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Fractional Calculus of Generalized P-K Wright Function

K.S. Gehlot, Chena Ram and Anita

Department of Mathematics and Statistics, Jai Narayan Vyas University Jodhpur-342005,India.

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ABSTRACT

In this paper we studied fractional calculus properties. Riemann-Liouville fractional integral and derivative of the Generalized p-k Wright function. Certain particular cases of the derived results are considered and indicated to further reduce to some known results.

1. Introduction

The two parameter pochhammer symbol is recently introduce by [6], equation 2.1, in the form,

1.1 Definition

Let $x \in C; k, p \in R^{+} - \{0\}$ and $Re(x) > 0, n \in N,$ the p - k Pochhammer Symbol (i.e. Two Parameter Pochhammer Symbol), ${}_p(x)_{n,k}$ is given by,

$$ {}_p(x)_{n,k} = (\frac{xp}{k})(\frac{xp}{k}+p)(\frac{xp}{k}+2p).........(\frac{xp}{k}+(n-1)p). \tag{1} $$

And the Two Parameter Gamma Function is given by [6], equation 2.6, 2.7 and 2.14,

1.2 Definition

For $x \in C/kZ^{-}; k, p \in R^{+} - \{0\}$ and $Re(x) > 0, n \in N,$ the p - k Gamma Function (i.e. Two Parameter Gamma Function), ${}_p\Gamma_k(x)$ as,

$$ {}_p\Gamma_k(x) = \frac{1}{k} \lim_{n\to\infty} \frac{n!\, p^{n+1} (np)^{\frac{x}{k}}}{{}_p(x)_{n+1,k}}. \tag{2} $$

or,

$$ {}_p\Gamma_k(x) = \frac{1}{k} \lim_{n\to\infty} \frac{n!\, p^{n+1} (np)^{\frac{x}{k}-1}}{{}_p(x)_{n,k}}. \tag{3} $$

The integral representation of p - k Gamma Function is given by,

$$ {}_p\Gamma_k(x) = \int_0^{\infty} e^{-\frac{t^k}{p}} t^{x-1} dt. \tag{4} $$

The Generalized p - k Wright function [5], denoted by, ${}^{p}_{r}\psi^{k}_{s}[(a_i, \alpha_i)_{1,r}; (b_j, \beta_j)_{1,s}; z]$ and defined as,

$$ {}^{p}_{r}\psi^{k}_{s}(z) = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i + \alpha_i n)}{\prod_{j=1}^{s} {}_p\Gamma_k(b_j + \beta_j n)} \frac{z^n}{n!}. \tag{5} $$

Tele:
E-mail address: drksgehlot@rediffmail.com

Where ${}_p\Gamma_k(x)$ is the two parameter Gamma function given by equation (1.3).

The left and right hand sided fractional integral operators are defined for $\alpha > 0$ and $a = 0$ as (samko et al [11]),

$$(I_{0+}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt; \tag{6}$$

and

$$(I_{-}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^{\infty} \frac{f(t)}{(t-x)^{1-\alpha}} dt; \tag{7}$$

and corresponding fractional differentiation operators defined as,

$$(D_{0+}^{\alpha} f)(x) = (\frac{d}{dx})^{[Re(\alpha)]+1} (I_{0+}^{1-\alpha+[Re(\alpha)]} f)(x)$$

$$= \frac{1}{\Gamma(1-\alpha+[Re(\alpha)])} (\frac{d}{dx})^{[Re(\alpha)]+1} \int_0^x \frac{f(t)}{(x-t)^{\alpha-[Re(\alpha)]}} dt. \tag{8}$$

and

$$(D_{-}^{\alpha} f)(x) = (-\frac{d}{dx})^{[Re(\alpha)]+1} (I_{-}^{1-\alpha+[Re(\alpha)]} f)(x)$$

$$= \frac{1}{\Gamma(1-\alpha+[Re(\alpha)])} (-\frac{d}{dx})^{[Re(\alpha)]+1} \int_x^{\infty} \frac{f(t)}{(t-x)^{\alpha-[Re(\alpha)]}} dt. \tag{9}$$

and

The next assertion is well known (Samko et al [11]),

For $\alpha \in C$ $(Re(\alpha) > 0)$ and $\gamma \in C$;

