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On Two Stage Optional Randomized Response Model

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ABSTRACT

In this paper, we propose a new optional randomized response model based on Gupta and Shabbir (2007) two-stage randomized response model which unbound the assumption that the mean of the scrambling variable S is 'unity' [i.e. $\theta = 1$]. We derive the estimator of the sensitive variable, mean, and show that our method is more efficient than other randomized response models suggested by Eichhorn and Hayre (1983) and Gupta and shabbir (2007) estimators. A numerical illustration is given in support of the present study.

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1. Introduction

Asking questions about sensitive and stigmatizing characteristics in surveys of human populations is not an easy matter. Gathering information on issues like sexual orientation, drunkenness, HIV positivity, experience in induced abortion, maltreatment of spouse, habits of wilful tax evasion, bribery, cheating, and fraud by means of direct questions and conventional survey methodology is likely to produce large non-sampling errors particularly due to nonresponse. People are not willing to provide information which might be considered as incriminating and stigmatizing. In cases they agreed to participate in such a survey, it is very reasonable to assume that many of them give false answers and provide misleading information.

Warner (1965) was the first to introduce an ingenuous procedure to estimate the incidence of attributes of sensitive nature through a randomization device. A rich growth of literature on randomized response techniques can be found in Tracy and Mangat (1996), Zou(1997), Singh and Joarder (1997), Bhargava and Singh (2001,2002), Singh and Mathur (2003), Gjestvang and Singh (2006), Singh and Tarray (2014), Tarray et al (2015), Tarray and Singh (2017).

Eichhorn and Hayre (1983) proposed a scrambled randomized response method for estimating the mean μ_x and the variance σ_x^2 of the sensitive quantitative variable, say X. Following them, each respondent selected in the sample is instructed to use a randomization device and generate a random number, say S, from some pre-assigned distribution. The distribution of the scrambling variable S is assumed to be known. The mean θ and variance σ_s^2 of the scrambling variable S are known. The ith respondent selected in the sample of size n, drawn by using simple random sampling with replacement (SRSWR), is requested to report the value $Z = \frac{SX_i}{\theta}$ as a scrambled response on the sensitive variable X. Eichhorn and Hayre (1983) suggested an unbiased estimator of the population mean μ_x of X, as

$$\hat{\mu}_{x(EH)} = \frac{1}{n}\sum_{i=1}^{n} Z_i \quad (1)$$

with variance

$$Var\left(\hat{\mu}_{x(EH)}\right) = (1/n)\left[\sigma_x^2 + \frac{\sigma_s^2}{\theta^2}\left(\sigma_x^2 + \mu_x^2\right)\right], \quad (2)$$

In some situations, a section of respondents feels that the subject of inquiry is not confidential enough and would like to respond directly. To accommodate such respondents, optional randomized response technique (ORRT) has been proposed by Chaudhuri and Mukherjee (1988), Gupta et al. (2002), Arnab (2004), Huang (2010), Gupta et al (2010, 2013) and Tarray and Singh (2017) among others.

Ryu et al (2006) attempted to combine the models introduced by Mangat and Singh (1990) and by Gupta et al (2002) to introduce an estimator of the mean of a quantitative sensitive variable and show that their estimator of the mean is more efficient than that of the estimator by Gupta et al (2002). However, the model by Ruy et al (2006) is not an optional RRT model and does not estimate the sensitivity level of the sensitive question, unlike Gupta et al (2002) whose model estimated simultaneously both mean and sensitivity of the sensitive variable.

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In this paper we discussed Gupta and Shabbir (2007) randomized response model and introducing an optional two-stage randomized response model based on it

- **Gupta and Shabbir (2007) Randomized Response Model**

Gupta and Shabbir (2007) randomized response model is based on Ryu et al (2006) model which is given as

$$Z=\begin{cases}X & \text{with the probability } T+(1-T)(1-w)\\ SX & \text{with the probaibility } (1-T)w\end{cases} \quad (3)$$

Gupta and Shabbir (2007) model is not different than the partial randomized respond model for quantitative response, discussed by Gupta and Thorntorn (2003), if *w* is assumed to be known, except that the proportion of the respondents providing truthful responses has been increased from *T* to *T+(1-T)(1-w).*

A true two-stage optional randomized response model was given as:

Stage 1: A randomly selected, pre-determined proportion *(T)* of the respondents, respond truthfully. Other respondent are instructed to go to Stage 2

Stage 2: These respondent are asked to provide a randomized response *SX* if they think the question to be sensitive. Otherwise they are asked to report the true response *X*.

The interviewer does not know in which stage and how the response is provided.

The reported response under this model was given by

$$Z=\{X^V\}\{XS^U\}^{1-V} \quad (4)$$

where $U \sim Bernoulli(w), V \sim Bernoulli(T)$

They assumed that the $E(S)=1$ and X, S, U, V are mutually independent.

An unbiased estimator for population mean $\boldsymbol{\mu_x}$ of the sensitive characteristics *X* as

$$\hat{\mu}_{x(G)}=\frac{\sum Z_i}{n} \quad (5)$$

whose variance is given by

$$V(\hat{\mu}_{x(G)})=\frac{1}{n}\left[\sigma_x^2+(1-T)w\sigma_S^2(\sigma_x^2+\mu_x^2)\right] \quad (6)$$

Gupta and Shabbir (2007) further suggested an estimator of the sensitivity level *w* as

$$\hat{w}_G=\frac{\frac{1}{n}\sum_{i=1}^{n}ln(Z_i)-ln\left(\frac{1}{n}\sum_{i=1}^{n}Z_i\right)+\frac{\hat{V}(Z)}{2\hat{\mu}_Z^2}}{(1-T)\delta} \quad (7)$$

- **Logical Reason Behind Assumption of $\theta=1$ in Gupta and Shabbir (2007) Optional Randomized Response Model**

In Gupta and Shabbir (2007) procedure it is assumed that the value of the mean $\boldsymbol{\theta}$ of the scrambled variable *S* is unity (i.e. $E(S)=1$). Thereby meaning is that the optional randomized response model due to Gupta and Shabbir (2007) will work for $E(S)=1$.

In Gupta and Shabbir (2007) model, the expected value of the observed response *Z* is given by

$$\begin{aligned}E(Z)&=\mu_x\{\theta w+(1-w)\}(1-T)+\mu_x T\\ &=\mu_x[\theta w(1-T)+(1-w)(1-T)+T]\\ &=\mu_x[w(1-T)\{\theta-1\}+1-T+T]\\ &=\mu_x[w(1-T)(\theta-1)+1]\end{aligned}$$

To get unbiased estimator of $\boldsymbol{\mu_x}$ the condition E(S)= $\boldsymbol{\theta}$ =1is must. That is why Gupta and Shabbir (2007) assumed $E(S)=1$ in their model that was the main hitch of Gupta and Shabbir (2007) model. To overcome this problem we have suggested an alternative to Gupta and Shabbir (2007) model in Section 2 which enhanced the result for different values of $E(S)=\theta$.

2. Suggested Procedure

It is known that the distribution of the scrambling variable *S* is known (i.e. the mean $\boldsymbol{\theta}$ and variance $\boldsymbol{\sigma_s^2}$ of the scrambling variable *S* are known). Thus using the known value of mean $\boldsymbol{\theta}$, if *w* is assumed known, except that the proportion of respondents providing truthful responses has been increased from *T* to *T+(1-T)(1-w).* We have suggested a modified, optional randomized procedure, we assume that a sample of size *n* is selected by SRSWR. The method is described as follows:

Stage 1: A randomly selected, pre-determined proportion (T) of the respondents, respond truthfully. Other respondent are instructed to go to stage 2.

Stage 2: These respondents are asked to provide a randomized response $\boldsymbol{X=\frac{x}{\theta}}$ if they think the question to be sensitive. Otherwise they are asked to report the true response *X*.

The interviewer does not know in which stage and how the response is provided.

Here we have assumed that both *S* and *X* are positive valued random variables. Thus the proposed optional randomized response model can be written as

$$Z=(X^V)\left\{X\left(\frac{S}{\theta}\right)^U\right\}^{1-V} \quad (8)$$

where *V = 1 or 0* according to as the response is scrambled or not.

