}\right]Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]$$

$$= \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\left[\binom{N}{x_0}\left(\frac{N^{x_0}\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^{x_0}}{N^{x_0}}\right)\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N \times \frac{\lambda^{x_0}}{N^{x_0}\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^{x_0}}\right]Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]$$

$$= \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\left[\binom{N}{x}\lambda^{x_0}\frac{N^{x_0}}{N^x}\left(\frac{\frac{\breve{A}}{\breve{A}+\breve{B}}}{N\frac{\breve{A}}{\breve{A}+\breve{B}}}\right)^{x_0}\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N\right]Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]$$

$$= \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\left[\frac{N(N-1)(N-2)\ldots\ldots(N-x+1)(N-x)..(2)(1)}{x_0(x_0-1)(x_0-2)\ldots\ldots(2)(1).(n-x_0)..(2)(1)}\frac{\lambda^{x_0}e^{-\lambda}}{N^x}\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N e^{\lambda}\right]Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]$$

$$= \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\left[\frac{N(N-1)(N-2)\ldots\ldots(N-x+1)}{N^{x_0}}\frac{e^{-\lambda}\lambda^{x_0}}{x!}\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N e^{\lambda}\right]Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]$$

$$C = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\binom{N}{x}\frac{e^{-\lambda}\lambda^{x_0}}{N^x}\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N e^{\lambda}\left[Max\left[u^{x_0}d^{N-x_0}S_{(0)}-K,0\right]\right] \quad (28)$$

Where $\lambda = \frac{nA}{B}$ and $x \in (0,1,..n)$Put option follows exactly the same derivation as the call option (10) implies

$$P = \frac{1}{e^{r\Delta t}}\sum_{x=0}^{N}\binom{N}{x}\frac{e^{-\lambda}\lambda^{x_0}}{N^{x_0}}\left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N e^{\lambda}\left[Max\left[K-u^{x_0}d^{N-x_0}S_{(0)},0\right]\right] \quad (29)$$

4. NUMERICAL RESULTS

The following numerical examples are given to illustrate how well improved Poisson distribution can approximate generalized binomial distribution.

Example 4.1: Le $n = 20$, $A = 25, A + B = 1000, \tilde{\lambda} = 0.512820513, \lambda = 0.5$ t

Table 4.1 .A Generalized Binomial distribution approximation of an improved Poisson distribution when $\alpha = 0$.

x_0	$Gbd(A,B,n)$	$\tilde{\wp}_{\tilde{\lambda}}$	$\wp_\lambda$	$\lvert Gbd - \tilde{\wp}_{\tilde{\lambda}}\rvert$	$\lvert Gbd - \wp_\lambda\rvert$
0	0.602687680	0.602687680	0.606530660	0.000000000	0.003842980
1	0.309070605	0.309070605	0.303265330	0.000000000	0.005805275
2	0.075286429	0.075474117	0.075816332	0.000188688	0.000529903
3	0.011582528	0.011779840	0.012636055	0.000197312	0.001053527
4	0.001262199	0.001335978	0.001579507	0.000073779	0.000317308
5	0.000103565	0.000118754	0.000157951	0.000015189	0.000054386
6	0.000006639	0.000008700	0.000013163	0.000002061	0.000006524
7	0.000000340	0.000000544	0.000000940	0.000000204	0.000000600
8	0.000000014	0.000000030	0.000000059	0.000000016	0.000000045

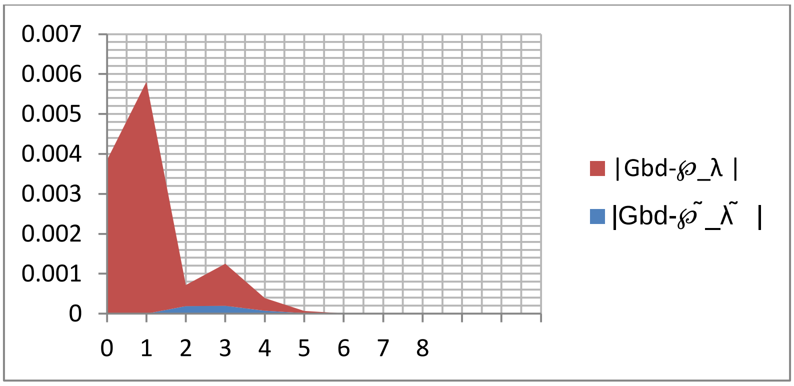

Figure4.1.A graph $\lvert Gbd - \tilde{\wp}_{\tilde{\lambda}}\rvert$ against $\lvert Gbd - \wp_\lambda\rvert$.