If $Re(\gamma) > 0$, then,

$$(I_{0+}^{\alpha} t^{\gamma-1})(x) = \frac{\Gamma(\gamma)}{\Gamma(\alpha+\gamma)} x^{\alpha+\gamma-1}. \tag{10}$$

If $Re(\gamma) > Re(\alpha) > 0$, then,

$$(I_{-}^{\alpha} t^{-\gamma})(x) = \frac{\Gamma(\gamma-\alpha)}{\Gamma(\gamma)} x^{\alpha-\gamma}. \tag{11}$$

2. Fractional Integration of the Generalized p-k Wright Function

In this section, we establishes fractional integration of the Generalized p-k Wright Function.

Theorem 1 Let $\alpha, \gamma, \beta \in C$ such that $Re(\alpha) > 0$, $Re(\gamma) > 0$ $Re(\beta) > 0$; $a \in C$, $\mu > 0$ then for $\Delta > -1$ fractional integration I_{0+}^{α} of Generalized p-k Wright function is given by,

$$(I_{0+}^{\alpha} (t^{\frac{\gamma}{k}-1})\, {}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{\mu}{k}}])(x)$$

$$= x^{\frac{\gamma}{k}+\alpha-1}\ p^{\alpha}$$

$${}_{r+1}^{p}\psi_{s+1}^{k}[(a_i,\alpha_i)_{1,r},(\gamma,\mu);(b_j,\beta_j)_{1,s},(\gamma+\alpha k,\mu);(ax^{\frac{\mu}{k}})]. \tag{12}$$

Proof Consider the right-hand side of (12) and using the definition (5),

we have,

$$A \equiv (I_{0+}^{\alpha} (t^{\frac{\gamma}{k}-1})\, {}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};(at^{\frac{\mu}{k}})](x)$$

$$A \equiv (I_{0+}^{\alpha} (t^{\frac{\gamma}{k}-1}) \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{(at^{\frac{\mu}{k}})^n}{n!})(x)$$

Using term by term integration of the series in the right-hand side of above equation and using (9),

we obtain,

$$A \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{(a)^n}{n!} (I_{0+}^{\alpha}(t^{\frac{\mu n}{k}+\frac{\gamma}{k}-1}))(x)$$

$$A \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{(a)^n}{n!} \frac{\Gamma(\frac{\gamma+\mu n}{k})}{\Gamma(\frac{\gamma+\mu n}{k}+\alpha)} x^{\frac{\gamma}{k}+\frac{\mu n}{k}+\alpha-1}$$

. Using the relation between Generalized p-k gamma function and classical gamma function [6], we have,

$$A \equiv x^{\frac{\gamma}{k}+\alpha-1} p^{\alpha} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{{}_p\Gamma_k(\gamma+\mu n)}{{}_p\Gamma_k(\gamma+\mu n+\alpha k)} \frac{(ax^{\frac{\mu}{k}})^n}{n!}$$

$$(I_{0+}^{\alpha}(t^{\frac{\gamma}{k}-1})\,{}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{\mu}{k}}])(x)$$
$$= x^{\frac{\gamma}{k}+\alpha-1}\; p^{\alpha}$$
$${}_{r+1}^{p}\psi_{s+1}^{k}[(a_i,\alpha_i)_{1,r},(\gamma,\mu);(b_j,\beta_j)_{1,s},(\gamma+\alpha k,\mu);(ax^{\frac{\mu}{k}})]. \tag{13}$$

Hence Proved.

Particular case: If we put $p=k$ in equation (13), then,

$${}_r^k\psi_s^k = x^{\frac{\gamma}{k}+\alpha-1} k^{\alpha}$$

$${}_{r+1}^{k}\psi_{s+1}^{k}[(a_i,\alpha_i)_{1,r},(\gamma,\mu);(b_j,\beta_j)_{1,s},(\gamma+\alpha k,\mu);(ax^{\frac{\mu}{k}})]$$

Which is known result given by [7].