Taking expectation of both sides of (8) we have

$$E(Z)=E\left[X^{V}\left\{X\left(\frac{S}{\theta}\right)^{U}\right\}^{1-V}\right]P(V=1)+E\left[X^{V}\left\{X\left(\frac{S}{\theta}\right)^{U}\right\}^{1-V}\right]P(V=0)$$

$$=E(X)P(V=1)+E(X)\left\{\frac{E(S)}{\theta}P(U=1)+P(U=0)\right\}P(V=0)$$

$$=E(X)\left[P(V=1)+\left\{\frac{\theta}{\theta}w+(1-w)\right\}(1-T)\right]$$

$$=E(X)[T+(w+1-w)(1-T)]$$

$$=E(X)[T+(1-T)]$$

$$=E(X)$$

$$=\mu_x .$$

Thus an unbiased estimator of the population mean $\boldsymbol{\mu_x}$ is given by

$$\hat{\mu}_{x(HG)}=\frac{1}{n}\sum_{i=1}^{n}Z_i=\bar{Z} \tag{9}$$

Theorem 1: The variance of the proposed estimator $\hat{\boldsymbol{\mu}}_{x(HG)}$ is given by

$$V(\hat{\mu}_{x(HG)})=\frac{1}{n}\left[\sigma_x^2+\frac{\sigma_s^2}{\theta}(\sigma_x^2+\mu_x^2)(1-T)w\right] \tag{10}$$

Proof: We have

$$V(\hat{\mu}_{x(HG)})=V\left(\frac{1}{n}\sum_{i=1}^{n}Z_i\right)$$

$$=\frac{V(Z_i)}{n} \tag{11}$$

Now,

$$V(Z_i)=E(Z_i^2)-(E(Z_i))^2$$

$$=E(Z_i^2)-\mu_x^2 \tag{12}$$

Here

$$E(Z^2)=E\left[X^{V}\left\{X\left(\frac{S}{\theta}\right)^{U}\right\}^{1-V}\right]^2P(V=1)+E\left[X^{V}\left\{X\left(\frac{S}{\theta}\right)^{U}\right\}^{1-V}\right]^2P(V=0)$$

$$=E(X^2)P(V=1)+E\left[X\left(\frac{S}{\theta}\right)^{U}\right]^2P(V=0)$$

$$=E(X^2)P(V=1)+E(X^2)\left\{E\left(\frac{S}{\theta}\right)^{2U}P(U=1)+E\left(\frac{S}{\theta}\right)^{2U}P(U=0)\right\}P(V=0)$$

$$=E(X^2)\left[P(V=1)+\left\{E\left(\frac{S}{\theta}\right)^{2U}P(U=1)+E\left(\frac{S}{\theta}\right)^{2U}P(U=0)\right\}P(V=0)\right]$$

$$=E(X^2)\left[P(V=1)+\left\{\frac{1}{\theta^2}E(S^2)P(U=1)+P(U=0)\right\}P(V=0)\right]$$

$$=E(X^2)\left[P(V=1)+\left\{\frac{1}{\theta^2}(\sigma_s^2+\theta^2)w+(1-w)\right\}P(V=0)\right]$$

$$=E(X^2)\left[T+\left\{w\left(\frac{\sigma_s^2}{\theta^2}+1\right)+(1-w)\right\}(1-T)\right]$$

$$=E(X^2)\left[T+\left\{w\left(\frac{\sigma_s^2}{\theta^2}\right)+1\right\}(1-T)\right]$$

$$=(\sigma_x^2+\mu_x^2)\left[\left(\frac{\sigma_s^2}{\theta^2}\right)w(1-T)+1\right]$$

$$=\sigma_x^2+\mu_x^2+\frac{\sigma_s^2}{\theta^2}(\sigma_x^2+\mu_x^2)(1-T)w \tag{13}$$

From (12) and (13) we have

$$\sigma_Z^2=V(Z_i)=\sigma_x^2+\frac{\sigma_s^2}{\theta^2}(\sigma_x^2+\mu_x^2)(1-T)w \tag{14}$$

Thus from (11) and (14) we get

$$V\left(\hat{\mu}_{x(HG)}\right)=\frac{1}{n}\left[\sigma_x^2+\frac{\sigma_s^2}{\theta^2}\left(\sigma_x^2+\mu_x^2\right)(1-T)w\right] \tag{15}$$

which proves the theorem.

The variance σ_x^2 of the sensitive variable x under the proposed randomization response procedure is obtained as follows:

$$V\left(\hat{\mu}_{x(HG)}\right)=\frac{\sigma_Z^2}{n}=\frac{1}{n}\left[\sigma_x^2+\frac{\sigma_s^2}{\theta^2}\left(\sigma_x^2+\mu_x^2\right)(1-T)w\right]$$

$$\sigma_Z^2=\sigma_x^2\left(1+\frac{\sigma_s^2}{\theta^2}(1-T)w\right)+\mu_x^2\frac{\sigma_s^2}{\theta}(1-T)w$$

or

$$\sigma_x^2=\frac{\sigma_Z^2-\mu_x^2\frac{\sigma_s^2}{\theta^2}(1-T)w}{\left(1+\frac{\sigma_s^2}{\theta^2}(1-T)w\right)} \tag{16}$$

where σ_Z^2 is defined in (14).

An estimator for $V\left(\hat{\mu}_{x(HG)}\right)$ is given by

$$\hat{V}\left(\hat{\mu}_{x(HG)}\right)=\frac{1}{n}\left[s_x^2+\frac{\sigma_s^2}{\theta^2}\left(s_x^2+\hat{\mu}_{x(HG)}^2\right)\left(1-\hat{T}\right)\hat{w}\right],$$

where

$$s_x^2=\frac{\left(s_z^2-\hat{w}\left(1-\hat{T}\right)\frac{\sigma_s^2}{\theta^2}\hat{\mu}_{x(HG)}^2\right)}{\left(1+\hat{w}\frac{\sigma_s^2}{\theta^2}\left(1-\hat{T}\right)\right)} \tag{17}$$

with $s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(Z_i-\bar{Z}\right)^2$ is an unbiased estimator of σ_Z^2.

The next section has been devoted to estimating *w* based on the information gathered through proposed randomized response procedure.

3. Estimation of *w*

Taking logarithm on both sides of (8) we have

$$log(Z)=V\,log(X)+(1-V)\{log(X)+U\,log(S/\theta)\}. \tag{18}$$

Taking expectation of both sides of (18) we have

$$\begin{aligned}E(log(Z))&=E(log(X))+E(1-V)E(U)E\{log(S)-log(\theta)\}.\\&=E(log(X))+(1-T)w(\delta-log\,\theta)\end{aligned} \tag{19}$$

Replacing *X* by $\hat{\mu}_{x(HG)}$ in (19), we get

$$\begin{aligned}E(log(Z))&\cong E\left(log\left(\hat{\mu}_{x(HG)}\right)\right)+(1-T)wE\{\delta-log(\theta)\}\\&=E\left(log\left(\bar{Z}\right)\right)+(1-T)wE\{\delta-log(\theta)\}\end{aligned} \tag{20}$$

$$w=\frac{E(log(Z))-E\left(log\left(\bar{Z}\right)\right)}{(1-T)(\delta-log\,\theta)} \tag{21}$$

Estimating $E(log\,Z)$ by $\frac{1}{n}\sum_{i=1}^{n}log\,Z_i$ and $E\left[log\left(\bar{Z}\right)\right]$ by $\frac{1}{n}\sum_{i=1}^{n}log\left(\bar{Z}\right)=log\left(\bar{Z}\right)$ in (21) leads to the estimator of *w* given by

$$\hat{w}=\frac{\frac{1}{n}\sum_{i=1}^{n}log(Z_i)-log\left(\frac{1}{n}\sum_{i=1}^{n}Z_i\right)}{(1-T)(\delta-log\,\theta)},\delta\neq\theta \tag{22}$$

where $\delta=E[log(S)]$ denotes the known expected value of the logarithm of the scrambling variable *S*.