Table 4.2. Generalized binomial distribution approximation of an improved Poisson when $\alpha = 1$

x_0	$Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right)$	$\tilde{\wp}_{\tilde{\lambda}}$	$\wp_\lambda$	$\left\lvert Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right) - \tilde{\wp}_{\tilde{\lambda}}\right\rvert$	$\left\lvert Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right) - \wp_\lambda\right\rvert$
0	0.599719601	0.602687680	0.606530660	0.002968079	0.006811089
1	0.313660879	0.309070605	0.303265330	0.004590274	0.010395549
2	0.074727984	0.075475117	0.075816332	0.000747133	0.001088340
3	0.010764574	0.011779840	0.012636055	0.001015266	0.001871481
4	0.001049518	0.001335978	0.001579507	0.000286460	0.000529989
5	0.000073466	0.000118754	0.000157951	0.000045288	0.000084485
6	0.000003822	0.000008700	0.000013163	0.000004874	0.000009341
7	0.000000151	0000000544	0.000000940	0.000000393	0.000000789
8	0.000000005	0.000000030	0.000000059	0.000000025	0.000000054

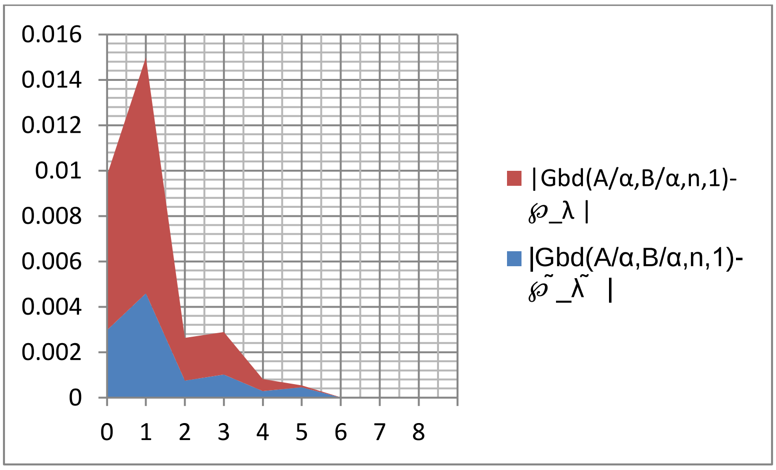

Figure 4.2: A graph of $\left\lvert Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right) - \tilde{\wp}_{\tilde{\lambda}}\right\rvert$ against $\left\lvert Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right) - \wp_\lambda\right\rvert$.

Table 4.3. A generalized Binomial approximation of an improved Poisson distribution when $\alpha = -1$.

x_0	$Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right)$	$\widetilde{\wp}_{\tilde{\lambda}}$	$\wp_{\lambda}$	$\left\lvert Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right) - \widetilde{\wp}_{\tilde{\lambda}}\right\rvert$	$\left\lvert Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right) - \wp_{\lambda}\right\rvert$
0	0.60559234	0.602687680	0.606530660	0.002905054	0.000937926
1	0.304624112	0.309070605	0.303265330	0.004446493	0.001358782
2	0.075772864	0.075475117	0.075816332	0.000297447	0.000043768
3	0.012374148	0.011779840	0.012636055	0.000594304	0.000261907
4	0.001485897	0.001335978	0.001579507	0.000149919	0.000093613
5	0.000139284	0.000118754	0.000157951	0.000020530	0.000018667
6	0.000010562	0.000008700	0.000013163	0.000001862	0.000002601
7	0.000000663	0.000000544	0.000000940	0.000000119	0.000000277
8	0.000000035	0.000000030	0.000000059	0.000000005	0.000000024