Theorem 2 Let $\alpha,\gamma,\beta \in C$ such that $Re(\alpha)>0$, $Re(\gamma)>0$ $Re(\beta)>0$; $a \in C$, $\mu>0$ then for $\Delta>-1$ fractional integration I_{-}^{α} of Generalized p-k Wright function is given by,

$$(I_{-}^{\alpha}(t^{\frac{-\gamma}{k}})\,{}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{-\mu}{k}}])(x)$$
$$= x^{\alpha-\frac{\gamma}{k}} p^{\alpha}\,{}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{-\mu}{k}}]. \tag{14}$$

Proof Consider the right-hand side of (14) and using the definition (5), we have,

$$B \equiv (I_{-}^{\alpha}(t^{\frac{-\gamma}{k}})\,{}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{-\mu}{k}}]))(x)$$

$$= (I_{-}^{\alpha}(t^{\frac{-\gamma}{k}})\sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{(at^{\frac{-\mu}{k}})^n}{n!})(x)$$

. Using term by term integration of the series in the right-hand side of above equation and using (11), we obtain,

$$B \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{(a)^n}{n!} (I_{-}^{\alpha}(t^{-\frac{\gamma}{k}-\frac{\mu n}{k}})(x)$$

$$B \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{(a)^n}{n!} \frac{\Gamma(\frac{\gamma+\mu n}{k}-\alpha)}{\Gamma(\frac{\gamma+\mu n}{k})} x^{\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}}$$

. Using the relation between p-k gamma function and classical gamma function [6], we have,

$$B \equiv x^{\alpha-\frac{\gamma}{k}} p^{\alpha} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)}{{}_p\Gamma_k(b_j+\beta_j n)} \frac{{}_p\Gamma_k(\gamma+\mu n-\alpha k)}{{}_p\Gamma_k(\gamma+\mu n)} \frac{(ax^{\frac{-\mu}{k}})^n}{n!}$$

$$(I_{-}^{\alpha}(t^{-\frac{\gamma}{k}})\,{}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{-\mu}{k}}])(x)$$
$$= x^{\alpha-\frac{\gamma}{k}} p^{\alpha}\,{}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{-\mu}{k}}]. \tag{15}$$

Hence Proved.

3. Fractional Differentiation of Generalized p-k Wright function

This section deals with fractional differentiation of the Generalized p-k Wright function.

Theorem 3 Let $\alpha,\gamma,\beta \in C$ such that $Re(\alpha)>0$, $Re(\gamma)>0$ $Re(\beta)>0$; $a\in C$, $\mu>0$ then for $\Delta>-1$ fractional differentiation D^{α}_{0+} of Generalized p-k Wright function is given by,

$$(D^{\alpha}_{0+}(t^{\frac{\gamma}{k}-1}\, {}^{p}_{r}\psi^{k}_{s}[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{\mu}{k}}])(x))$$
$$= x^{\frac{\gamma}{k}-\alpha-1}\; p^{-\alpha}$$
$${}^{p}_{r+1}\psi^{k}_{s+1}[(a_i,\alpha_i)_{1,r},(\gamma,\mu);(b_j,\beta_j)_{1,s},(\gamma-\alpha k,\mu);ax^{\frac{\mu}{k}}]. \quad (16)$$

Proof Using left side of (16), we have,

$$C\equiv(D^{\alpha}_{0+}(t^{\frac{\gamma}{k}-1}\, {}^{p}_{r}\psi^{k}_{s}[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{\mu}{k}}])(x))$$
$$\equiv(\frac{d}{dx})^{r}(I^{r-\alpha}_{0+}(t^{\frac{\gamma}{k}-1}\, {}^{p}_{r}\psi^{k}_{s}[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{\mu}{k}}]))(x)$$

. Using the result (12), we have,

$$\equiv(\frac{d}{dx})^{r}(x^{\frac{\gamma}{k}+r-\alpha-1}\, p^{r-\alpha}\, {}^{p}_{r+1}\psi^{k}_{s+1}[(a_i,\alpha_i)_{1,r},(\gamma,\mu);(b_j,\beta_j)_{1,s},(\gamma+(r-\alpha)k,\mu);ax^{\frac{\mu}{k}}])$$