4. Efficiency comparison

From (2) and (10) we have

$$V\left(\hat{\mu}_{x(EH)}\right)-V\left(\hat{\mu}_{x(HG)}\right)=(1/n)\frac{\sigma_s^2}{\theta^2}\left(\sigma_x^2+\mu_x^2\right)\left\{\frac{1}{\theta^2}-(1-T)w\right\} \tag{23}$$

>0 if

$$\frac{1}{\theta^2}>(1-T)w$$

i.e. if $1>\theta^2(1-T)w$

i.e. if $\theta^2<\frac{1}{(1-T)w}$

i.e. if $-\frac{1}{(1-T)w}>\theta<\frac{1}{(1-T)w}$

i.e. if $|\theta|<\frac{1}{(1-T)w}$

Thus we have the inequality:

$$V(\hat{\mu}_{x(HG)}) < V(\hat{\mu}_{x(EH)}) \quad (24)$$

which follows that the proposed estimator $\hat{\mu}_{x(HG)}$ is more efficient than the Eichhorn and Hayre's (1983) estimator $\hat{\mu}_{x(EH)}$.
Further, from (6) and (10) we have

$$V(\hat{\mu}_{x(G)}) - V(\hat{\mu}_{x(HG)}) = \frac{1}{n}\left[\sigma_s^2(\sigma_x^2 + \mu_x^2)(1-T)w\left(1 - \frac{1}{\theta^2}\right)\right] \quad (25)$$

which is positive if

$$1 - \frac{1}{\theta^2} > 0$$

i.e. if $\theta^2 > 1$

i.e. if $\theta > 1$ (26)

Thus we have the inequality:

$$V(\hat{\mu}_{x(HG)}) < V(\hat{\mu}_{x(G)}) \quad , \theta > 1 \quad (27)$$

It follows that the suggested randomization response procedure is always superior to Gupta and Shabbir (2007) randomized response procedure as long as the condition θ >1(i.e. the mean θ of the scrambling variable *S* is larger than the 'unity').
It follows that when the mean θ of the scrambling variable *S* is greater than the 'unity' (i.e. θ >1) the proposed estimator $\hat{\mu}_{x(HG)}$ would be always better than the Eichhorn and Hayre (1983) estimator $\hat{\mu}_{x(EH)}$ and Gupta and Shabbir (2007) estimator $\hat{\mu}_{x(G)}$. However, we note that the proposed randomized response model can be used in practice without imposing condition over the mean θ of the scrambling variables *S*.

5. Empirical Study

To judge the merits of the suggested estimator $\hat{\mu}_{x(HG)}$ over Eichhorn and Hayre's estimator $\hat{\mu}_{x(EH)}$ and Gupta and Shabbir (2007) estimator $\hat{\mu}_{x(G)}$ we have computed the percent relative efficiency (PRE) of $\hat{\mu}_{x(HG)}$ with respect to $\hat{\mu}_{x(EH)}$ and $\hat{\mu}_{x(G)}$ respectively by using following formulae:

$$PRE(\hat{\mu}_{x(HG)}, \hat{\mu}_{x(EH)}) = \frac{[C_x^2 + C_\gamma^2(1+C_x^2)]}{[C_x^2 + (1-T)wC_\gamma^2(1+C_x^2)]} \times 100 \quad (30)$$

$$PRE(\hat{\mu}_{x(SG)}, \hat{\mu}_{x(G)}) = \frac{[C_x^2 + (1-T)w\sigma_s^2(1+C_x^2)]}{[C_x^2 + (1-T)wC_\gamma^2(1+C_x^2)]} \times 100 \quad (31)$$

for T= 0.1, 0.3, 0.5 $w = 0.2(0.2)0.8,\ C_x = 0.1(0.5)1.5,\ \sigma_s^2 = 5, 10, 15, 20$ and $\theta = 0.5(0.5)3.$

where $C_x^2 = \frac{\sigma_x^2}{\mu_x^2}$ and $C_\gamma^2 = \frac{\sigma_s^2}{\theta^2}$

Findings are shown in Table 5.1, Table 5.2, and Table 5.3.

Table 5.1. Value of $PRE(\hat{\mu}_x, \hat{\mu}_{x(EH)})$ and $PRE(\hat{\mu}_x, \hat{\mu}_{x(G)})$ for T=0.1

$C_x = 0.1$				
σ_s^2	θ	w	$PRE(\hat{\mu}_{x(HG)}, \hat{\mu}_{x(EH)})$	$PRE(\hat{\mu}_{x(HG)}, \hat{\mu}_{x(G)})$
5	0.5	0.2	185.11	25.41
		0.4	138.86	25.10
		0.6	550.60	25.07
		0.8	276.81	25.05
5	1	0.2	184.87	100.00
		0.4	138.78	100.00
		0.6	544.55	100.00
		0.8	275.60	100.00
5	1.5	0.2	184.49	221.98
		0.4	138.65	223.47
		0.6	536.35	223.98
		0.8	273.95	224.23
5	2	0.2	183.95	387.36
		0.4	138.47	393.54
		0.6	526.25	395.66
		0.8	271.87	396.74
5	2.5	0.2	183.28	591.22
		0.4	138.23	607.55
		0.6	514.51	613.24
		0.8	269.39	616.13

Table 5.1 Continued

5	3	0.2	182.46	827.93
		0.4	137.95	862.26
		0.6	185.11	874.44
		0.8	138.86	880.68
10	0.5	0.2	1108.34	25.21
		0.4	277.66	25.05
		0.6	185.15	25.03
		0.8	138.88	25.03
10	1	0.2	553.06	100.00
		0.4	277.29	100.00
		0.6	185.03	100.00
		0.8	138.84	100.00
10	1.5	0.2	549.99	223.47
		0.4	276.68	224.23
		0.6	184.84	224.49
		0.8	138.77	224.61
10	2	0.2	545.75	393.54
		0.4	275.84	396.74
		0.6	184.56	397.82
		0.8	138.68	398.36
10	2.5	0.2	540.41	607.55
		0.4	274.77	616.13
		0.6	184.22	619.05
		0.8	138.56	620.53
10	3	0.2	534.07	862.26
		0.4	273.48	880.68
		0.6	183.80	887.01
		0.8	138.41	890.22
15	0.5	0.2	1109.26	25.14
		0.4	277.70	25.03
		0.6	185.16	25.02
		0.8	138.88	25.02
15	1	0.2	553.89	100.00
		0.4	277.45	100.00
		0.6	185.08	100.00
		0.8	138.85	100.00
15	1.5	0.2	551.83	223.98
		0.4	277.05	224.49
		0.6	184.95	224.66
		0.8	138.81	224.74
15	2	0.2	548.97	395.66
		0.4	276.48	397.82
		0.6	184.77	398.54
		0.8	138.75	398.90
15	2.5	0.2	545.35	613.24
		0.4	275.76	619.05
		0.6	184.54	621.02
		0.8	138.67	622.01
15	3	0.2	541.00	874.44
		0.4	274.89	887.01
		0.6	184.26	891.29
		0.8	138.57	893.45
20	0.5	0.2	1109.72	25.10
		0.4	277.72	25.03
		0.6	185.17	25.02
		0.8	138.88	25.01
20	1	0.2	554.31	100.00
		0.4	277.53	100.00
		0.6	185.11	100.00
		0.8	138.86	100.00