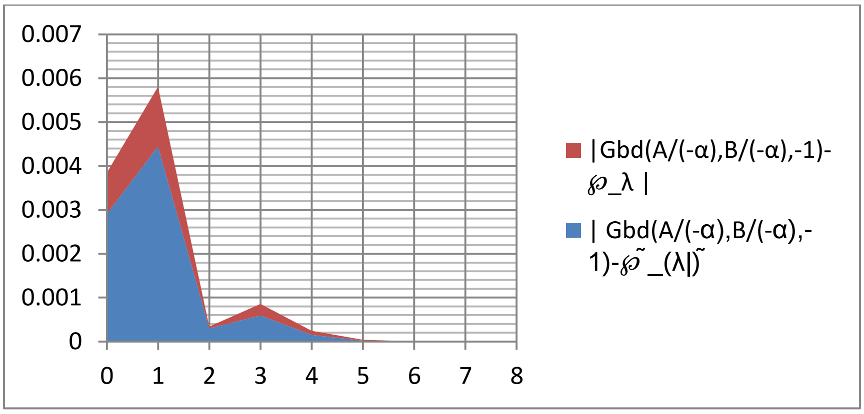

Figure 4.3.A graph of $\left\lvert Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right) - \widetilde{\wp}_{\tilde{\lambda}}\right\rvert$ against $\left\lvert Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right) - \wp_{\lambda}\right\rvert$.

Example 4.4 : Let $n = 30,\ A = 50\ \ A + B = 1000, \lambda = \frac{nA}{B} = 1.578947368,\ \frac{A}{A+B} = 0.05$

Table 4.4.A generalized binomial approximation of an Improved Poisson when $\alpha = 0$.

x_0	$Gbd(A, B, n)$	$\widetilde{\wp}_{\lambda}$	$\wp_{\lambda}$	$\lvert Gbd - \widetilde{\wp}_{\lambda}\rvert$	$\lvert Gbd - \wp_{\lambda}\rvert$
2	**0.255636738**	**0.258924431**	**0.251021450**	**0.000287693**	**0.007615308**
3	**0.127049626**	**0.128016864**	**0.125510715**	**0.000967238**	**0.001538911**
4	**0.045136051**	**0.046321892**	**0.047066518**	**0.001185841**	**0.001930467**
5	**0.012353025**	**0.013651691**	**0.014119955**	**0.000812144**	**0.001766930**
6	**0.002708997**	**0.003079572**	**0.003529989**	**0.000370575**	**0.000820992**
7	**0.000488841**	**0.000612918**	**0.000756426**	**0.000124077**	**0.000143508**
8	**0.000073969**	**0.000106371**	**0.000141830**	**0.000032402**	**0.000074875**

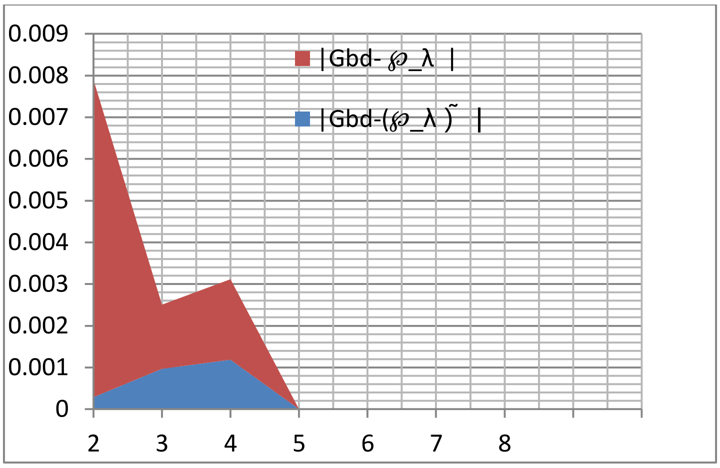

Figure 4.4.A graph of $\lvert Gbd - \widetilde{\wp}_{\lambda}\rvert$ against $\lvert Gbd - \wp_{\lambda}\rvert$.

Table 4.5.A generalized binomial approximation of an improved Poisson when $\alpha = -1$.

x_0	$Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right)$	$\widetilde{\wp}_{\lambda}$	$\wp_{\lambda}$	$\left\lvert Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right) - \widetilde{\wp}_{\tilde{\lambda}}\right\rvert$	$\left\lvert Gbd\left(\frac{A}{-\alpha}, \frac{B}{-\alpha}, -1\right) - \wp_{\lambda}\right\rvert$
2	0.254309554	0.258924431	0.251021430	0.004614766	0.007903388
3	0.126330560	0.128016864	0.125510715	0.001711264	0.002505049
4	0.046306810	0.046321820	0.047066518	0.000015010	0.000744698
5	0.013336158	0.013165169	0.014119955	0.000170989	0.000954786
6	0.003137785	0.003079572	0.003529981	0.000058213	0.000392196
7	0.000656938	0.0000612918	0.000756426	0.000044020	0.000143508
8	0.000104396	0.0000106371	0.00141830	0.000001975	0.000035459