.Using the equation (5), we obtain,

$$\equiv\sum_{n=0}^{\infty}p^{r-\alpha}\frac{\prod_{i=1}^{r}{}_{p}\Gamma_k(a_i+\alpha_i n)\,{}_{p}\Gamma_k(\gamma+\mu n)}{\prod_{j=1}^{s}{}_{p}\Gamma_k(b_j+\beta_j n)\,{}_{p}\Gamma_k(\gamma+(r-\alpha)k+\mu n)}\frac{a^n}{n!}(\frac{d}{dx})^{r}(x^{\frac{\gamma}{k}+\frac{\mu n}{k}+r-\alpha-1})$$

$$\equiv\sum_{n=0}^{\infty}p^{r-\alpha}\frac{\prod_{i=1}^{r}{}_{p}\Gamma_k(a_i+\alpha_i n)\,{}_{p}\Gamma_k(\gamma+\mu n)}{\prod_{j=1}^{s}{}_{p}\Gamma_k(b_j+\beta_j n)\,{}_{p}\Gamma_k(\gamma+(r-\alpha)k+\mu n)}\frac{a^n}{n!}$$

$$\times\frac{\Gamma(\frac{\gamma}{k}+\frac{\mu n}{k}+r-\alpha)}{\Gamma(\frac{\gamma}{k}+\frac{\mu n}{k}-\alpha)}(x^{\frac{\gamma}{k}+\frac{\mu n}{k}-\alpha-1})$$

$$\equiv p^{-\alpha}x^{\frac{\gamma}{k}-\alpha-1}\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p}\Gamma_k(a_i+\alpha_i n)\,{}_{p}\Gamma_k(\gamma+\mu n)}{\prod_{j=1}^{s}{}_{p}\Gamma_k(b_j+\beta_j n)\,{}_{p}\Gamma_k(\gamma-\alpha k+\mu n)}\frac{(ax^{\frac{\mu}{k}})^n}{n!}$$

$$\equiv x^{\frac{\gamma}{k}-\alpha-1}\; p^{-\alpha}$$
$${}^{p}_{r+1}\psi^{k}_{s+1}[(a_i,\alpha_i)_{1,r},(\gamma,\mu);(b_j,\beta_j)_{1,s},(\gamma-\alpha k,\mu);ax^{\frac{\mu}{k}}]. \quad (17)$$

Hence Proved.

Theorem 4 Let $\alpha,\gamma,\beta\in C$ such that $Re(\alpha)>0$, $Re(\gamma)\ >[Re(\alpha)]+1-Re(\alpha)$ $Re(\beta)>0$; $a\in C$, $\mu>0$ then for $\Delta>-1$ fractional differentiation D^{α}_{-} of Generalized p-k Wright function is given by

$$(D^{\alpha}_{-}(t^{\frac{-\gamma}{k}}\, {}^{p}_{r}\psi^{k}_{s}[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{\frac{-\mu}{k}}])(x))$$
$$= x^{\frac{-\gamma}{k}-\alpha}$$
$$p^{-\alpha}\, {}^{p}_{r+1}\psi^{k}_{s+1}[(a_i,\alpha_i)_{1,r},(\gamma+\alpha k,\mu);(b_j,\beta_j)_{1,s},(\gamma,\mu);ax^{-\frac{\mu}{k}}] \quad (18)$$

Proof Using left side of (18), we have,

$$D \equiv (D_-^{\alpha}(t^{-\frac{\gamma}{k}}\, {}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{-\frac{\mu}{k}}])(x))$$

$$\equiv (-\frac{d}{dx})^r (I_-^{r-\alpha}(t^{-\frac{\gamma}{k}}\, {}_r^p\psi_s^k[(a_i,\alpha_i)_{1,r};(b_j,\beta_j)_{1,s};at^{-\frac{\mu}{k}}]))(x)$$