Table 5.1 Continued

20	1.5	0.2	552.75	224.23
		0.4	277.23	224.61
		0.6	185.01	224.74
		0.8	138.83	224.81
20	2	0.2	550.60	396.74
		0.4	276.81	398.36
		0.6	184.87	398.90
		0.8	138.78	399.18
20	2.5	0.2	547.86	616.13
		0.4	276.26	620.53
		0.6	184.70	622.01
		0.8	138.72	622.75
20	3	0.2	544.55	880.68
		0.4	275.60	890.22
		0.6	184.49	893.45
		0.8	138.65	895.08
$C_x = 0.5$				
5	0.5	0.2	531.58	28.95
		0.4	272.97	27.03
		0.6	183.64	26.36
		0.8	138.36	26.03
5	1	0.2	472.73	100.00
		0.4	260.00	100.00
		0.6	179.31	100.00
		0.8	136.84	100.00
5	1.5	0.2	403.70	183.33
		0.4	242.22	200.00
		0.6	173.02	207.14
		0.8	134.57	211.11
5	2	0.2	341.18	258.82
		0.4	223.08	307.69
		0.6	165.71	331.43
		0.8	131.82	345.45
5	2.5	0.2	290.70	319.77
		0.4	204.92	409.84
		0.6	158.23	458.86
		0.8	128.87	489.69
5	3	0.2	251.85	366.67
		0.4	188.89	500.00
		0.6	151.11	580.00
		0.8	125.93	633.33
10	0.5	0.2	543.24	27.03
		0.4	275.34	26.03
		0.6	184.40	25.69
		0.8	138.62	25.52
10	1	0.2	510.00	100.00
		0.4	268.42	100.00
		0.6	182.14	100.00
		0.8	137.84	100.00
10	1.5	0.2	464.44	200.00
		0.4	258.02	211.11
		0.6	178.63	215.38
		0.8	136.60	217.65
10	2	0.2	415.38	307.69
		0.4	245.45	345.45
		0.6	174.19	361.29
		0.8	135.00	370.00
10	2.5	0.2	368.85	409.84
		0.4	231.96	489.69
		0.6	169.17	526.32
		0.8	133.14	547.34

Table 5.1 Continued

10	3	0.2	327.78	500.00
		0.4	218.52	633.33
		0.6	163.89	700.00
		0.8	131.11	740.00
15	0.5	0.2	547.27	26.36
		0.4	276.15	25.69
		0.6	184.66	25.46
		0.8	138.71	25.35
15	1	0.2	524.14	100.00
		0.4	271.43	100.00
		0.6	183.13	100.00
		0.8	138.18	100.00
15	1.5	0.2	490.48	207.14
		0.4	264.10	215.38
		0.6	180.70	218.42
		0.8	137.33	220.00
15	2	0.2	451.43	331.43
		0.4	254.84	361.29
		0.6	177.53	373.03
		0.8	136.21	379.31
15	2.5	0.2	411.39	458.86
		0.4	244.36	526.32
		0.6	173.80	554.81
		0.8	134.85	570.54
15	3	0.2	373.33	580.00
		0.4	233.33	700.00
		0.6	169.70	754.55
		0.8	133.33	785.71
20	0.5	0.2	549.32	26.03
		0.4	276.55	25.52
		0.6	184.79	25.35
		0.8	138.75	25.26
20	1	0.2	531.58	100.00
		0.4	272.97	100.00
		0.6	183.64	100.00
		0.8	138.36	100.00
20	1.5	0.2	504.94	211.11
		0.4	267.32	217.65
		0.6	181.78	220.00
		0.8	137.71	221.21
20	2	0.2	472.73	345.45
		0.4	260.00	370.00
		0.6	179.31	379.31
		0.8	136.84	384.21
20	2.5	0.2	438.14	489.69
		0.4	251.48	547.34
		0.6	176.35	570.54
		0.8	135.78	583.07
20	3	0.2	403.70	633.33
		0.4	242.22	740.00
		0.6	173.02	785.71
		0.8	134.57	811.11
$C_x = 1.0$				
5	0.5	0.2	500.00	34.15
		0.4	266.23	29.87
		0.6	181.42	28.32
		0.8	137.58	27.52
5	1	0.2	392.86	100.00
		0.4	239.13	100.00
		0.6	171.88	100.00
		0.8	134.15	100.00

Table 5.1 Continued

5	1.5	0.2	302.47	155.56
		0.4	209.40	176.92
		0.6	160.13	188.24
		0.8	129.63	195.24
5	2	0.2	241.38	193.10
		0.4	184.21	242.11
		0.6	148.94	272.34
		0.8	125.00	292.86
5	2.5	0.2	201.86	217.39
		0.4	164.97	291.88
		0.6	139.48	343.35
		0.8	120.82	381.04
5	3	0.2	175.93	233.33
		0.4	150.79	328.57
		0.6	131.94	400.00
		0.8	117.28	455.56
10	0.5	0.2	525.97	29.87
		0.4	271.81	27.52
		0.6	183.26	26.70
		0.8	138.23	26.28
10	1	0.2	456.52	100.00
		0.4	256.10	100.00
		0.6	177.97	100.00
		0.8	136.36	100.00
10	1.5	0.2	380.34	176.92
		0.4	235.45	195.24
		0.6	170.50	203.45
		0.8	133.63	208.11
10	2	0.2	315.79	242.11
		0.4	214.29	292.86
		0.6	162.16	318.92
		0.8	130.43	334.78
10	2.5	0.2	266.50	291.88
		0.4	195.17	381.04
		0.6	153.96	432.55
		0.8	127.12	466.10
10	3	0.2	230.16	328.57
		0.4	179.01	455.56
		0.6	146.46	536.36
		0.8	123.93	592.31
15	0.5	0.2	535.40	28.32
		0.4	273.76	26.70
		0.6	183.89	26.14
		0.8	138.44	25.86
15	1	0.2	484.38	100.00
		0.4	262.71	100.00
		0.6	180.23	100.00
		0.8	137.17	100.00
15	1.5	0.2	421.57	188.24
		0.4	247.13	203.45
		0.6	174.80	209.76
		0.8	135.22	213.21
15	2	0.2	361.70	272.34
		0.4	229.73	318.92
		0.6	168.32	340.59
		0.8	132.81	353.13
15	2.5	0.2	311.16	343.35
		0.4	212.61	432.55
		0.6	161.47	478.84
		0.8	130.16	507.18

Table 5.1 Continued

15	3	0.2	270.83	400.00
		0.4	196.97	536.36
		0.6	154.76	614.29
		0.8	127.45	664.71
20	0.5	0.2	540.27	27.52
		0.4	274.74	26.28
		0.6	184.21	25.86
		0.8	138.55	25.65
20	1	0.2	500.00	100.00
		0.4	266.23	100.00
		0.6	181.42	100.00
		0.8	137.58	100.00
20	1.5	0.2	447.09	195.24
		0.4	253.75	208.11
		0.6	177.15	213.21
		0.8	136.07	215.94
20	2	0.2	392.86	292.86
		0.4	239.13	334.78
		0.6	171.88	353.13
		0.8	134.15	363.41
20	2.5	0.2	343.87	381.04
		0.4	223.97	466.10
		0.6	166.07	507.18
		0.8	131.95	531.38
20	3	0.2	302.47	455.56
		0.4	209.40	592.31
		0.6	160.13	664.71
		0.8	129.63	709.52
$C_x = 1.5$				
5	0.5	0.2	482.08	37.10
		0.4	262.18	31.58
		0.6	180.05	29.52
		0.8	137.10	28.44
5	1	0.2	357.49	100.00
		0.4	228.40	100.00
		0.6	167.80	100.00
		0.8	132.62	100.00
5	1.5	0.2	266.82	145.77
		0.4	195.30	167.01
		0.6	154.02	179.27
		0.8	127.14	187.25
5	2	0.2	211.74	173.58
		0.4	170.03	218.18
		0.6	142.05	248.10
		0.8	121.98	269.57
5	2.5	0.2	178.44	190.40
		0.4	152.23	254.24
		0.6	132.73	301.72
		0.8	117.66	338.43
5	3	0.2	157.50	200.97
		0.4	139.85	279.31
		0.6	125.75	341.86
		0.8	114.24	392.96
10	0.5	0.2	515.59	31.58
		0.4	269.62	28.44
		0.6	182.54	27.33
		0.8	137.98	26.76
10	1	0.2	429.01	100.00
		0.4	249.10	100.00
		0.6	175.51	100.00
		0.8	135.48	100.00