x_0	Gbd	$\mathbb{HY}d$	$\mathbb{P}_{\mathbb{Y}}$	$\tilde{\wp}_{\tilde{\lambda}}$	$\wp_\lambda$	$\lvert Gbd - \tilde{\wp}_{\tilde{\lambda}}\rvert$	$HYd - \tilde{\wp}_{\tilde{\lambda}}\rvert$	$\lvert \mathbb{P}_{\mathbb{Y}} - \tilde{\wp}_{\tilde{\lambda}}\rvert$	$\lvert Gbd - \wp_\lambda\rvert$	$\lvert HYd - \wp_\lambda\rvert$	$\lvert \mathbb{P}_{\mathbb{Y}} - \wp_\lambda\rvert$
1	0.028632	0.007237	X	0.028632	0.033690	0.000000	0.021395	X	0.005058	0.026453	X
2	0.077943	0.037993	X	0.077974	0.084224	0.000031	0.039981	X	0.006281	0.046231	X
3	0.138565	0.113096	X	0.138948	0.110480	0.000383	0.025851	X	0.001809	0.027277	X
4	0.180905	0.211413	X	0.182644	0.175467	0.001739	0.028769	X	0.005457	0.035946	X
5	0.184926	0.259334	X	0.189409	0.175467	0.004483	0.069925	X	0.009459	0.083866	X
6	0.154104	0.211413	X	0.161888	0.104486	0.007784	0.022004	X	0.007881	0.065190	X
7	0.10728	0.11396	X	0.117625	0.104445	0.009997	0.004529	X	0.003179	0.008651	X

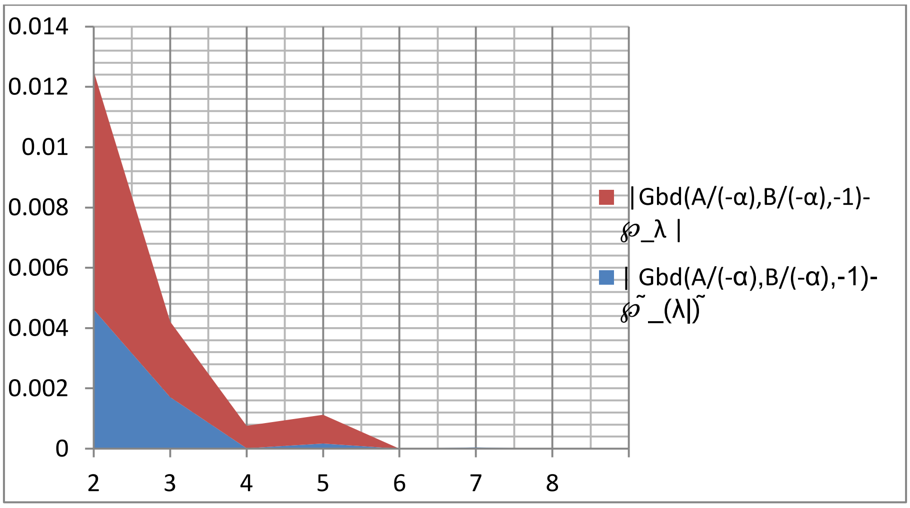

Figure 4.5.A graph of $\lvert \left(\frac{A}{-\alpha},\frac{B}{-\alpha},-1\right) - \tilde{\wp}_{\tilde{\lambda}}\rvert$ **against** $\lvert Gbd\left(\frac{A}{-\alpha},\frac{B}{-\alpha},-1\right) - \wp_\lambda\rvert$.

Table 4.6.A generalized binomial approximation of an improved Poisson when $\alpha = 1$.

x_0	$Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right)$	$\tilde{\wp}_{\tilde{\lambda}}$	$\wp_\lambda$	$\lvert Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right) - \tilde{\wp}_{\tilde{\lambda}}\rvert$	$\lvert Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right) - \wp_\lambda\rvert$
2	0.263162818	0.258924431	0.251021430	0.004238387	0.013448842
3	0.127732332	0.128016864	0.125510715	0.000284532	0.002506149
4	0.043856150	0.046321892	0.047066518	0.002465742	0.003209030
5	0.011340963	0.013165169	0.014119955	0.003209030	0.002778992
6	0.002296361	0.003079872	0.003529989	0.000783211	0.001233628
7	0.000373703	0.000612918	0.000756426	0.001233628	0.000382723
8	0.000049783	0.000106371	0.000141830	0.000056588	0.000092047

Example 4.5 .Let $n = 100, A = 50,\ A + B = 1000, \frac{A}{A+B} = 0.05, \lambda = 5.263157895$.