.Using the result (14), we have,

$$\equiv (-\frac{d}{dx})^r (x^{-\frac{\gamma}{k}+r-\alpha} p^{r-\alpha}\, {}_{r+1}^{\ p}\psi_{s+1}^k[(a_i,\alpha_i)_{1,r},(\gamma-(r-\alpha)k,\mu);(b_j,\beta_j)_{1,s},(\gamma,\mu);ax^{-\frac{\mu}{k}}])$$

. Using the equation (5), we obtain,

$$\equiv \sum_{n=0}^{\infty} p^{r-\alpha} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)\, {}_p\Gamma_k(\gamma+\mu n-(r-\alpha)k)}{\prod_{j=1}^{s} {}_p\Gamma_k(b_j+\beta_j n)\, {}_p\Gamma_k(\gamma+\mu n)} \frac{a^n}{n!} (-\frac{d}{dx})^r (x^{-\frac{\gamma}{k}-\frac{\mu n}{k}+r-\alpha})$$

$$\equiv \sum_{n=0}^{\infty} p^{r-\alpha} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)\, {}_p\Gamma_k(\gamma+\mu n-(r-\alpha)k)}{\prod_{j=1}^{s} {}_p\Gamma_k(b_j+\beta_j n)\, {}_p\Gamma_k(\gamma+\mu n)}$$

$$\times \frac{a^n}{n!} \frac{\Gamma(-\frac{\gamma}{k}-\frac{\mu n}{k}+r-\alpha+1)}{\Gamma(-\frac{\gamma}{k}-\frac{\mu n}{k}-\alpha+1)} (-1)^r (x^{-\frac{\gamma}{k}-\frac{\mu n}{k}-\alpha})$$

$$\equiv \sum_{n=0}^{\infty} p^{r-\alpha} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)\, p^{\frac{\gamma-(r-\alpha)k+\mu n}{k}} \Gamma(\frac{\gamma-(r-\alpha)k+\mu n}{k})}{\prod_{j=1}^{s} {}_p\Gamma_k(b_j+\beta_j n)\, p^{\frac{\gamma+\mu n}{k}} \Gamma(\frac{\gamma+\mu n}{k})}$$

$$\times \frac{a^n}{n!} \frac{\Gamma(\frac{k+(r-\alpha)k-\gamma-\mu n}{k})}{\Gamma\frac{k-\alpha k-\gamma-\mu n}{k}} (-1)^r x^{-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}}. \tag{19}$$

since,

$$\frac{1}{\Gamma(1-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k})} = \frac{\Gamma(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k}) sin(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k})\pi}{\pi}. \tag{20}$$

and

$$\Gamma(\alpha-r+\frac{\gamma}{k}+\frac{\mu n}{k}) \quad \Gamma(1-(\alpha-r+\frac{\gamma}{k}+\frac{\mu n}{k})) = \frac{\pi}{sin(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k})\pi - r\pi}$$

$$= \frac{\pi}{sin(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k})\pi cos r\pi} = \frac{\pi(-1)^r}{sin(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k})\pi}. \tag{21}$$

Using equation (20) and (21) in equation (19), we have,

$$\equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)\Gamma(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k})}{\prod_{j=1}^{s} {}_p\Gamma_k(b_j+\beta_j n)\Gamma(\frac{\gamma}{k}+\frac{\mu n}{k})} \frac{a^n}{n!} x^{-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}}$$

Using relation between p-k gamma function and classical gamma function, we have,

$$= x^{-\frac{\gamma}{k}-\alpha}\ p^{-\alpha} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_p\Gamma_k(a_i+\alpha_i n)\, {}_p\Gamma_k(\alpha k+\gamma+\mu n)}{\prod_{j=1}^{s} {}_p\Gamma_k(b_j+\beta_j n)\, {}_p\Gamma_k(\gamma+\mu n)} \frac{(ax^{-\frac{\mu}{k}})^n}{n!}$$

$$= x^{-\frac{\gamma}{k}-\alpha}\ p^{-\alpha}\ {}^{p}_{r+1}\psi^{k}_{s+1}[(a_i,\alpha_i)_{1,r},(\gamma+\alpha k,\mu);(b_j,\beta_j)_{1,s},(\gamma,\mu);ax^{-\frac{\mu}{k}}]. \tag{22}$$

Hence Proved.

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