Table 5.1 Continued

10	1.5	0.2	344.22	167.01
		0.4	224.09	187.25
		0.6	166.11	197.01
		0.8	131.97	202.77
10	2	0.2	279.46	218.18
		0.4	200.48	269.57
		0.6	156.31	298.31
		0.8	128.09	316.67
10	2.5	0.2	233.84	254.24
		0.4	180.74	338.43
		0.6	147.29	391.46
		0.8	124.29	427.93
10	3	0.2	202.11	279.31
		0.4	165.10	392.96
		0.6	139.55	471.43
		0.8	120.85	528.87
15	0.5	0.2	528.11	29.52
		0.4	272.26	27.33
		0.6	183.40	26.57
		0.8	138.28	26.18
15	1	0.2	462.59	100.00
		0.4	257.58	100.00
		0.6	178.48	100.00
		0.8	136.55	100.00
15	1.5	0.2	388.89	179.27
		0.4	237.98	197.01
		0.6	171.45	204.84
		0.8	133.99	209.24
15	2	0.2	324.89	248.10
		0.4	217.51	298.31
		0.6	163.48	323.57
		0.8	130.95	338.78
15	2.5	0.2	275.04	301.72
		0.4	198.70	391.46
		0.6	155.52	442.20
		0.8	127.77	474.83
15	3	0.2	237.73	341.86
		0.4	182.54	471.43
		0.6	148.15	552.17
		0.8	124.66	607.32
20	0.5	0.2	534.66	28.44
		0.4	273.60	26.76
		0.6	183.84	26.18
		0.8	138.43	25.89
20	1	0.2	482.08	100.00
		0.4	262.18	100.00
		0.6	180.05	100.00
		0.8	137.10	100.00
20	1.5	0.2	417.97	187.25
		0.4	246.16	202.77
		0.6	174.45	209.24
		0.8	135.09	212.80
20	2	0.2	357.49	269.57
		0.4	228.40	316.67
		0.6	167.80	338.78
		0.8	132.62	351.61
20	2.5	0.2	306.89	338.43
		0.4	211.04	427.93
		0.6	160.82	474.83
		0.8	129.90	503.70
20	3	0.2	266.82	392.96
		0.4	195.30	528.87
		0.6	154.02	607.32
		0.8	127.14	658.39

Table 5.2- Value of $PRE(\hat{\mu}_x, \hat{\mu}_{x(EH)})$ and $PRE(\hat{\mu}_x, \hat{\mu}_{x(G)})$ for T=0.3

$C_x = 0.1$				
σ_s^2	θ	w	$PRE(\hat{\mu}_{x(HG)}, \hat{\mu}_{x(EH)})$	$PRE(\hat{\mu}_{x(HG)}, \hat{\mu}_{x(G)})$
5	0.5	0.2	1419.24	25.53
		0.4	356.69	25.13
		0.6	237.93	25.09
		0.8	178.50	25.07
5	1	0.2	705.72	100.00
		0.4	355.34	100.00
		0.6	237.45	100.00
		0.8	178.29	100.00
5	1.5	0.2	695.34	221.14
		0.4	353.12	223.04
		0.6	236.65	223.69
		0.8	177.95	224.01
5	2	0.2	681.39	383.94
		0.4	350.07	391.75
		0.6	235.54	394.45
		0.8	177.48	395.82
5	2.5	0.2	664.39	582.36
		0.4	346.26	602.78
		0.6	234.14	609.97
		0.8	176.87	613.65
5	3	0.2	644.92	809.66
		0.4	341.76	852.13
		0.6	232.47	867.44
		0.8	176.15	875.33
10	0.5	0.2	1423.89	25.26
		0.4	356.92	25.07
		0.6	238.01	25.04
		0.8	178.54	25.03
10	1	0.2	709.97	100.00
		0.4	356.24	100.00
		0.6	237.77	100.00
		0.8	178.43	100.00
10	1.5	0.2	704.66	223.04
		0.4	355.11	224.01
		0.6	237.37	224.34
		0.8	178.26	224.50
10	2	0.2	697.39	391.75
		0.4	353.56	395.82
		0.6	236.81	397.20
		0.8	178.02	397.89
10	2.5	0.2	688.28	602.78
		0.4	351.58	613.65
		0.6	236.09	617.38
		0.8	177.71	619.26
10	3	0.2	677.53	852.13
		0.4	349.21	875.33
		0.6	235.23	883.38
		0.8	177.34	887.47
15	0.5	0.2	1425.45	25.18
		0.4	356.99	25.04
		0.6	238.04	25.03
		0.8	178.55	25.02
15	1	0.2	711.40	100.00
		0.4	356.54	100.00
		0.6	237.88	100.00
		0.8	178.48	100.00

Table 5.2 Continued

15	1.5	0.2	707.84	223.69
		0.4	355.79	224.34
		0.6	237.61	224.56
		0.8	178.36	224.67
15	2	0.2	702.92	394.45
		0.4	354.74	397.20
		0.6	237.23	398.13
		0.8	178.20	398.59
15	2.5	0.2	696.70	609.97
		0.4	353.41	617.38
		0.6	236.75	619.89
		0.8	178.00	621.16
15	3	0.2	689.28	867.44
		0.4	351.80	883.38
		0.6	236.17	888.84
		0.8	177.75	891.60
20	0.5	0.2	1426.23	25.13
		0.4	357.03	25.03
		0.6	238.05	25.02
		0.8	178.55	25.02
20	1	0.2	712.12	100.00
		0.4	356.69	100.00
		0.6	237.93	100.00
		0.8	178.50	100.00
20	1.5	0.2	709.44	224.01
		0.4	356.12	224.50
		0.6	237.73	224.67
		0.8	178.42	224.75
20	2	0.2	705.72	395.82
		0.4	355.34	397.89
		0.6	237.45	398.59
		0.8	178.29	398.94
20	2.5	0.2	701.00	613.65
		0.4	354.33	619.26
		0.6	237.09	621.16
		0.8	178.14	622.12
20	3	0.2	695.34	875.33
		0.4	353.12	887.47
		0.6	236.65	891.60
		0.8	177.95	893.69
$C_x = 0.5$				
5	0.5	0.2	673.33	30.00
		0.4	348.28	27.59
		0.6	234.88	26.74
		0.8	177.19	26.32
5	1	0.2	577.78	100.00
		0.4	325.00	100.00
		0.6	226.09	100.00
		0.8	173.33	100.00
5	1.5	0.2	473.91	176.09
		0.4	294.59	194.59
		0.6	213.73	202.94
		0.8	167.69	207.69
5	2	0.2	386.67	240.00
		0.4	263.64	290.91
		0.6	200.00	317.24
		0.8	161.11	333.33
5	2.5	0.2	320.51	288.46
		0.4	235.85	377.36
		0.6	186.57	429.10
		0.8	154.32	462.96

Table 5.2 Continued

5	3	0.2	272.00	324.00
		0.4	212.50	450.00
		0.6	174.36	530.77
		0.8	147.83	586.96
10	0.5	0.2	693.10	27.59
		0.4	352.63	26.32
		0.6	236.47	25.88
		0.8	177.88	25.66
10	1	0.2	637.50	100.00
		0.4	340.00	100.00
		0.6	231.82	100.00
		0.8	175.86	100.00
10	1.5	0.2	564.86	194.59
		0.4	321.54	207.69
		0.6	224.73	212.90
		0.8	172.73	215.70
10	2	0.2	490.91	290.91
		0.4	300.00	333.33
		0.6	216.00	352.00
		0.8	168.75	362.50
10	2.5	0.2	424.53	377.36
		0.4	277.78	462.96
		0.6	206.42	504.59
		0.8	164.23	529.20
10	3	0.2	368.75	450.00
		0.4	256.52	586.96
		0.6	196.67	660.00
		0.8	159.46	705.41
15	0.5	0.2	700.00	26.74
		0.4	354.12	25.88
		0.6	237.01	25.59
		0.8	178.11	25.44
15	1	0.2	660.87	100.00
		0.4	345.45	100.00
		0.6	233.85	100.00
		0.8	176.74	100.00
15	1.5	0.2	605.88	202.94
		0.4	332.26	212.90
		0.6	228.89	216.67
		0.8	174.58	218.64
15	2	0.2	544.83	317.24
		0.4	316.00	352.00
		0.6	222.54	366.20
		0.8	171.74	373.91
15	2.5	0.2	485.07	429.10
		0.4	298.17	504.59
		0.6	215.23	538.08
		0.8	168.39	556.99
15	3	0.2	430.77	530.77
		0.4	280.00	660.00
		0.6	207.41	722.22
		0.8	164.71	758.82
20	0.5	0.2	703.51	26.32
		0.4	354.87	25.66
		0.6	237.28	25.44
		0.8	178.22	25.33
20	1	0.2	673.33	100.00
		0.4	348.28	100.00
		0.6	234.88	100.00
		0.8	177.19	100.00