Where $\mathbb{HY}d = Gbd\left(\frac{A}{\alpha},\frac{B}{\alpha},n,1\right)$, $Gbd = Gbd(A,B,n)$, $\mathbb{P}_{\mathbb{Y}} = Gbd\left(\frac{A}{-\alpha},\frac{B}{-\alpha},-1\right)$ and $X = N0\ result$

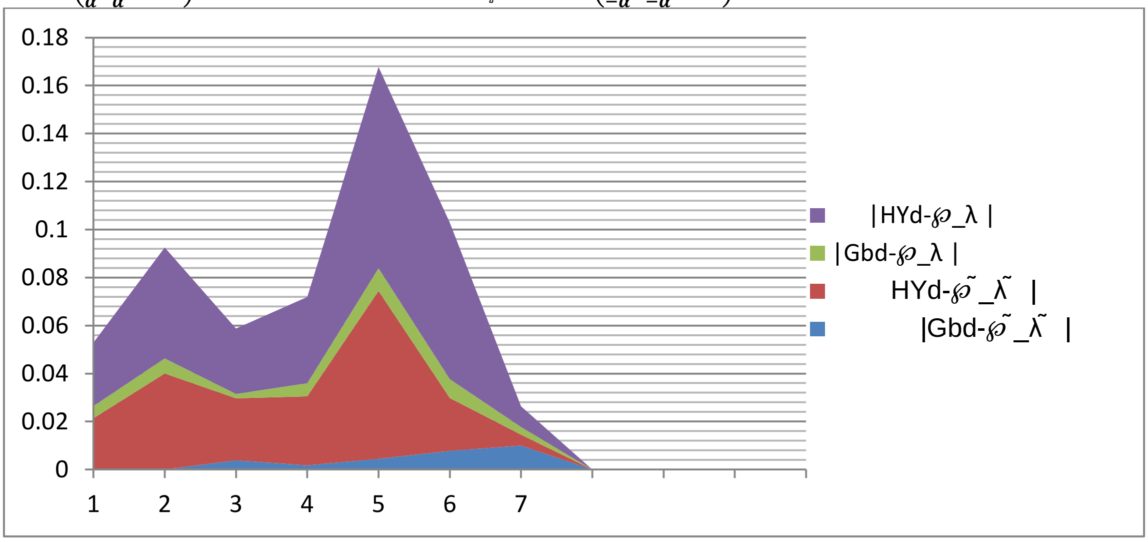

Figure 4.6: A graph of a generalized binomial approximation of an improved Poisson .

5.APPLICATION OF IMPROVED POISSON IN OPTION PRICING

The following numerical results show how an proved Poisson approaches CRR binomial model, to validate the theoretical results in comparison with CRR binomial model [**4**]

Example 5.1

Suppose a non –dividend pay stock is selling at Rs100 and stock's volatility is 24%. Assume that the continuously compounded risk-free is 5% .A European call and put option is offered on this stock and time of maturity is 4years and strike price Rs125. Calculate the prince of the options

Solution

$$S_0 = Rs100, \quad K = Rs125, T = 4yrs, r = 0.05, \sigma = 0.24, \quad u = \frac{1}{d} = 1.4041, d = 0.7122$$

$\frac{\breve{A}}{\breve{A}+\breve{B}} = 0.5680, \frac{\breve{B}}{\breve{A}+\breve{B}} = 0.4320$ and $\lambda = 2.6296$. $fS(N) = max[u^x d^{N-x} S_0 - K, 0]$

$C_{uu} = 72.1479$, $C_{ud} = 0$, and $C_{dd} = 0$

By (16)

$$C = \frac{1}{e^{r\Delta t}} \sum_{x=0}^{N} \binom{N}{x} \frac{e^{-\lambda}\lambda^{x_0}}{N^x} \left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N e^{\lambda} \left[Max\left[u^{x_0} d^{N-x_0} S_{(0)} - K, 0\right]\right]$$

$= e^{-0.1}\Big[2C_0 e^{-2.6296} \times \frac{(2.6296)^0}{2^0} \times e^{2.6298} \times (0.4320)^2 \times 0 + 2C_1 \times e^{-2.6296} \times \frac{(2.6296)^1}{2^1} \times e^{2.6296} \times (0.4320)^2 \times 0 + 2C_2 \times e^{-2.6296} \times \frac{(2.6296)^2}{2^2} \times e^{32.6296} \times (0.4320)^2 \times 96.0664\Big] = Rs21.056$