Table 5.2 Continued

20	1.5	0.2	629.23	207.69
		0.4	338.02	215.70
		0.6	231.07	218.64
		0.8	175.54	220.17
20	2	0.2	577.78	333.33
		0.4	325.00	362.50
		0.6	226.09	373.91
		0.8	173.33	380.00
20	2.5	0.2	524.69	462.96
		0.4	310.22	529.20
		0.6	220.21	556.99
		0.8	170.68	572.29
20	3	0.2	473.91	586.96
		0.4	294.59	705.41
		0.6	213.73	758.82
		0.8	167.69	789.23
$C_x = 1.0$				
5	0.5	0.2	621.21	36.36
		0.4	336.07	31.15
		0.6	230.34	29.21
		0.8	175.21	28.21
5	1	0.2	458.33	100.00
		0.4	289.47	100.00
		0.6	211.54	100.00
		0.8	166.67	100.00
5	1.5	0.2	335.62	147.95
		0.4	242.57	169.31
		0.6	189.92	181.40
		0.8	156.05	189.17
5	2	0.2	259.26	177.78
		0.4	205.88	223.53
		0.6	170.73	253.66
		0.8	145.83	275.00
5	2.5	0.2	212.42	196.08
		0.4	179.56	262.43
		0.6	155.50	311.00
		0.8	137.13	348.10
5	3	0.2	182.69	207.69
		0.4	161.02	289.83
		0.6	143.94	354.55
		0.8	130.14	406.85
10	0.5	0.2	663.93	31.15
		0.4	346.15	28.21
		0.6	234.10	27.17
		0.8	176.86	26.64
10	1	0.2	552.63	100.00
		0.4	318.18	100.00
		0.6	223.40	100.00
		0.8	172.13	100.00
10	1.5	0.2	440.59	169.31
		0.4	283.44	189.17
		0.6	208.92	198.59
		0.8	165.43	204.09
10	2	0.2	352.94	223.53
		0.4	250.00	275.00
		0.6	193.55	303.23
		0.8	157.89	321.05
10	2.5	0.2	290.06	262.43
		0.4	221.52	348.10
		0.6	179.18	401.02
		0.8	150.43	436.96

Table 5.2 Continued

10	3	0.2	245.76	289.83
		0.4	198.63	406.85
		0.6	166.67	486.21
		0.8	143.56	543.56
15	0.5	0.2	679.78	29.21
		0.4	349.71	27.17
		0.6	235.41	26.46
		0.8	177.42	26.10
15	1	0.2	596.15	100.00
		0.4	329.79	100.00
		0.6	227.94	100.00
		0.8	174.16	100.00
15	1.5	0.2	500.00	181.40
		0.4	302.82	198.59
		0.6	217.17	206.06
		0.8	169.29	210.24
15	2	0.2	414.63	253.66
		0.4	274.19	303.23
		0.6	204.82	327.71
		0.8	163.46	342.31
15	2.5	0.2	346.89	311.00
		0.4	247.44	401.02
		0.6	192.31	450.93
		0.8	157.27	482.65
15	3	0.2	295.45	354.55
		0.4	224.14	486.21
		0.6	180.56	566.67
		0.8	151.16	620.93
20	0.5	0.2	688.03	28.21
		0.4	351.53	26.64
		0.6	236.07	26.10
		0.8	177.70	25.83
20	1	0.2	621.21	100.00
		0.4	336.07	100.00
		0.6	230.34	100.00
		0.8	175.21	100.00
20	1.5	0.2	538.22	189.17
		0.4	314.13	204.09
		0.6	221.78	210.24
		0.8	171.40	213.59
20	2	0.2	458.33	275.00
		0.4	289.47	321.05
		0.6	211.54	342.31
		0.8	166.67	354.55
20	2.5	0.2	390.30	348.10
		0.4	265.04	436.96
		0.6	200.65	482.65
		0.8	161.43	510.47
20	3	0.2	335.62	406.85
		0.4	242.57	543.56
		0.6	189.92	620.93
		0.8	156.05	670.70
$C_x = 1.5$				
5	0.5	0.2	592.51	39.87
		0.4	328.85	33.25
		0.6	227.58	30.71
		0.8	174.00	29.37
5	1	0.2	408.84	100.00
		0.4	272.06	100.00
		0.6	203.86	100.00
		0.8	163.00	100.00

Table 5.2 Continued

5	1.5	0.2	290.46	138.76
		0.4	221.72	159.17
		0.6	179.28	171.77
		0.8	150.49	180.32
5	2	0.2	223.95	160.53
		0.4	186.35	200.74
		0.6	159.56	229.38
		0.8	139.50	250.83
5	2.5	0.2	185.54	173.11
		0.4	162.86	228.34
		0.6	145.12	271.54
		0.8	130.87	306.26
5	3	0.2	162.04	180.80
		0.4	147.18	246.77
		0.6	134.81	301.66
		0.8	124.36	348.04
10	0.5	0.2	646.70	33.25
		0.4	342.17	29.37
		0.6	232.63	27.97
		0.8	176.22	27.25
10	1	0.2	511.03	100.00
		0.4	306.17	100.00
		0.6	218.55	100.00
		0.8	169.93	100.00
10	1.5	0.2	390.77	159.17
		0.4	265.23	180.32
		0.6	200.73	191.18
		0.8	161.47	197.80
10	2	0.2	306.27	200.74
		0.4	229.28	250.83
		0.6	183.22	280.79
		0.8	152.57	300.74
10	2.5	0.2	250.17	228.34
		0.4	201.03	306.26
		0.6	168.02	358.59
		0.8	144.32	396.16
10	3	0.2	212.70	246.77
		0.4	179.73	348.04
		0.6	155.60	422.12
		0.8	137.19	478.67
15	0.5	0.2	667.51	30.71
		0.4	346.97	27.97
		0.6	234.40	27.01
		0.8	176.99	26.51
15	1	0.2	561.98	100.00
		0.4	320.75	100.00
		0.6	224.42	100.00
		0.8	172.59	100.00
15	1.5	0.2	452.68	171.77
		0.4	287.58	191.18
		0.6	210.72	200.22
		0.8	166.28	205.45
15	2	0.2	364.93	229.38
		0.4	254.97	280.79
		0.6	195.93	308.40
		0.8	159.09	325.62
15	2.5	0.2	300.72	271.54
		0.4	226.66	358.59
		0.6	181.87	411.24
		0.8	151.86	446.51

Table 5.2 Continued

15	3	0.2	254.85	301.66
		0.4	203.54	422.12
		0.6	169.43	502.21
		0.8	145.11	559.31
20	0.5	0.2	678.53	29.37
		0.4	349.43	27.25
		0.6	235.31	26.51
		0.8	177.38	26.14
20	1	0.2	592.51	100.00
		0.4	328.85	100.00
		0.6	227.58	100.00
		0.8	174.00	100.00
20	1.5	0.2	494.70	180.32
		0.4	301.18	197.80
		0.6	216.49	205.45
		0.8	168.98	209.74
20	2	0.2	408.84	250.83
		0.4	272.06	300.74
		0.6	203.86	325.62
		0.8	163.00	340.53
20	2.5	0.2	341.34	306.26
		0.4	245.06	396.16
		0.6	191.15	446.51
		0.8	156.68	478.70
20	3	0.2	290.46	348.04
		0.4	221.72	478.67
		0.6	179.28	559.31
		0.8	150.49	614.03

Table 5.3- Value of $PRE(\hat{\mu}_x, \hat{\mu}_{x(EH)})$ and $PRE(\hat{\mu}_x, \hat{\mu}_{x(G)})$ for T=0.5

$C_x = 0.1$				
σ_s^2	θ	w	$PRE(\hat{\mu}_{x(HG)}, \hat{\mu}_{x(EH)})$	$PRE(\hat{\mu}_{x(HG)}, \hat{\mu}_{x(G)})$
5	0.5	0.2	1981.37	25.74
		0.4	499.01	25.19
		0.6	332.95	25.12
		0.8	249.81	25.09
5	1	0.2	982.52	100.00
		0.4	496.08	100.00
		0.6	331.80	100.00
		0.8	249.26	100.00
5	1.5	0.2	961.61	219.67
		0.4	491.28	222.28
		0.6	329.92	223.17
		0.8	248.35	223.62
5	2	0.2	933.94	377.98
		0.4	484.76	388.57
		0.6	327.33	392.28
		0.8	247.09	394.17
5	2.5	0.2	900.88	567.18
		0.4	476.69	594.41
		0.6	324.09	604.20
		0.8	245.50	609.24
5	3	0.2	863.87	778.99
		0.4	467.27	834.55
		0.6	320.25	855.14
		0.8	243.60	865.88
10	0.5	0.2	1990.64	25.37
		0.4	499.51	25.09
		0.6	333.14	25.06
		0.8	249.91	25.05

Table 5.3 Continued

10	1	0.2	991.18	100.00
		0.4	498.03	100.00
		0.6	332.57	100.00
		0.8	249.63	100.00
10	1.5	0.2	980.39	222.28
		0.4	495.59	223.62
		0.6	331.61	224.08
		0.8	249.17	224.31
10	2	0.2	965.71	388.57
		0.4	492.23	394.17
		0.6	330.29	396.09
		0.8	248.53	397.06
10	2.5	0.2	947.55	594.41
		0.4	488.00	609.24
		0.6	328.62	614.39
		0.8	247.71	617.00
10	3	0.2	926.36	834.55
		0.4	482.94	865.88
		0.6	326.60	876.92
		0.8	246.73	882.57
15	0.5	0.2	1993.75	25.25
		0.4	499.67	25.06
		0.6	333.21	25.04
		0.8	249.94	25.03
15	1	0.2	994.10	100.00
		0.4	498.68	100.00
		0.6	332.82	100.00
		0.8	249.75	100.00
15	1.5	0.2	986.83	223.17
		0.4	497.05	224.08
		0.6	332.18	224.38
		0.8	249.45	224.54
15	2	0.2	976.85	392.28
		0.4	494.79	396.09
		0.6	331.30	397.38
		0.8	249.02	398.03
15	2.5	0.2	964.34	604.20
		0.4	491.92	614.39
		0.6	330.17	617.88
		0.8	248.47	619.64
15	3	0.2	949.53	855.14
		0.4	488.46	876.92
		0.6	328.80	884.47
		0.8	247.80	888.29
20	0.5	0.2	1995.31	25.19
		0.4	499.75	25.05
		0.6	333.24	25.03
		0.8	249.95	25.02
20	1	0.2	995.57	100.00
		0.4	499.01	100.00
		0.6	332.95	100.00
		0.8	249.81	100.00
20	1.5	0.2	990.09	223.62
		0.4	497.78	224.31
		0.6	332.47	224.54
		0.8	249.58	224.65
20	2	0.2	982.52	394.17
		0.4	496.08	397.06
		0.6	331.80	398.03
		0.8	249.26	398.52

Table 5.3 Continued

20	2.5	0.2	972.99	609.24
		0.4	493.91	617.00
		0.6	330.95	619.64
		0.8	248.85	620.97
20	3	0.2	961.61	865.88
		0.4	491.28	882.57
		0.6	329.92	888.29
		0.8	248.35	891.19
$C_x = 0.5$				
5	0.5	0.2	918.18	31.82
		0.4	480.95	28.57
		0.6	325.81	27.42
		0.8	246.34	26.83
5	1	0.2	742.86	100.00
		0.4	433.33	100.00
		0.6	305.88	100.00
		0.8	236.36	100.00
5	1.5	0.2	573.68	165.79
		0.4	375.86	186.21
		0.6	279.49	196.15
		0.8	222.45	202.04
5	2	0.2	446.15	215.38
		0.4	322.22	266.67
		0.6	252.17	295.65
		0.8	207.14	314.29
5	2.5	0.2	357.14	250.00
		0.4	277.78	333.33
		0.6	227.27	386.36
		0.8	192.31	423.08
5	3	0.2	295.65	273.91
		0.4	242.86	385.71
		0.6	206.06	463.64
		0.8	178.95	521.05
10	0.5	0.2	957.14	28.57
		0.4	490.24	26.83
		0.6	329.51	26.23
		0.8	248.15	25.93
10	1	0.2	850.00	100.00
		0.4	463.64	100.00
		0.6	318.75	100.00
		0.8	242.86	100.00
10	1.5	0.2	720.69	186.21
		0.4	426.53	202.04
		0.6	302.90	208.70
		0.8	234.83	212.36
10	2	0.2	600.00	266.67
		0.4	385.71	314.29
		0.6	284.21	336.84
		0.8	225.00	350.00
10	2.5	0.2	500.00	333.33
		0.4	346.15	423.08
		0.6	264.71	470.59
		0.8	214.29	500.00
10	3	0.2	421.43	385.71
		0.4	310.53	521.05
		0.6	245.83	600.00
		0.8	203.45	651.72
15	0.5	0.2	970.97	27.42
		0.4	493.44	26.23
		0.6	330.77	25.82
		0.8	248.76	25.62

Table 5.3 Continued

15	1	0.2	894.12	100.00
		0.4	475.00	100.00
		0.6	323.40	100.00
		0.8	245.16	100.00
15	1.5	0.2	792.31	196.15
		0.4	447.83	208.70
		0.6	312.12	213.64
		0.8	239.53	216.28
15	2	0.2	686.96	295.65
		0.4	415.79	336.84
		0.6	298.11	354.72
		0.8	232.35	364.71
15	2.5	0.2	590.91	386.36
		0.4	382.35	470.59
		0.6	282.61	510.87
		0.8	224.14	534.48
15	3	0.2	509.09	463.64
		0.4	350.00	600.00
		0.6	266.67	671.43
		0.8	215.38	715.38
20	0.5	0.2	978.05	26.83
		0.4	495.06	25.93
		0.6	331.40	25.62
		0.8	249.07	25.47
20	1	0.2	918.18	100.00
		0.4	480.95	100.00
		0.6	325.81	100.00
		0.8	246.34	100.00
20	1.5	0.2	834.69	202.04
		0.4	459.55	212.36
		0.6	317.05	216.28
		0.8	242.01	218.34
20	2	0.2	742.86	314.29
		0.4	433.33	350.00
		0.6	305.88	364.71
		0.8	236.36	372.73
20	2.5	0.2	653.85	423.08
		0.4	404.76	500.00
		0.6	293.10	534.48
		0.8	229.73	554.05
20	3	0.2	573.68	521.05
		0.4	375.86	651.72
		0.6	279.49	715.38
		0.8	222.45	753.06
$C_x = 1.0$				
5	0.5	0.2	820.00	40.00
		0.4	455.56	33.33
		0.6	315.38	30.77
		0.8	241.18	29.41
5	1	0.2	550.00	100.00
		0.4	366.67	100.00
		0.6	275.00	100.00
		0.8	220.00	100.00
5	1.5	0.2	376.92	138.46
		0.4	288.24	158.82
		0.6	233.33	171.43
		0.8	196.00	180.00
5	2	0.2	280.00	160.00
		0.4	233.33	200.00
		0.6	200.00	228.57
		0.8	175.00	250.00