For put price by (15)

$$fS(N) = Max[K - u^x d^{N-x} S_0, 0]$$
$$P_{uu} = 0, P_{ud} = 25 \ , and\ P_{dd} = 74.2771$$

By $P = \frac{1}{e^{r\Delta t}} \sum_{x=0}^{N} \binom{N}{x} \frac{e^{-\lambda}\lambda^{x_0}}{N^{x_0}} \left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N e^{\lambda}\left[Max\left[K - u^{x_0} d^{N-x_0} S_{(0)}, 0\right]\right]$ we obtained

$$C = Rs\ 23.64$$

By CRR binomial model $[4]$ for call price

$$C = e^{-r\Delta t}[\breve{p}^2 C_{uu} + 2\breve{p}(1-\breve{p})C_{ud} + (1-\breve{p})^2 C_{dd}] = Rs21.056$$

For Put price we have $fS(N) = max[K - u^x d^{N-x} S_0]$

$P_{uu} = 0, P_{ud} = 25$ and $P_{dd} = 74.2771$

By CRR binomial model $[4]$ for put price

$$e^{-r\Delta t}[\breve{p}^2 C_{uu} + 2\breve{p}(1-\breve{p})C_{ud} + (1-\breve{p})^2 C_{dd}] = Rs23.64$$

By CRR binomial model $[4]$ for call price

$$C = e^{-r\Delta t}[\breve{p}^2 C_{uu} + 2\breve{p}(1-\breve{p})C_{ud} + (1-\breve{p})^2 C_{dd}] = Rs21.056$$

For Put price we have $fS(N) = max[K - u^x d^{N-x} S_0]$

$P_{uu} = 0, P_{ud} = 25$ and $P_{dd} = 74.2771$

By CRR binomial model $[4]$ for put price

$$e^{-r\Delta t}[\breve{p}^2 C_{uu} + 2\breve{p}(1-\breve{p})C_{ud} + (1-\breve{p})^2 C_{dd}] = Rs23.64$$

Example 5.2

A non-dividend paying stock is currently selling at Rs100 with annual volatility 20% .Assume that the continuously compound risk free interest rate is 5% . Find the price of European call option on this stock with a strike price of Rs 80 and time to expiration 4 years .Using a two period CRR binomial option model and improved Poisson distribution model.

Solution

Given $S_0 = 100, K = 80, T = 4, r\ 0.05, \sigma = 0.2, \lambda = \frac{nA}{B} = 3.1706$ .Then a fixed up factor and down factor $u = e^{\sigma\sqrt{\Delta t}} = 1.3269$, $d = \frac{1}{u} = 0.7536$, $\frac{\breve{A}}{\breve{A}+\breve{B}} = 0.6132$ and $\frac{\breve{B}}{\breve{A}+\breve{B}} = 0.3868$

Now Payoff values $f\big(S(N)\big) = Max\ [u^x d^{N-x} S_0 - K, 0]$

$C_{uu} = 96.0664$, $C_{ud} = 20$ and $C_{dd} = 0$

By (14) for call price

$C = e^{-r\Delta t} \sum_{x=0}^{N} \frac{e^{-\lambda}\lambda^x e^{\lambda}}{N^x} \left(\frac{\breve{B}}{\breve{A}+\breve{B}}\right)^N Max\left[u^x d^{N-x} S_0 - K, 0\right] = 2C_0 e^{-3.1706} \times \frac{(3.1706)^0}{2^0} \times e^{3.1706} \times (0.3868)^2 \times 20 + 2C_1 \times e^{-3.1706} \times \frac{(3.1706)^1}{2^1} \times e^{3.1706} \times (0.3868)^2 \times 20 + 2C_2 \times e^{-3.1706} \times \frac{(3.1706)^2}{2^2} \times e^{3.1706} \times (0.3868)^2 \times 96.0664$

$C = e^{-0.1}[45.6090] = Rs\ 41.27$. Chandra[4] for call price. $C_{(0)} = e^{-r\Delta t}[\breve{p}^2 C_{uu} + 2\breve{p}(1-\breve{p})C_{ud} + (1-\breve{p})^2 C_{dd}]$

$$= e^{-0.1}[(0.3760)(96.0664) + (0.4744)(20.00) + 0]$$

=Rs 41.27.

To calculate the put price is left as an exercise for the reader.