Table 5.3 Continued

5	2.5	0.2	224.14	172.41
		0.4	196.97	227.27
		0.6	175.68	270.27
		0.8	158.54	304.88
5	3	0.2	190.00	180.00
		0.4	172.73	245.45
		0.6	158.33	300.00
		0.8	146.15	346.15
10	0.5	0.2	900.00	33.33
		0.4	476.47	29.41
		0.6	324.00	28.00
		0.8	245.45	27.27
10	1	0.2	700.00	100.00
		0.4	420.00	100.00
		0.6	300.00	100.00
		0.8	233.33	100.00
10	1.5	0.2	523.53	158.82
		0.4	356.00	180.00
		0.6	269.70	190.91
		0.8	217.07	197.56
10	2	0.2	400.00	200.00
		0.4	300.00	250.00
		0.6	240.00	280.00
		0.8	200.00	300.00
10	2.5	0.2	318.18	227.27
		0.4	256.10	304.88
		0.6	214.29	357.14
		0.8	184.21	394.74
10	3	0.2	263.64	245.45
		0.4	223.08	346.15
		0.6	193.33	420.00
		0.8	170.59	476.47
15	0.5	0.2	930.77	30.77
		0.4	484.00	28.00
		0.6	327.03	27.03
		0.8	246.94	26.53
15	1	0.2	775.00	100.00
		0.4	442.86	100.00
		0.6	310.00	100.00
		0.8	238.46	100.00
15	1.5	0.2	614.29	171.43
		0.4	390.91	190.91
		0.6	286.67	200.00
		0.8	226.32	205.26
15	2	0.2	485.71	228.57
		0.4	340.00	280.00
		0.6	261.54	307.69
		0.8	212.50	325.00
15	2.5	0.2	391.89	270.27
		0.4	295.92	357.14
		0.6	237.70	409.84
		0.8	198.63	445.21
15	3	0.2	325.00	300.00
		0.4	260.00	420.00
		0.6	216.67	500.00
		0.8	185.71	557.14
20	0.5	0.2	947.06	29.41
		0.4	487.88	27.27
		0.6	328.57	26.53
		0.8	247.69	26.15

Table 5.3 Continued

20	1	0.2	820.00	100.00
		0.4	455.56	100.00
		0.6	315.38	100.00
		0.8	241.18	100.00
20	1.5	0.2	676.00	180.00
		0.4	412.20	197.56
		0.6	296.49	205.26
		0.8	231.51	209.59
20	2	0.2	550.00	250.00
		0.4	366.67	300.00
		0.6	275.00	325.00
		0.8	220.00	340.00
20	2.5	0.2	451.22	304.88
		0.4	324.56	394.74
		0.6	253.42	445.21
		0.8	207.87	477.53
20	3	0.2	376.92	346.15
		0.4	288.24	476.47
		0.6	233.33	557.14
		0.8	196.00	612.00
$C_x = 1.5$				
5	0.5	0.2	768.57	44.29
		0.4	440.98	36.07
		0.6	309.20	32.76
		0.8	238.05	30.97
5	1	0.2	477.42	100.00
		0.4	336.36	100.00
		0.6	259.65	100.00
		0.8	211.43	100.00
5	1.5	0.2	318.69	130.37
		0.4	256.39	148.87
		0.6	214.47	161.32
		0.8	184.32	170.27
5	2	0.2	237.65	145.88
		0.4	206.12	179.59
		0.6	181.98	205.41
		0.8	162.90	225.81
5	2.5	0.2	193.23	154.38
		0.4	175.09	198.56
		0.6	160.07	235.15
		0.8	147.42	265.96
5	3	0.2	166.86	159.43
		0.4	155.32	210.64
		0.6	145.27	255.22
		0.8	136.45	294.39
10	0.5	0.2	867.21	36.07
		0.4	468.14	30.97
		0.6	320.61	29.09
		0.8	243.78	28.11
10	1	0.2	631.82	100.00
		0.4	397.14	100.00
		0.6	289.58	100.00
		0.8	227.87	100.00
10	1.5	0.2	451.88	148.87
		0.4	324.86	170.27
		0.6	253.59	182.28
		0.8	207.96	189.97
10	2	0.2	338.78	179.59
		0.4	267.74	225.81
		0.6	221.33	256.00
		0.8	188.64	277.27

Table 5.3 Continued

10	2.5	0.2	268.95	198.56
		0.4	226.44	265.96
		0.6	195.54	314.96
		0.8	172.06	352.19
10	3	0.2	224.47	210.64
		0.4	197.20	294.39
		0.6	175.83	360.00
		0.8	158.65	412.78
15	0.5	0.2	906.90	32.76
		0.4	478.18	29.09
		0.6	324.69	27.78
		0.8	245.79	27.10
15	1	0.2	715.79	100.00
		0.4	425.00	100.00
		0.6	302.22	100.00
		0.8	234.48	100.00
15	1.5	0.2	541.51	161.32
		0.4	363.29	182.28
		0.6	273.33	192.86
		0.8	219.08	199.24
15	2	0.2	416.22	205.41
		0.4	308.00	256.00
		0.6	244.44	285.71
		0.8	202.63	305.26
15	2.5	0.2	331.68	235.15
		0.4	263.78	314.96
		0.6	218.95	367.65
		0.8	187.15	405.03
15	3	0.2	274.63	255.22
		0.4	230.00	360.00
		0.6	197.85	435.48
		0.8	173.58	492.45
20	0.5	0.2	928.32	30.97
		0.4	483.41	28.11
		0.6	326.79	27.10
		0.8	246.82	26.59
20	1	0.2	768.57	100.00
		0.4	440.98	100.00
		0.6	309.20	100.00
		0.8	238.05	100.00
20	1.5	0.2	605.95	170.27
		0.4	387.89	189.97
		0.6	285.24	199.24
		0.8	225.55	204.63
20	2	0.2	477.42	225.81
		0.4	336.36	277.27
		0.6	259.65	305.26
		0.8	211.43	322.86
20	2.5	0.2	384.50	265.96
		0.4	292.15	352.19
		0.6	235.57	405.03
		0.8	197.35	440.72
20	3	0.2	318.69	294.39
		0.4	256.39	412.78
		0.6	214.47	492.45
		0.8	184.32	549.73

From Table 5.1, Table 5.2 and Table 5.3, we illustrate that, the percent relative efficiency of the proposed estimator $\hat{\boldsymbol{\mu}}_{x(HG)}$ with respect to Eichhorn and Hayre's (1983) estimator $\hat{\boldsymbol{\mu}}_{x(EH)}$ is larger than 100%. It follows that the proposed estimator $\hat{\boldsymbol{\mu}}_{x(HG)}$ is always better than the Eicchorn and Hayre's (1983) estimator $\hat{\boldsymbol{\mu}}_{x(EH)}$. The proposed estimator $\hat{\boldsymbol{\mu}}_{x(HG)}$ is: (a) inferior (b) equally efficient, and (c) always superior; to the Gupta and Shabbir (2007) estimator $\hat{\boldsymbol{\mu}}_{x(G)}$ when $\theta < 1, \theta = 1$ and $> \mathbf{1}$. For different values of $\boldsymbol{\sigma_s^2}, \boldsymbol{\theta}$, the percent relative efficiency of $\hat{\boldsymbol{\mu}}_{x(HG)}$ with respect to $\hat{\boldsymbol{\mu}}_{x(EH)}$increases as the value of T increases and decreases as the value of *w* increases whereas the percent relative efficiency of $\hat{\boldsymbol{\mu}}_{x(HG)}$ with respect to $\hat{\boldsymbol{\mu}}_{x(G)}$ decreases as the value of T increases and increases with the increase in the value of *w*. It is also observed that the values of *PRE* $(\hat{\boldsymbol{\mu}}_{x(HG)}, \hat{\boldsymbol{\mu}}_{x(EH)})$ always decrease with increasing values of the coefficient of variation C_x and all values of $\boldsymbol{\theta}$ while the value of *PRE* $(\hat{\boldsymbol{\mu}}_{x(HG)}, \hat{\boldsymbol{\mu}}_{x(EH)})$ also decreases for increasing values of $\boldsymbol{C_x}$ and only when $\boldsymbol{\theta}$ is larger than one. The larger gain in efficiency is observed by using the proposed estimator $\hat{\boldsymbol{\mu}}_{x(HG)}$ over Gupta and shabbir (2007) estimator $\hat{\boldsymbol{\mu}}_{x(G)}$ for the values of $\boldsymbol{\theta > 1}$. We can also perceive a loss in efficiency for the values of $\theta < 1$. Thus we conclude that the proposed randomized response model is superior to the Eichhorn and Hayre's (1983) and Gupta and Shabbir (2007) randomized response models for $\theta > 1$ both theoretically and numerically.

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