6.DISCUSSION

From example 4.1-4.7 (table4.1-table4.7) ,it is clear that an improved Poisson approximate generalized binomial more better than Poisson distribution, when $\alpha = 0$ and $\alpha = 1$. From figure 4.1-figure 4.7

If the blue or green dotted line graph is close to X $axise$ it shows a good approximation. And a good approximation is obtained when $\alpha = 0$ and $\alpha = 1$. The improved Poisson distribution in this study approximates more better than the improved Poisson discussed in Teeraporlan et al $[8]$ .From example 5.1-5.2, it was found that CRR binomial model discussed in Chandra$[4]$ for two period model of non-dividend paying stock of a European (call and put) gives exactly the same numerical results with an improved Poisson distribution model , when equipped with financial terms, under the same conditions on its parameters.

7.CONCLUSION

The option pricing model for two periods of non- dividend paying was discussed in Chandra et al$[4]$, Oduro et al $[10]$ and Rutkowoki $[9]$. This work is much interested in Chandra et al$[4]$, Chandra et al $[4]$ gave a two periods model by matching CRR model with a multi-period binomial model. The applicability of the model in Chandra et al$[4]$ works in good agreement with the proposed model. An improved Poisson with mean $\tilde{\lambda} = \frac{nA}{B}$ was used to approximate a generalized binomial with parameters A, B, n and α . In view of the approximation, it shows that an improved Poisson distribution with the mean $\tilde{\lambda} = \frac{nA}{B}$ can approximates generalized binomial more better than Poisson when $\alpha = 0$ and 1 .And approximate sufficiently enough when n ,$\frac{B}{A+B}$ is larger and $\alpha = 0$.An improved Poisson in this study gives a better result when used to approximates binomial distribution then the improved Poisson discussed in Jaioun et al [8].

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APPENDIX

Proof of Case (i)

The form (15) can be expressed as of the form

$$P_x(x_0) = \binom{n}{x_0}\frac{[A(A-\alpha)\ldots.A-(x_0-1)\alpha][B(B-\alpha)\ldots B-(n-x_0-1)\alpha]}{(A+B)(A+B-\alpha)\ldots..A+B-(n-1)\alpha}$$

Where $x_0 \in \mathbb{N} \cup (0)$

$$P_x(x_0) = \binom{n}{x}\frac{[A(A-\alpha)\ldots.A-(\beta)\alpha][B(B-\alpha)\ldots B-(n-x_0-1)\alpha]}{(A+B)(A+B-\alpha)\ldots..A+B-(n-1)\alpha}$$

Where $\beta = \begin{cases} 0 \; if \; x_0 = 0 \\ (x_0-1) if \; x_0 = 1,2 \ldots.n \end{cases}$ and $\delta = \begin{cases} 0 \; if \; x_0 = 1,2 \ldots n \\ (n-x_0-1) if \; x_0 \end{cases}$

Now setting $\alpha = 0$

$$P_x(x_0) = \binom{n}{x}\frac{\overbrace{A(A)(A)\ldots\ldots..A}^{x_0 times}\overbrace{B(B)(B)\ldots\ldots.B}^{n-x_0 times}}{\underbrace{A+B(A+B)\ldots\ldots A+B}_{ntimes}} = \binom{n}{x}\frac{A^{x_0}B^{n-x_0}}{A+B^n} = \binom{n}{x}\frac{A^{x_0}B^{n-x_0}}{(A+B)^{x_0\circ}(A+B)^{n-x_0}}$$

$$P_x(x_0) = \binom{n}{x}\left(\frac{A}{A+B}\right)^{x_0}\left(\frac{B}{A+B}\right)^{n-x_0}$$

For case ii

With

$$P_x(x_0) = \binom{n}{x_0}\frac{[A(A-\alpha)\ldots.A-(x_0-1)\alpha][B(B-\alpha)\ldots B-(n-x_0-1)\alpha]}{(A+B)(A+B-\alpha)\ldots..A+B-(n-1)\alpha}$$

$$= \binom{n}{x_0}\frac{A\alpha(A/\alpha-1)\ldots\ldots.\alpha(A/\alpha-(x_0-1)(B\alpha(B/\alpha-1)\ldots\ldots.\alpha(B/\alpha-(n-x_0-1))}{(A+B)\alpha(A/\alpha+B/\alpha-1)\ldots\ldots\alpha(A/\alpha+B/\alpha-n-1)}$$

$$\binom{n}{x_0}\frac{-\alpha^{x}\left[{}^{A}\!/\!-\alpha\left({}^{A}\!/_{\alpha}+1\right)\ldots\ldots\ldots\,{}^{A}\!/\!-\alpha+(x_0-1)\right]-\alpha^{n-x_0}\left[{}^{B}\!/_{\alpha}\left({}^{B}\!/\!-\alpha+1\right)\ldots\,{}^{B}\!/\!-\alpha+(n-x_0-1)\right]}{-\alpha^{n}\left[\left({}^{A}\!/\!-\alpha+{}^{B}\!/\!-\alpha\right)\left({}^{A}\!/\!-\alpha+{}^{B}\!/\!-\alpha+1\right)\ldots\ldots\ldots\,{}^{A}\!/\!-\alpha+{}^{B}\!/\!-\alpha+(n-1)\right]}$$

Note $\frac{-\alpha^{x_0}-\alpha^{n-x_0}}{-\alpha^{n}}=\frac{\alpha^{x_0+n-x_0}}{-\alpha^{n}}=\frac{\alpha^{n}}{\alpha^{n}}=-\alpha^{n-n}=-\alpha^{0}=1$

$$=\binom{n}{x_0}\frac{{}^{A}\!/\!-\alpha\left({}^{A}\!/\!-\alpha\right)\ldots{}^{A}\!/\!-\alpha+(x_0-1)\left({}^{B}\!/\!-\alpha\left({}^{B}\!/\!-\alpha+1\right)\ldots{}^{B}\!/\!-\alpha+(n-x_0-1)\right)}{{}^{A}\!/\!-\alpha+{}^{B}\!/\!-\alpha\left({}^{A}\!/\!-\alpha+{}^{B}\!/\!-\alpha+1\right)\ldots{}^{A}\!/\!-\alpha+{}^{B}\!/\!-\alpha+(n-1)}$$

$$P_{\mathbb{Y}}(x_0)=\frac{\binom{{}^{A}\!/\!-\alpha+x_0-1}{x}\binom{{}^{B}\!/\!-\alpha+n-x_0-1}{n-x}}{\binom{{}^{A}\!/\!-\alpha+{}^{B}\!/\!-\alpha+n-1}{n}}$$

If

$$\sum_{j=1}^{3}\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}=1$$

Proof

Let

$\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_1=\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2,\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_2=2\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)\left(\frac{e^{\sigma\sqrt{\sigma}}-e^{r\Delta t}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)$ and $\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_3=\left(\frac{e^{\sigma\sqrt{\sigma}}-e^{r\Delta t}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2$

$$\sum_{j=1}^{3}\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}=\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2+2\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)\left(1-\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)+\left(1-\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2$$

$$=\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2+2\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)-2\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2+\left(1-\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2$$

$$=\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2-\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)^2+1=1$$

1. $E_{\frac{\breve{A}}{\breve{A}+\breve{B}}}(S_2)=e^{2r\Delta t}S_0$

2. $\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)_j>0$ where $j=1,2,..n$

Proof (1): For $S(2)$ implies $t=2$ so that

$$\left(\frac{\breve{A}}{\breve{A}+\breve{B}}+1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2=\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2+2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)+\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2.$$

By defining

$\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_1=\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2,\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)_2=2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)$ and $\left(\frac{\tilde{A}}{\breve{A}+\breve{B}}\right)_3=\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2,$

and

$$E_{\frac{\breve{A}}{\breve{A}+\breve{B}}}S(2)=\left[\left(\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2u^2S_0+2\frac{\breve{A}}{\breve{A}+\breve{B}}\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}udS_0\right)+\left(1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right)^2dS_0\right].$$

Therefore

$$S_0\left[\frac{\breve{A}}{\breve{A}+\breve{B}}+1-\frac{\breve{A}}{\breve{A}+\breve{B}}\right]^2=S_0\left[\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)u+\left(\frac{e^{\sigma\sqrt{\Delta t}}-e^{r\Delta t}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)d\right]^2=S_0\left[\frac{e^{r\Delta t}u-e^{-\sigma\sqrt{\Delta t}}u+e^{\sigma\sqrt{\Delta t}}d-e^{r\Delta t}d}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right]=S_0e^{2r\Delta t}$$

Proof(2): Since $e^{\sigma\sqrt{\Delta t}}>e^{r\Delta t}>e^{-\sigma\sqrt{\Delta t}}>0$, it follows that

$\left(\frac{e^{r\Delta t}-e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}}\right)_i>0\forall\ i=1,2..n.$